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Image quality criteria of the apidized optical systems with spherical aberrations for one- and two-point imaging*

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This paper is concerned with the performance of apodized optical systems with spherical aberrations. The assumed apodization was radially symmetrical and the taken into account aberrations of the third, fifth, and seventh orders have Zernike representation. The following performance criteria were considered: the point intensity distribution, second moment of the point intensity distribution, encircled energy distribution, Strehl ratio, two-point intensity distribution in partially coherent illumination, point image contrast, and the resolutions due to both Rayleigh and Sparrow conditions. For all the mentioned criteria analytical formulae were derived.

1. Introduction

The purpose of apodization is to improve performance of the image forming optical system. Many apodizing problems and their solutions have been reviewed by JACQUINOT and ROIZEN-DOSSIER [1]. These problems can be devided into two general groups. First group contains all the problems in which the performance of apodized systems was examined by trial and error method, i.e. was investigated for the selected filters [2, 3]. Second groups [4, 5] includes the problems which allows to calculate the optimal apodizing filters under specified assumptions. Most authors have considered only apodizers in diffraction limited systems.

In the paper [6] we have calculated the optimal filter, for which the second moment of the point spread function in the optical systems with spherical aberrations takes the minimum value. BISWAS and BOIVIN [7] examined the influence of the Straubel and Lansraux–Boivin apodizers in optical system with the first and second orders spherical aberrations. The only performance criteria considered in [7] were the point spread function, Strehl ratio, the relative encircled energy and the resolution due to Huber–Hopkins condition.

In the present paper we have derived the formulae for the second moment of the point spread function, the two-point intensity distribution in partially coherent illumination, the point image contrast, and, finally, the equations which must be satisfied by Sparrow and Rayleigh resolutions.

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As the third and fifth order spherical aberrations are not satisfactory for all optical systems, these criteria were derived not only for primary and secondary but also for tertiary aberrations. The criteria considered in [7] were additionally derived for seventh order spherical aberration. They were also recalculated for secondary aberrations, since the series expansion of the point spread function up to the seventh term contains the linear combinations of the Zernike polynomials up to the $R_{24}^0[\varrho]$ and not up to R_{14}^0 only as was written in [7].

2. General considerations

The far-field effects due to a circular aperture in an optical system can be derived from its amplitude response. The diffracted light amplitude associated with a rotationally symmetric pupil is given in [8]

$$A(u) \propto \int_0^1 f(\varrho) J_0(u\varrho)\varrho d\varrho, \tag{1}$$

where $f(\varrho)$ is the pupil function, J_0 is the zero-order Bessel function of the first kind, ϱ denotes the normalized radial coordinate in the pupil plane, and u describes the normalized distance from the diffraction head expressed in diffraction units.

For systems with spherical aberration the pupil function has the form

$$f(\varrho) = f_0(\varrho)\exp[ikW(\varrho)], \tag{2}$$

where $f_0(\varrho)$ denotes the transmission of the pupil, $W(\varrho)$ describes wave aberration and k is the wave number.

If, for diffraction limited system with uniform transmission of the pupil, the amplitude $A(u)$ at $u = 0$ is normalized to unity, then eqs. (1) and (2) yield

$$A(u) = 2\int_0^1 f_0(\varrho)\exp[ikW(\varrho)]J_0(u\varrho)\varrho d\varrho. \tag{3}$$

Let us consider optical systems with radially symmetrical transparences of the general form

$$f_0(\varrho) = \sum_{p=0}^{\infty} a_p \varrho^{2p}, \tag{4}$$

where a_p — the power expansion coefficient.

The spherical wave aberration, according to the ZERNIKE-NIJBOER [9, 10] theory, can be written as follows:

$$W(\varrho) = \frac{\beta_{ln0}}{k} R_n^0(\varrho), \tag{5}$$

where β_{ln0} is the aberration coefficient, and $R_n^0(\varrho)$ is Zernicke circle polynomial. Indices $n = 4, 6, 8\ldots$ correspond to primary secondary, tertiary, and higher aberrations. Inserting (4) and (5) into (3) we obtain

$$A(u) = 2\sum_{p=0}^{\infty} a_p \int_0^1 \exp[i\beta_{ln0}R_n^0(\varrho)]J_0(u\varrho)\varrho^{2p+1}d\varrho. \tag{6}$$

This formula will be the fundamental one for further calculations.

