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Letters to the Editor

Two-point Sparrow of resolution with phase and antiphase partially coherent illumination in apodized system*

ANNA MAGIERA, KAZIMIERZ PIETRASZKIEWICZ

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

When a two-point object is illuminated coherently and there exists a phase difference $\varphi = \pi$ between the two object points, an infinite resolution may be realized [1, 2]. This means that in the anti-phase-coherent case the points are always resolved, since the intensity in the mid-point between the geometrical images is always zero independently of separation. This fact has been exploited in the holographic spectroscopy to split the Hg doublets 577 and 579 nm [3, 4]. The image of the Hg spectrum recorded on the Fourier hologram with the fringe densities of $80 \frac{\text{fringes}}{\text{mm}}$ and $10 \frac{\text{fringes}}{\text{mm}}$, was next reconstructed with the He–Ne laser beam. When the density of fringes diminished, the dispersion diminished too and the doublet observations met some difficulties. The application of two gratings of slightly differing spacings allowed to evoke the phase difference $\varphi = \pi$ and, consequently, enabled to split the doublet of wavelengths 577 mm and 579 nm, respectively.

One of the techniques applied to improve the resolution (especially for the noncoherent light) is the apodization. An amplitude apodizer of given transmittance can be realized experimentally with the help of an aperture diaphragm (of the width proportional to the value of the apodizing function) and a cylindric lens [5].

In the present paper the influence of two types of apodization was examined: $1 - r^2$, and $\frac{1}{2}(1 + r)^2$. These types are used most frequently in the apodization examinations [6] concerning the critical value of the distance between two object points for phase ($\varphi = 0$), and antiphase ($\varphi = \pi$) partial coherence.

The amplitude distribution $A(v)$ in the image of an object-point located on the optical axis of the optical system with a circular aperture is equal to [7]:

$$A(v) = \int_0^1 T(r)\, J_0(vr)r\, dr, \tag{1}$$

where v – transversal shift in the image plane equal to $2\pi(x^2 + y^2)^{1/2}\varrho$,
x,y – dimensionless coordinates in the pupil plane,
ϱ – numerical aperture,
$T(r)$ – pupil function, $T(r) \leqslant 1$,
$r = (x^2 + y^2)^{1/2}$, $0 < r \leqslant 1$.

Then the intensity in the image of two object points of the same luminance shifted in phase by φ, each distant by β from the optical axis and illuminated with a partially coherent source, is equal to

$$I(v, \beta) = |A(v - \beta)|^2 + |A(v + \beta)|^2 \tag{2}$$
$$+ \mathrm{Re}\,\{2\gamma(\beta - \beta)\, A(v - \beta)\, A^*(v + \beta)\}\cos\varphi.$$

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where φ — phase shift between the object points,
γ — complex degree of coherence, $0 < |\gamma| \leqslant 1$.

According to Sparrow criterion two points are resolved when the derivative of the intensity distribution in the two-point image disappears in the midpoint, i.e.,

$$\frac{\partial^2}{\partial v^2}[I(v,\beta)]_{v=0} = 0. \tag{3}$$

By substituting (2) to the formula (3) the equation for the limiting distance value $\delta_{\min} = (2\beta)_{\min}$, for the points to be resolved is obtained in the form

$$\frac{\partial^2 A\left(\frac{\delta_{\min}}{2}\right)}{\partial v^2} A\left(\frac{\delta_{\min}}{2}\right) + \left(\frac{1-\gamma_r\cos\varphi}{1+\gamma_r\cos\varphi}\right) \times \left[\frac{\partial A\left(\frac{\delta_{\min}}{2}\right)}{\partial v}\right]^2 = 0, \tag{4}$$

where γ_r — real part of γ.

An integral form will be obtained after substituting (1) to the formula (4).

$$\int_0^1 T(r) J_0''\left(\frac{\delta_{\min} r}{2}\right) r^3 dr \int_0^1 T(r) J_0\left(\frac{\delta_{\min} r}{2}\right) r dr + \left(\frac{1-\gamma_r\cos\varphi}{1+\gamma_r\cos\varphi}\right) \times \left\{\int_0^1 T(r) J_0'\left(\frac{\delta_{\min} r}{2}\right) r^2 dr\right\}^2 = 0. \tag{5}$$

