

Field curvature and astigmatism in holographic imaging*

JERZY NOWAK, GRAŻYNA MULAŁ

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

By using the coefficients determining the higher order aberrations two methods of calculating the sagittal and meridional curvature are given. The analytic discussion has been confronted with the results of numerical calculations of positions of the meridional and sagittal foci for selected examples of holographic recording and reconstructions. The results obtained have been discussed.

1. Introduction

One of the problems occurring in the holographic imaging is that of aberration estimation. Usually the procedure of aberration determination is analogous to that employed in classical optical systems where first the coefficients characterizing the particular aberrations are determined and the next the transversal aberrations estimated. An essential problem here lies in accuracy of the results obtained. The coefficients characterizing the particular aberrations have been determined by MEIER [1] and next modified by CHAMPAGNE [2]. The theories of III order may be applied only when the angles made by the means forming the image are small. This condition is more difficult to satisfy when using the expansion due to Meier than that of Champagne. Of course, when calculating the aberrations, it is always possible to take account also of the coefficients determining the aberrations of V, VII and the higher orders [3, 4] (in addition to the third order coefficients). However, this procedure would be usually tedious one due to great number of coefficients. For instance, in the Meier expansion the field curvature and astigmatism of VII order are determined by 17 coefficients [3]. Therefore, a compromise should be found to achieve the needed accuracy on one hand, and reasonable comfort of calculations, on the other.

The examination of the field curvature and astigmatism may be found in the works [5-8], among the others. The purpose of this paper is to give a method of determining the meridional and sagittal curvatures in the holographic imaging. We will derive the expressions for the meridional

* This work was carried on under the Research Project M.R. I.5.

curvature K_m and sagittal curvature K_s by using both the above expansion. In the case of Meier expansion we will calculate the aberrations up to VII order (in the classical optical imaging the restriction to the VII order aberrations appears to be sufficient [9]). In the case of Champagne expansion the calculations will be restricted to the V order, since the angles made by the rays in the respective beams with the central ray of the beam are considerably less than those created by the rays with the axis of the Cartesian coordinate system (the angles essential in the Meier expansion). However, in some problems the Meier expansion proves to be more convenient, for instance, if the aberrations of the optical system composed of lenses and hologram are calculated. Therefore, in this work both methods of calculation of the field curvature and astigmatism will be exploited.

2. Calculation of the meridional curvature K_m and sagittal curvature K_s

— Method due to Meier

The part of the eikonal corresponding to the field curvature and astigmatism in the III, V and VII orders take the forms [1, 3]:

$$\Delta W_{\text{III}} = -\frac{1}{2} \rho^2 \left(S_{3y} \sin^2 \Theta - \frac{1}{2} S_{3y} \right), \quad (1)$$

$$\Delta W_{\text{V}} = \frac{1}{16} \rho^2 (12 T_{3y} \sin^2 \Theta + 3 T_{3y} + 12 \rho^2 T'_{3y} \sin^2 \Theta + 3 \rho^2 T'_{3y}), \quad (2)$$

$$\begin{aligned} \Delta W_{\text{VII}} = & -\frac{5}{128} \rho^2 (24 R_{3y} \sin^2 \Theta + 4 \rho^2 R_{3y} + 24 \rho^4 R'_{3y} \sin^2 \Theta \\ & + 4 \rho^2 R'_{3y} + 6 \rho^2 R''_{3y} + 48 \rho^2 R''_{3y} \sin^2 \Theta + 16 \rho^2 R''_{3y} \sin^4 \Theta), \end{aligned} \quad (3)$$

where ρ denotes the running radius in the hologram plane, Θ is the angle made by ρ with the X axis, and the coefficients S_{3y} , T_{3y} , T'_{3y} , R_{3y} , R'_{3y} , R''_{3y} characterizing the aberrations have the forms

$$S_{3y} = \frac{y_c^2}{z_c^3} \pm \frac{\mu}{m^2} \left(\frac{y_1^2}{z_1^3} - \frac{y_r^2}{z_r^3} \right) - \frac{y_3^2}{z_3^3}, \quad (4)$$

$$T_{3y} = \frac{y_c^4}{z_c^5} \pm \frac{\mu}{m^2} \left(\frac{y_1^4}{z_1^5} - \frac{y_r^4}{z_r^5} \right) - \frac{y_3^4}{z_3^5}, \quad (5)$$

$$T'_{3y} = \frac{y_c^2}{z_c^5} \pm \frac{\mu}{m^4} \left(\frac{y_1^2}{z_1^5} - \frac{y_r^2}{z_r^5} \right) - \frac{y_3^2}{z_3^5}, \quad (6)$$