3. Primary aberration

For primary aberration we must set $n = 4$ in formulae (6). Expanding the exp function up to the fifth term we have

$$A(u) = 2\sum_{p=1}^{\infty} a_p \int_0^1 \Bigg[1+i\beta_{ln0}R_4^0(\varrho)+\frac{(i\beta_{l40})^2}{2!}(R_4^0(\varrho))^2 + \frac{(i\beta_{l40})^3}{3!}(R_4^0(\varrho))^3+\frac{(i\beta_{l14})^4}{4!}(R_4^0(\varrho))^4+\ldots\Bigg]J_0(u\varrho)\varrho^{2p+1}d\varrho. \tag{7}$$

Using the following relations [8] for circle polynomials:

$$R_{2s}^0R_2^0 = \frac{(s+1)}{(2s+1)}R_{2s+2}^0+\frac{s}{(2s+1)}R_{2s-2}^0,$$

$$R_{2s}^0R_4^0 = \frac{s(s+2)(s+1)}{3(2s+3)(2s+1)}R_{2s+4}^0+\frac{s(s+1)}{(2s+3)(2s-1)}R_{2s}^0 + \frac{3}{2}\,\frac{s(s-1)}{(2s+1)(2s-1)}R_{2s-4}, \tag{8}$$

where s — integer number $\geqslant 0$; all the powers of R_4^0 can be rewritten as a linear combination of circle polynomials. Hence, form (7) and (8) we, finally, have

$$A(u) = 2\sum_{p=0}^{\infty} a_p \sum_{\nu=0}^{8(4)} d_\nu \int_0^1 R_\nu^0(\varrho)J_0(u\varrho)\varrho^{2p+1}d\varrho, \tag{9}$$

where 8 (4) over the summation sign denotes that the summation over ν goes with step 4 (i.e. for $\nu = 0, 4, 8$) only, and coefficients d_ν have the form

$$\begin{aligned} d_0 &= 1-\frac{1}{10}\beta_{l40}^2+\frac{1}{280}\beta_{l40}^4-\frac{i}{105}\beta_{l40}^3,\\ d_4 &= -\frac{1}{7}\beta_{l40}^2+\frac{5}{462}\beta_{l40}^4+i\left(\beta_{l40}-\frac{1}{14}\beta_{l40}^3\right),\\ d_8 &= -\frac{9}{35}\beta_{l40}^2+\frac{153}{10010}\beta_{l40}^4-i\frac{18}{385}\beta_{l40}^3. \end{aligned} \tag{9a}$$

To calculate the integral in (9) we use the exact forms of $R_\nu^0(\varrho)$ and $J_0(u\varrho)$ [8, 11]

$$R_\nu^0(\varrho) = \sum_{s=0}^{\frac{1}{2}\nu} (-1^s) \frac{s(\nu-s)!}{s!\left[\left(\frac{\nu}{2}-s\right)!\right]^2} \varrho^{\nu-2s} \tag{10}$$

and

$$J_0(u\varrho) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{u\varrho}{2}\right)^{2k}. \tag{10}$$

Substitution of (10) in (9) and integration yields

$$A(u) = 2 \sum_{\nu=0}^{8(4)} \sum_{s=0}^{\frac{1}{2}\nu} \sum_{p,k=0}^{\infty} a_p d_\nu \varepsilon_{\nu s} a_{pk}\nu s (u/2)^{2k}, \tag{11}$$

where $\varepsilon_{\nu s}$, $a_{pk\nu s}$ are general coefficients of the form

$$\varepsilon_{\nu s} = (-1)^s \frac{(\nu-s)!}{s!\left[\left(\frac{\nu}{2} s\right)!\right]^2},$$

$$a_{pk\nu s} = (-1)^k \frac{1}{(k!)^2[2(p+k-s+1)+\nu]}. \tag{11a}$$

Hence, the point intensity distribution, as a square modulus of $A(u)$, has the form

$$I(u) = 4 \sum_{p,q,k,m=0}^{\infty} \sum_{\mu,\nu=0}^{8(4)} \sum_{r=0}^{\frac{1}{2}\mu} \sum_{s=0}^{\frac{1}{2}\nu} a_p a_q \varepsilon_{\nu s} \varepsilon_{\mu r}$$

$$a_{pk\nu s} a_{qm\mu r} d_\mu d_\nu^* (u/2)^{2(k+m)}, \tag{12}$$

where * — complex conjugation.