By introducing an apodizing filter of transmittance $T(r) = 1-r^2$ (curve I, fig. 1) into the exit pupil of the optical system the formula (5) yields

$$\left[\frac{J_1(\delta)}{2\delta} - \frac{4J_2(\delta)}{\delta^2} + \frac{J_3(\delta)}{\delta}\left(\frac{1}{2}+\frac{10}{\delta^2}\right)\right] \times \left[\frac{4J_1(\delta)}{\delta^3} - \frac{4}{\delta^2}J_0(\delta)\right]\left(\frac{1-\gamma_r\cos\varphi}{1+\gamma_r\cos\varphi}\right) \times \left[\frac{J_2(\delta)}{\delta} - \frac{4J_3(\delta)}{\delta^2}\right]^2 = 0, \tag{6}$$

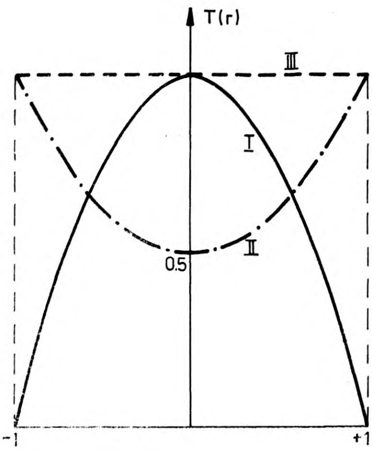

Fig. 1. Transmittance of apodizers analyzed in this paper

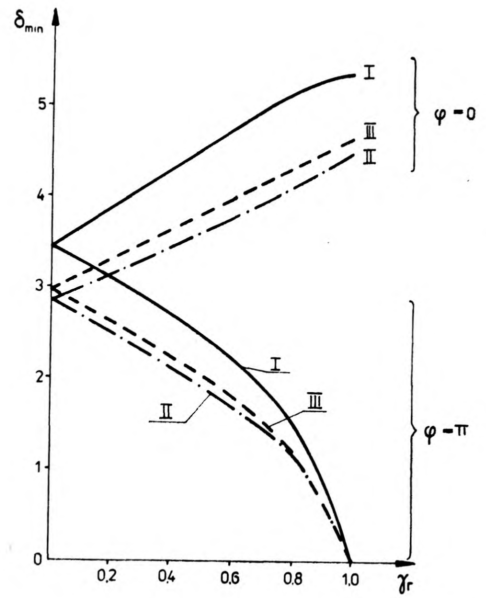

Fig. 2. Dependence between the minimal spacing in the two-point object $\delta_{\min}$, defined by the Sparrow criterion and the coherence degree γ_r.

where $\delta = \delta_{\min}/2$.

For the apodizer of amplitude transmittance $T(r) = \frac{1}{2}(1+r^2)$ (curve II, fig. 1) eq. (5) yields

$$\left[-\frac{3}{4}\frac{J_1(\delta)}{\delta}+\frac{3J_2(\delta)}{\delta^2}+\frac{J_3(\delta)}{\delta}\right.$$

$$\left.\left(\frac{1}{4}-\frac{5}{\delta^2}\right)\right]\left[\frac{J_1(\delta)}{\delta}-\frac{2J_1(\delta)}{\delta^3}+\frac{2J_0(\delta)}{\delta^2}\right)$$

$$+\left(\frac{1-\gamma_r\cos\varphi}{1+\gamma_r\cos\varphi}\right)\left[\frac{2J_3(\delta)}{\delta^2}\right.$$

$$\left.-\frac{3}{2}\frac{J_2(\delta)}{\delta}\right]^2=0. \qquad (7)$$

For a nonapodized system (curve III, fig. 1) the equation for a limiting distance between the two-points takes the form

$$\frac{1}{2}\left[\frac{J_3(\delta)}{\delta}+\frac{2J_2(\delta)}{\delta^2}-\frac{J_1(\delta)}{\delta}\right]\left[\frac{J_1(\delta)}{\delta}\right]$$

$$+\left(\frac{1-\gamma_r\cos\varphi}{1+\gamma_r\cos\varphi}\right)\left[-\frac{J_2(\delta)}{\delta}\right]^2=0. \qquad (8)$$

The results obtained numerically from the eqs. (6)–(8) are show in fig. 2, for the phase $\varphi = 0$, and antiphase ($\varphi = \pi$) partial coherent ($0 \leqslant \gamma_r \leqslant 1$) of the apodized (fig. 2, curves I and II) and nonapodized (fig. 2, curve III) optical systems. For the definite coherence an improvement of the resolution may be obtained by applying an apodizer of transmittance (II) both for $\varphi = 0$ and $\varphi = \pi$. An opposite result is obtained for an apodizer of type I. Though theoretically an infinite resolution is obtained for $\varphi = \pi$ (fig. 2), it cannot be achieved in practice, due to restricted sensitivity of the detector and the noise level. Consequently, the ratio of the measured spacing to the real one should be examined. Also, the influence of the ratio of the measured-to-real separations should be found and the image contrast of a periodic object determined.

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