$$R_{3y} = \frac{y_c^6}{z_c^7} \pm \frac{\mu}{m^2} \left(\frac{y_1^6}{z_1^7} - \frac{y_r^6}{z_r^7} \right) - \frac{y_3^6}{z_3^7}, \quad (7)$$

$$R'_{3y} = \frac{y_c^2}{z_c^7} \pm \frac{\mu}{m^6} \left(\frac{y_1^2}{z_1^7} - \frac{y_r^2}{z_r^7} \right) - \frac{y_3^2}{z_3^7}, \quad (8)$$

$$R''_{3y} = \frac{y_c^4}{z_c^7} \pm \frac{\mu}{m^4} \left(\frac{y_1^4}{z_1^7} - \frac{y_r^4}{z_r^7} \right) - \frac{y_3^4}{z_3^7}, \quad (9)$$

where $y_1, z_1, y_r, z_r, y_c, z_c, y_3, z_3$ being the coordinates of the object, reference wave source, reconstructing wave source and paraxial image, respectively, and μ is the reconstructing-to-recording wavelength ratio. The upper sign corresponds to the primary image while the lower sign — to the secondary image. In order to simplify the calculation it has been assumed that the object, reference wave and the reconstructing wave sources are positioned in the YZ plane. This assumption simplifies the calculations; thus, for instance, the number of coefficients describing the field curvature and astigmatism of VII order is reduced from seventeen to three. Also the notations of the coefficients (as compared to those used in the work [3]) have been changed, and since there exists no uniform systems of notation which would be generally accepted in the works concerning holographic imaging, hereafter it has been assumed that the given aberration order will be denoted by the same letter (similarly as it has been proposed by Buchdahl in the theory of classical systems).

The transversal aberrations of the image $\delta x_3, \delta y_3$ are determined by the relations [10]

$$\delta x_3 = -R_3 \frac{\partial W}{\partial x}, \quad (10)$$

$$\delta y_3 = -R_3 \frac{\partial W}{\partial y}, \quad (11)$$

where $R_3 = \sqrt{z_3^2 + y_3^2}$. If $y_3 \ll z_3$, then it suffices to substitute z_3 for R_3 in the formulae (10) and (11).

If we examine the field curvature and astigmatism the respective ensemble of rays should be considered in the meridional and sagittal planes. The transversal aberrations are expressed as follows:

$$\delta x_{3s}^{\text{III}} = \frac{1}{2} R_3 \varrho S_{3y}, \quad (12)$$

$$\delta y_{3m}^{\text{III}} = \frac{3}{2} R_3 \varrho S_{3y}, \quad (13)$$

$$\delta x_{3s}^{\text{V}} = -\frac{6}{16} R_3 \varrho (T_{3y} + 2 \varrho^2 T'_{3y}), \quad (14)$$

$$\delta y_{3m}^{\text{V}} = -\frac{6}{16} R_3 \varrho (5 T_{3y} + 10 \varrho^2 T'_{3y}), \quad (15)$$

$$\delta x_{3s}^{\text{VII}} = \frac{5}{128} R_3 \varrho (8 R_{3y} + 24 \varrho^4 R'_{3y} + 24 \varrho^2 R''_{3y}), \quad (16)$$

$$\delta y_{3m}^{\text{VII}} = \frac{5}{128} R_3 \varrho (56 R_{3y} + 168 R'_{3y} + 280 \varrho^2 R''_{3y}), \quad (17)$$

$$\delta x_{3m}^{\text{III}} = \delta y_{3s}^{\text{III}} = \delta x_{3m}^{\text{V}} = \delta y_{3s}^{\text{V}} = \delta x_{3m}^{\text{VII}} = \delta y_{3s}^{\text{VII}} = 0. \quad (18)$$

The meridional and sagittal curvatures may be computed from the relations [8]:

$$K_m = \frac{-z_3 \delta y_{3m}}{-\varrho + \delta y_{3m}}, \quad (19)$$

$$K_s = \frac{-z_3 \delta x_{3s}}{-\varrho + \delta x_{3s}}. \quad (20)$$

— *Method due to Champagne*

Under the same assumptions, as assumed previously, the part of eikonal corresponding to astigmatism has the form [2, 4]:

$$\Delta W_{\text{III}} = -\frac{1}{2} \varrho^2 \bar{S}_{3y} \sin^2 \Theta, \quad (21)$$

$$\Delta W_{\text{V}} = \frac{3}{4} \varrho^4 \bar{T}_{3y} \sin^2 \Theta - \frac{1}{2} \varrho^3 \bar{T}'_{3y} \sin^3 \Theta. \quad (22)$$