In order to asses imaging properties of the apodized optical system some important criteria will be derived from the fundamental formula (12).

3.1. One-point object criteria

These criteria can be obtained directly form (12), Strehl ratio (S.R.) being equal to the intensity at $u = 0$ has the form

$$\text{S.R.} = I(0) = 4 \sum_{p,q=0}^{\infty} \sum_{\mu,\nu=0}^{8(4)} \sum_{s=0}^{\frac{1}{2}\mu} \sum_{r=0}^{\frac{1}{2}\nu} a_p a_q a_{p0\nu s} a_{q0\mu r} d_\mu d_\nu^*. \tag{13}$$

The function describing the energy scattered in the point object image is the encircled energy distribution

$$E(v) = \int_0^v I(u)udu. \tag{14}$$

From (12) and (14) we have

$$E(v) = 8 \sum_{p,q,k,m=0}^{\infty} \sum_{\mu,\nu=0}^{8(4)} \sum_{s=0}^{\frac{1}{2}\mu} \sum_{r=0}^{\frac{1}{2}\nu} a_p a_q \varepsilon_{\mu r} \varepsilon_{\nu s} a_{pk\nu s} a_{qm\mu r} d_\mu d_\nu^* \frac{1}{k+m+1} (v/2)^{2(k+m+1)}. \tag{15}$$

The second moment σ of the point intensity distribution can be alternatively used as a measure of the scattered energy, and is defined as follows:

$$\sigma = \int_0^\infty I(u)u^3 du. \tag{16}$$

As it was shown in [6] it can be presented in form of integral in the pupil plane

$$\sigma = \int_0^1 \left\{\left[\frac{\partial f_0(\varrho)}{\partial \varrho}\right]^2 + k^2 f_0^2(\varrho)\left[\frac{\partial W(\varrho)}{\partial \varrho}\right]^2\right\} \varrho d\varrho. \tag{17}$$

It should be added that σ is convergent only for apodizers vanishing at the edge of the pupil. i.e. for $f_0(1) = 0$. By combining (4), (5), (10), and (17) we get

$$\sigma = 2 \sum_{p,q=1}^{\infty} a_p a_q \frac{pq}{p+q} + a_0 \beta_{l\mu 0}^2 \sum_{s=0}^{\frac{1}{2}\nu-1} \sum_{r=0}^{\frac{1}{2}\mu-1} \varepsilon_{\mu r} \varepsilon_{\nu s} (\nu-2s)(\mu-2r)$$
$$\left\{a_0 + \sum_{p=1}^{\infty} a_p \left[\frac{1}{\nu+\mu-2(s+r-p)} + \sum_{q=1}^{\infty} a_q \frac{1}{\nu+\mu-2(s+r-q-p)}\right]\right\}, \tag{18}$$

with $\nu, \mu = 4$ for primary aberrations.

The summation procedures over s, r were reduced to $\frac{1}{2}\nu-1$ and $\frac{1}{2}\mu-1$, respectively, because the last term in the derivative of $R_\nu^0(\varrho)$

$$\frac{d}{d\varrho}[R^0(\varrho)] = \sum_{s=0}^{\frac{1}{2}\nu} (-1)^s \frac{(\nu-s)!(\nu-2s)}{s!\left[\left(\frac{\nu}{2}-s\right)!\right]^2} \varrho^{\nu-2s-1} \tag{19}$$

equals zero.

3.2. Two-point object criteria

In this part resultant intensity in the images of two-point objects along the line joining geometrical images of the objects will be the fundamental relation. Assuming that the isoplanatism condition is satisfied and that the point objects are identical we have

$$J(u) = \left|A\left(u+\frac{1}{2}u_0\right)\right|^2 + \left|A\left(u-\frac{1}{2}u_0\right)\right|^2 + 2\mu_{12}Re\left\{A\left(u+\frac{1}{2}u_0\right)A^*\left(u-\frac{1}{2}u_0\right)\right\}, \tag{20}$$

where $A\left(u+\frac{1}{2}u_0\right)$, $A\left(u-\frac{1}{2}u_0\right)$ represent the complex amplitudes due to points objects whose geometrical images are at $\pm\frac{1}{2}u_0$, μ_{12} is coherency factor, Re is the real part and asterisk denotes a complex conjugation.

From (11) and (20) we obtain

$$J(u) = 4\left\{\sum_{p,q,k,m=0}^{\infty}\sum_{\mu,\nu=0}^{8(4)}\sum_{s=0}^{\frac{1}{2}\nu}\sum_{r=0}^{\frac{1}{2}\mu} a_p a_q \varepsilon_{\nu s}\varepsilon_{\mu r} a_{pk\nu s} a_{qm\mu r} d_\mu d_\nu^* \times\left[\left(u+\frac{u_0}{2}\right)^{2(k+m)} + \left(u-\frac{u_0}{2}\right)^{2(k+m)} + 2\mu_{12}\left(u+\frac{u_0}{2}\right)^{2k}\left(u-\frac{u_0}{2}\right)^{2m}\right]\frac{1}{2^{2(k+m)}}\right\}. \tag{21}$$

The point image contrast can be immediatelly calculated from (21). It is defined as follows

$$C(u_0) = \frac{J(u_0/2)-J(0)}{J(u_0/2)} = 1-\frac{J(0)}{J(u_0/2)}. \tag{22}$$

We see that in order to determine $C(u_0)$ the resultant intensity should be determined in two points only, i.e. $u = 0$, and $u = u_0/2$. From (21) we have

$$J(0) = 2(1+\mu_{12})J(u_0/2) \tag{23a}$$

and

$$J(u_0/2) = 4\sum_{p,q=0}^{\infty}\sum_{\mu,\nu=0}^{8(4)}\sum_{s=0}^{\frac{1}{2}\nu}\sum_{r=0}^{\frac{1}{2}\mu} a_p a_q \varepsilon_{\nu s}\varepsilon_{\mu r} d_\mu d_\nu^* \times\left\{2(1+\mu_{12})a_{p0\nu s}a_{q0\mu r} + \sum_{k,m=1}^{\infty} a_{pk\nu s}a_{qm\mu r}(u_0/2)^{2(k+m)}\right\}. \tag{23b}$$

The limit of resolution can be calculated from (23). If the Rayleigh criterion is applied then it suffices to solve the following fundamental equation

$$J(0) = 0.736\,J(u_0/2), \tag{24}$$

but the Sparrow resolution u_0 must satisfy the equation

$$\left(\frac{\partial^2 J}{\partial u^2}\right)_{u=0} = 0. \tag{25}$$

Combining (21) and (25) and carrying out simple calculations we get

$$\sum_{p,q=0}^{\infty}\sum_{\nu,\mu=0}^{8(4)}\sum_{s=0}^{\frac{1}{2}\nu}\sum_{r=0}^{\frac{1}{2}\mu}\sum_{k,m=1}^{\infty} a_p a_q \varepsilon_{\nu s}\varepsilon_{\mu r} a_{pk\nu s} a_{qm\mu r} d_\mu d_\nu^* \tag{26}$$
$$\times\{(m+k)[2(m+k)-1]+\mu_{12}[k(2k-m-1)-m(k+m-1)]\}(u_0/4)^{2(k+m-1)} = 0$$

4. Secondary aberration

For secondary spherical aberration the sum (6) takes the form

$$A(u) = 2\sum_{p=0}^{\infty} a_p \int_0^1 \exp[i\beta_{160}R_6^0(\varrho)]J_0(u\varrho)\varrho^{2p+1}d\varrho. \tag{27}$$