The coefficients characterizing the aberrations have the forms

$$\bar{S}_{3y} = \frac{y_c^2}{R_c^3} \pm \frac{\mu}{m^2} \left(\frac{y_1^2}{R_1^2} - \frac{y_r^2}{R_r^2} \right) - \frac{y_3^2}{R_3^3}, \quad (23)$$

$$\bar{T}_{3y} = \frac{y_c^2}{R_c^5} \pm \frac{\mu}{m^2} \left(\frac{y_1^2}{R_1^5} - \frac{y_r^2}{R_r^5} \right) - \frac{y_3^2}{R_3^5}, \quad (24)$$

$$\bar{T}'_{3y} = \frac{y_c^3}{R_c^5} \pm \frac{\mu}{m^2} \left(\frac{y_1^3}{R_1^5} - \frac{y_r^3}{R_r^5} \right) - \frac{y_3^3}{R_3^5}, \quad (25)$$

where R_1 , R_r , R_c , R_3 are the respective distances of the object, reference wave sources, reconstructing wave sources, and paraxial image from the hologram midpoint. It should be noticed that in the Champagne expansion the field curvature does not appear directly, the image is produced on a sphere. In order to determine the curvature it will be assumed that the image plane is distant from the hologram by z_3 (which coincides with the image plane determined by the Meier's formulas). The relations analogous to (12)–(18) have now the form

$$\delta y_{3m}^{\text{III}} = R_3 \varrho \bar{S}_{3y}, \quad (26)$$

$$\delta y_{3m}^V = -3R_3 \rho^2 \left(\rho \bar{T}_{3v} - \frac{1}{2} \bar{T}'_{3v} \right), \quad (27)$$

$$\delta y_{3s}^{III} = \delta x_{3m}^{III} = \delta x_{3s}^{III} = \delta y_{3s}^V = \delta x_{3m}^V = \delta x_{3s}^V = 0. \quad (28)$$

The meridional and sagittal curvatures may be calculated from the formulae:

$$K_s = R_3 \cos \alpha_3 - z_3, \quad (29)$$

$$K_m = \frac{-z_3 \delta y_{3m}}{-\rho + \delta y_{3m}} + K_s, \quad (30)$$

where $\sin \alpha_3 = y_3/R_3$.

3. The examples of calculations of K_m and K_s

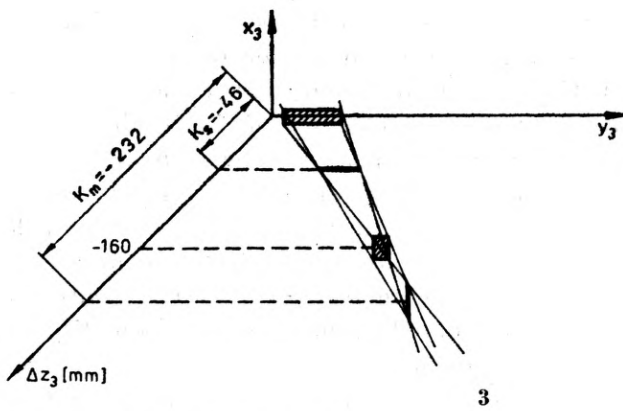
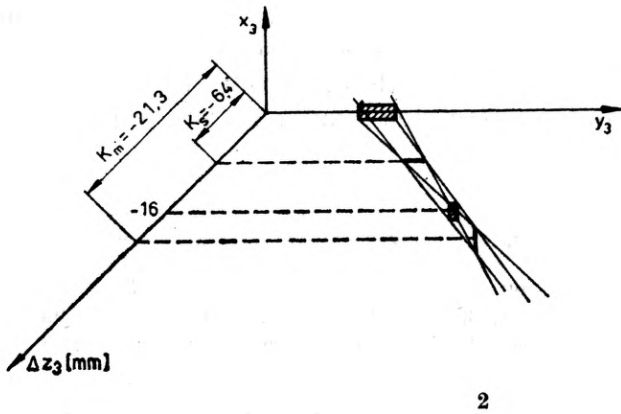
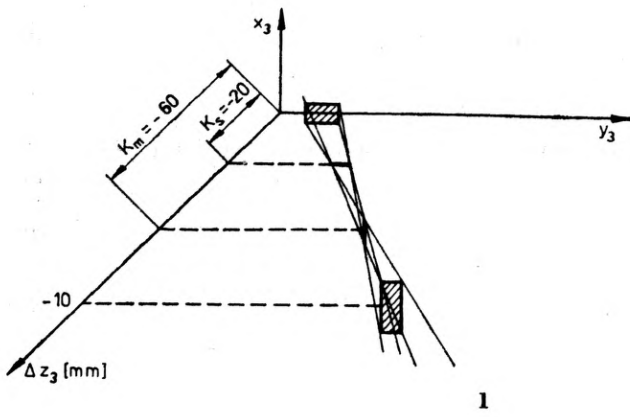
For the illustration of the discussed problems the values of K_m and K_s for three examples of holographic recording and reconstruction given in table 1 have been determined and compared with the results of calculations obtained by applying the formulae (19), (20), (29), and (30), respectively.