If we take the first seven terms in exponent function expansion and replace higher powers of $R_6^0(\varrho)$ in this expansion using eq. (8) and the following known formulae [12]

$$\begin{aligned} R_{2s}^0 R_6^0 = {} & \frac{5}{2}\frac{(s+1)(s+2)(s+3)}{(2s+1)(2s+3)(2s+5)}R_{2s+6}^0 + \frac{5}{2}\left\{\frac{(s+1)(s+2)^2}{(2s+1)(2s+3)(2s+5)}\right. \\ & \left. + \frac{s^2(s+1)}{(2s+1)(2s-1)} + \frac{(s+1)^3}{(2s+1)(2s+3)} - \frac{3}{5}\frac{(s+1)}{(2s+1)}\right\}R_{2s+2}^0 \\ & + \frac{5}{2}\left\{\frac{s^3}{(2s+1)^2(2s-1)} + \frac{s(s+1)^2}{(2s+1)^2(2s+3)} + \frac{s(s-1)^2}{(2s+1)(2s-1)(2s-3)}\right. \\ & \left. + \frac{3}{5}\frac{s}{(2s+1}\right\}R_{2s-2}^0 + \frac{5}{2}\frac{s(s-1)(s-2)}{(2s+1)(2s-1)(2s-3)}R_{2s-6}^0, \end{aligned} \tag{28}$$

then we have

$$A(u) = 2\sum_{p=0}^{\infty} a_p \sum_{\nu=0}^{24(2)} \tilde{d}_\nu \int_0^1 R_\nu^0(\varrho)J_0(u\varrho)\varrho^{2p+1}d\varrho. \tag{29}$$

Notation 24 (2) over summation sign means that the summation goes up to 24 with step 2. Coefficients $\tilde{d}$ depend on β_{160} and are related to those coefficients C_ν evaluated by Som [12]

$$\tilde{d}_\nu = \begin{cases} C_\nu \\ iC_\nu \end{cases} \text{for} \quad \nu = \begin{cases} 0,\ 4,\ 8,\ 12,\ 16,\ 20,\ 24 \\ 2,\ 6,\ 10,\ 14,\ 18,\ 22 \end{cases}. \tag{29a}$$

The integration of (29) gives

$$A(u) = 2\sum_{p,k=0}^{\infty}\sum_{\nu=0}^{24(2)}\sum_{s=1}^{\frac{1}{2}\nu} a_p\tilde{d}_\nu\varepsilon_{\nu s}a_{pk\nu s}(u/2)^{2k}. \tag{30}$$

The last relation is more general than that obtained for primary aberration. If $\tilde{d}_\nu$ is replaced by d_ν and additionally

$$\tilde{d}_\nu = 0 \quad \text{for all} \quad \nu, \quad \text{except} \quad \nu = 0,\ 4,\ 8, \tag{31}$$

then (30) is reduced to (11). This remark allows to rewrite automatically all desired expressions for secondary aberration. In order the relations (12)–(15), (18), (21), (23), and (26) could be applied in systems with secondary spherical aberration the notation 8(4) should be replaced by 24(2) and d_ν by $\tilde{d}_\nu$.

5. Tertiary aberrations

In the case of tertiary aberration we apply the alternative method of image criteria evaluation those considered in the previous part of this paper. To this end higher powers of the exponent function expansion will not be expressed as a linear combination of the Zernike polynomial but simply by a binomial expansion. This expansion has the form [11]

$$(A_1+A_2+\ldots+A_6)^2 = \sum \frac{n!}{k_1!k_2!\ldots k_s!} A_1^{k_1} A_2^{k_2} \ldots A_s^{k_s} \tag{32}$$

under the following condition:

$$k_1+k_2+\ldots+k_s = n.$$

Since the 8-th order Zernike polynomial is equal to

$$R_8^0(\varrho) = 70\varrho^8 - 140\varrho^6 + 90\varrho^4 - 20\varrho^2 + 1, \tag{33}$$

therefore, the ν-th power of (33) gives

$$\begin{aligned}[R_8^0(\varrho)]^\nu = & \sum_{k_1,k_2,\ldots,k_5=0}^{\nu=k_1+k_2+\ldots+k_5} \Omega(A_1, A_2, \ldots, A_5, \nu, k_1, \nu, k_2, \ldots, k_5) \\ & \times \varrho^{2}4_{(k_1+3k_2+2k_3+k_4)}.\end{aligned} \tag{34}$$

where $A_1 = 70$, $A_2 = -140$, $A_3 = 90$, $A_4 = -20$, $A_5 = 1$, and

$$\Omega(A_1, A_2, \ldots, A_5, \nu, k_1, k_2, \ldots, k_5) = A_1^{k_1} A_2^{k_2} A_3^{k_3} A_4^{k_4} \frac{n!}{k_1!k_2!k_3!k_4!k_5!}. \tag{34a}$$

In evaluation of the diffracted light amplitude we apply n-th order expansion of $A(u)$. From (4), (6), (10), and (34) we finally have

$$\begin{aligned}A(u) = 2 \sum_{p,k}^{\infty} \sum_{\nu=0}^{n} \sum_{k_1,k_2,\ldots,k_5=0}^{\nu=k_1+k_2+,\ldots,k_5} & a_p \tilde{\tilde{d}}_\nu \tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_1, k_2, \ldots, \\ & \ldots, k_5, k, p)(u/2)^{2k},\end{aligned} \tag{35}$$

with

$$\tilde{\Omega} = \frac{1}{(k!)^2(4k_1+3k_2+2k_3+k_4+k+p)}\,\Omega,$$

and

$$\tilde{\tilde{d}}_\nu = \frac{(i\beta_{180})^\nu}{\nu!} \quad \text{for } \nu = 0, 1, 2, \ldots, n.$$

The required image criteria can be directly obtained from (35) and have the following forms:

— Intensity point spread function

$$I(u) = \sum_{p,q,k,m=0}^{\infty} \sum_{\mu,\nu=0}^{n} \sum_{\substack{k_1,k_2,\ldots,k_{10}=0}}^{\substack{\nu=k_1+k_2+\ldots k_5\\ \mu=k_6+k_7+\ldots k_{10}}} a_p a_q \tilde{\tilde{d}}_\nu \tilde{\tilde{d}}^*_\mu \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_1, k_2, \ldots, k_5, k, p) \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_6, k_7, \ldots, k_{10}, m, q)(u/2)^{2(k+m)} \tag{36}$$

— Strehl ratio

$$\text{S.R.} = \sum_{p,q=0}^{\infty} \sum_{u,\nu=0}^{n} \sum_{\substack{k_1,k_2,\ldots,k_{10}=0}}^{\substack{\nu=k_1+k_2+\ldots+k_5\\ \nu=k_6+k_7+\ldots+k_{10}}} a_p a_q \tilde{\tilde{d}}_\nu \tilde{\tilde{d}}^*_\mu \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_1, k_2, \ldots, k_5, 0, p) \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \mu, k_6, k_7, \ldots, k_{10}, 0, q). \tag{37}$$

— Encircled energy distribution

$$E(u) = 2\sum_{p,q=0}^{\infty} \sum_{u,\nu=0}^{n} \sum_{\substack{k_1,k_2,\ldots,k_{10}}}^{\substack{\nu=k_1+\ldots+k_5\\ \mu=k_6+\ldots+k_{10}}} a_p a_q \tilde{\tilde{d}}_\mu \tilde{\tilde{d}}^*_\nu \frac{1}{k+m+1} \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_1, k_2, \ldots, k_5, k, p) \times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \mu, k_6, k_7, \ldots, k_{10}, m, q)(u/2)^{2(k+m+1)}. \tag{38}$$

— The second moment

$$\sigma = 2\sum_{p,q=0}^{\infty} \sum_{\mu,\nu=0}^{n} \sum_{\substack{k_1,k_2,\ldots,k_{10}}}^{\substack{\mu=k_6+\ldots k_{10}\\ \nu=k_1+\ldots k_5}} a_p a_q \tilde{\tilde{d}}_\mu \tilde{\tilde{d}}^*_\nu \frac{1}{k+m+2} \tag{39}$$

$$\times\tilde{\Omega}(A_1, A_2, \ldots, A_5, \nu, k_1, \ldots, k_5, k, p)\tilde{\Omega}(A_1, A_2, \ldots, A_5, \mu, k_5, k_7, \ldots, k_{10}, m, q)(u/2)^{2(k+m+2)}.$$