Table 1

No.	z_1	y_1	z_r	y_r	z_c	y_c	z_3	y_3	μ/m	α_3
1	-100	0	-100	10	-100	30	-100	20	1	11.3°
2	-100	0	-200	20	-83.333	-2.5	-500	85	2	9.65°
3	-100	0	-200	50	-83.333	-6.25	-500	212.5	2	23.03°

In the first example we will calculate the field curvature for the primary image, while in the second and third ones — that from the secondary image.

In all the three curves the conditions of recording and reconstruction were chosen so that the spherical aberration and coma be corrected [11]. The distortion appearing in the imaging does not change the position of the meridional and sagittal foci. Since the accuracy of the results for the fixed diameter of the hologram depends upon the angles made by the rays of the object, reference, reconstructing and image forming beams with the central ray of each of these beams (in the Champagne approach) or with the axis from the Cartesian coordinate system (in the Meier approach) the value of the angle α_3 which is the greatest one of all the mentioned is given in table 2, for information reasons. In table 2 the values of K_m and K_s calculated by using the methods of Meier and Champagne have been given for those three cases, while for the second examples the field curvature of seventh order has not been computed due to its small value.



Ray-tracing in the image space

Table 2

No.	III order		V order		VII order		ΣK_m	ΣK_s	K_m	K_s	
	K_m	K_s	K_m	K_s	K_m	K_s					
1	<i>M</i>	-6.52	-2.04	1.25	0.25	-0.17	-0.03	-5.44	-1.96	-6.0	-2.0
	<i>Ch</i>	-5.84	-2.02	-0.09	0	-	-	-5.93	-2.02		
2	<i>M</i>	-21.01	-6.81	-0.22	-0.05	-	-	-21.2	-7.05	-21.3	-6.4
	<i>Ch</i>	-21.36	-6.74	0.11	0	-	-	-21.2	-6.70		
3	<i>M</i>	-184.88	-49.44	-11.98	-2.35	5.43	0.79	-191.44	-51.005	-232	-46
	<i>Ch</i>	-195.64	-46.80	0.96	0	-	-	-196.60	-46.797		

The results were obtained at the assumption of the hologram diameter equal to 10 mm. In the last columns the values of the field curvature found numerically have been given. The runs of four chosen rays in the image space which determine the position of sagittal and meridional foci have been shown in the figure. From the above results it follows that in Meier expansion it is sufficient to take account of aberrations up to seventh order, while in the Champagne expansion — the aberrations up to the fifth order should be considered. There exist, however, differences between the results obtained analytically and numerically. These differences appear for several reasons. First of all the correction of spherical aberration and coma was only in the third order approximation, which may be important especially in the third example when determining the meridional and sagittal foci. It should be remembered that the division into aberrations is a conventional one and the value of the transversal aberrations $\delta x_s, \delta y_s$ is influenced by all the aberrations. Also the formulae (10) and (11) are approximate only.

Concluding our considerations it may be stated that the suggested method allows to determine the field curvature and astigmatism in a simple way, these data being necessary for practical determination of the holographic imaging quality.

References

- [1] MEIER R. W., *J. Opt. Soc. Am.* **55** (1965), 987.
- [2] CHAMPAGNE E. B., *J. Opt. Soc. Am.* **57** (1967), 51.
- [3] MULA G., *Optica Applicata* (in press).
- [4] LATTA J. N., *Appl. Opt.* **10** (1971), 666.
- [5] LEITH E. N., UPATNIEKS J., HAINES K. A., *J. Opt. Soc. Am.* **55** (1965), 981.

- [6] SMITH R. W., *Optics Commun.* **21** (1977), 102.
- [7] SMITH R. W., *Optics Commun.* **21** (1977), 106.
- [8] NOWAK J., *Optica Applicata* (in press).
- [9] MAGIERA A., Doctor's Thesis, Technical University of Wrocław, Wrocław 1976.
- [10] BORN H., WOLF E., *Principles of Optics*, Pergamon Press, London 1964.
- [11] NOWAK J., ZAJAC M., *Optik* **55** (1979), 93.

Received April 26, 1980

Кривизна поля и астигматизм в голографическом отображении

Используя коэффициенты, определяющие aberrации высших порядков, были разработаны два метода расчёта меридиональной и сагитальной кривизны. Аналитические рассуждения сопоставлены для избранных примеров регистрации и голографической реконструкции с результатами численных расчётов положений меридионального и сагитального фокусов. Обсуждены полученные результаты.