The two-point Rayleigh resolution must satisfy the equation

$$\sum_{p,q=0}^{\infty}\sum_{\mu,\nu=0}^{n}\sum_{k_1,\ldots,k_{10}=0}^{\substack{k_6+\ldots+k_{10}=\mu\\ k_1+\ldots+k_5=\nu}} a_p a_q d_\mu d_\nu$$
$$\times\Big[(1.264-2\mu_{12})\tilde{\Omega}(A_1,\ldots,A_5,\nu,k_1,\ldots,k_5,0,p)$$
$$\times\tilde{\Omega}(A_1,\ldots,A_5,\mu,k_6,\ldots,k_{10},0,q) \tag{40}$$
$$+0.264\sum_{k,m=1}^{\infty}\tilde{\Omega}(A_1,\ldots,A_5,\nu,k_1,\ldots,k_5,k,p)$$
$$\times\tilde{\Omega}(A_1,\ldots,A_5,\mu,k_6,\ldots,k_{10},m,q)(u/2)^{2(k+m)}\Big]=0,$$

and finally, for Sparrow resolution we have

$$\sum_{p,q=0}^{\infty}\sum_{\mu,\nu=0}^{n}\sum_{k,m=1}^{\infty}\sum_{k_1,\ldots,k_{10}=0}^{\substack{k_1+\ldots+k_5=\\ k_6+\ldots+k_{10}=u}} a_p a_q \tilde{\tilde{d}}_\mu \tilde{\tilde{d}}_\nu^*$$
$$\times\tilde{\Omega}(A_1,\ldots,A_5,\nu,k_1,\ldots,k_5,k,p)\tilde{\Omega}(A_1,\ldots,A_5,\mu,k_6,\ldots,k_{10},m,q) \tag{41}$$
$$\times\{(m+k)[2(m+k)-1]+\mu_{12}[k(2-m-1)-m(k+m-1)]\}(u/4)^{2(k+m-1)}=0.$$

6. Final remarks

The performance criteria obtained in analytical forms are very useful and convenient for numerical investigations of the imaging properties of the symmetric optical system with circular apodization. This convenience lies in the fact that the calculation of the considered criteria requires only the summation procedures. By carrysing the summations over k and m it is easy to decide when they should be finished. Considering that two adjacent terms in the power expansion have opposite sign the error made by contraction of the mentioned summation procedures can be easily estimated. For all apodizers of the polynomial form the maximal index in the corresponding summation procedure is strictly determined. In order to compare the influence of two different apodizers with the expansion coefficients a_p, a_p' then in all derived criteria it suffices to replace the coefficients a_p by the differences Δa_p ($\Delta a_p = a_p - a_p'$). For systems with uniform aperture all a_p, expect for a_0, are equal to zero. In this case, the summations over p and q are automatically removed. When the performance of apodized optical systems without aberrations is explored, we have to set

$$d_\mu = \tilde{d}_\mu = \tilde{\tilde{d}}_\mu = 0, \quad \text{for} \quad \mu \neq 0$$

and

$$d_0 = \tilde{d}_0 = \tilde{\tilde{d}}_0 = 1.$$

It should be noted that the formulae derived enable a simultaneous calculation of the desired distributions for different values of the variable u (the general factors have the same values). Summing up, the derived formulae are a powerful tool for inspection of the imaging properties of circularly symmetrical optical systems.

The numerical illustration of the criteria proposed above are hoped to be delivered in the next future.

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Характеристки отображения аподизированных оптических систем со сферической абберацией при отображении одно-и двуточечнных предметов

Работа касается качества аподизированных оптических систем со сферической абберацией. Была принята аподизация с радиальной симметрией. Абберации приведены в представлении Зеринке. Обсуждены следующие критерии отображения: точечная функция размытия, второй момент точечного размытия, радиальное распределение энергии, число Стреля, распределение интенсивности изображения двуточечного предмета, точечная контрасность изображения, а также распределяемости Рейляйга и Спаррова. Для всех отмеченных критериев были получены аналитические зависимости.