Optica Applicata, Vol. XI, No. 1, 1981

Improvement of recording linearity of Fourier transform holograms by using a random phase modulator with the spectrum shaped by spatial filitering*

MAREK KOWALCZYK

Military Academy of Technology, 00-908 Warsaw, Poland.

The papers deals with the problems connected of holographic recording of binary transparencies containing a great number of information ($\simeq 10^4$ bits) in the Fourier scheme. The suggested method enables to achieve the linear recording, by introducing into the object beam a phase modulator based on a narrow-band coherent noise filtering. The enclosed photographs representing the object beams and reconstructions confirm theoretical considerations.

1. Introduction

A great interest in getting a high quality Fourier hologram is due to the fact that the Fourier holography is the most suitable for the purpose of binary and alphanumerical information storage in holographic computer memories as well as in archives [1, 2].

The storage process consists in holographic recording of the transparency in which the information is coded in form of photographic density variation as a function of spatial co-ordinates (fig. 1). The retrieval process

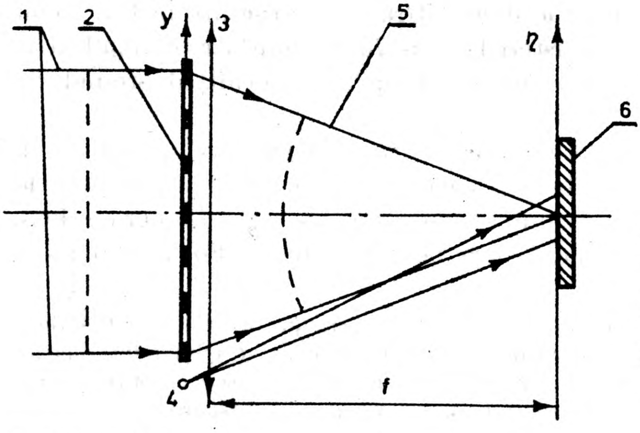


Fig. 1. Hologram recording system:
1 – plane wave, 2 – object (transparency in which a binary information is recorded), 3 – thin transforming lens, 4 – reference beam source, 5 – object beam, 6 – photoplate, f – focal length

* Work partially sponsored by Warsaw University (M.R. I.5).

ought to be performed by reconstructing a real transparency image from the chosen hologram at the plane of a matrix photodetectors or on a screen (fig. 2).

The amplitude transmittance t_a, corresponding to a given distribution of photographic density, is a bivalent periodic function of co-ordinates for the transparency in which a binary information had been coded*.

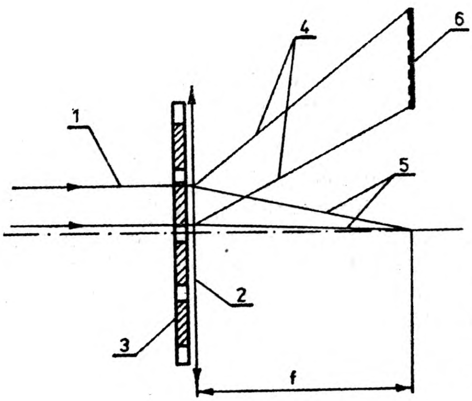


Fig. 2. Hologram reconstruction system: 1 – reconstructing beam (plane wave), 2 – transforming lens, 3 – matrix of holograms, 4 – reconstructed beam, 5 – zero-order beam, 6 – matrix of photodetectors

The effect of above is that the electric field amplitude distribution produced by an object beam in the hologram plane, which in the Fourier scheme is just a spatial frequency spectrum of the object, form an infinite sum of sharp and equidistant maxima. This kind of spectrum may be considered to be almost discrete (blurred due to limited extension of periodic function t_a). The amplitudes of Fourier components differ so much from one another that it is not possible to achieve a linear recording with good diffraction efficiency and without losses of information on the whole hologram area. The low diffraction efficiency of hologram results also from spectrum discretion, for the light diffracting structure in the reconstruction process is being registered in a finite number of small areas (in comparison with the whole hologram surface), localized around the spectrum components.

In spite of this disadvantage the Fourier holography owing to its other virtues [3], should not be replaced by another technique, say the Fresnel holography. Rather it requires some further search of other methods of smoothing out the spectrum of binary periodic objects. For that purpose

* Due to the finite sizes of transparency, t_a is periodic function on the finite interval only. Besides, the said periodicity appears in case of trivial information distributions, e.g. as it concerns only binary units. The above case is just taken into account, because it possesses the largest dynamic range of transparency spectrum. From the point of view of holographic registration in the Fourier scheme, it is the most difficult one.

the method of random phase shifter [4] and that of defocussed recording (with small shift from the back focal plane) had been developed [4, 5]. To construct a good phase shifter special materials and an advanced technology are needed, whereas defocussing can be easily realised and, moreover, gives advantageous results, provided, however, that extremely large recording densities are not expected.

The aim of this paper is to present a new method of smoothing out the transparency spectrum and the experimental results achieved. The idea of this method is to use in recording process such a purely phase object sticking to the transparency whose spectrum provides the highest degree of regular distribution of light intensity in an object beam, due to the convolution with the transparency spectrum.

2. Theoretical analysis

The amplitude transmittance of a binary object, e.g. an opaque screen with circular holes of diameter l placed in sites of a plane regular lattice whose constant is Δ and sizes are $(M-1)\Delta\times(M-1)\Delta$ (fig. 3), may be written as follows:

$$t_a(x, y) = \frac{1}{\Delta^2}\left[\operatorname{circ}\frac{\sqrt{x^2+y^2}}{l/2}\otimes\operatorname{comb}\left(\frac{x}{\Delta}-\beta\right)\operatorname{comb}\left(\frac{y}{\Delta}-\beta\right)\right]\operatorname{rect}\frac{x}{M\Delta}\operatorname{rect}\frac{y}{M\Delta}, \tag{1}$$

where M is a natural number,

$$\beta = \begin{cases} 0.5; & M - \text{even}, \\ 0; & M - \text{uneven}, \end{cases}$$

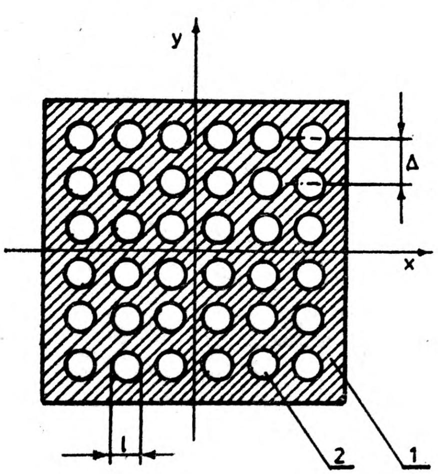


Fig. 3. Binary transparency in case when $M = 6$

1 – opaque area ($t_a = 0$), 2 – areas featured by amplitude transmittance equal to 1

and $\otimes$ denotes the convolution operation. Typical values for M, Δ, and l are 10^2, 1 mm and 0.5 mm, respectively.

If in the scheme shown in fig. 1 we place a transparency with a transmittance described by (1), then the distribution of field amplitude A in the object beam, in the (ξ, η) plane where the hologram had been placed, may be formulated as follows:

$$A(\xi, \eta) = \frac{ia}{\lambda f} \exp\left[-\frac{i\pi}{\lambda f}(\xi^2+\eta^2)\right] \mathscr{F}(t_a). \tag{2}$$

Note that $\mathscr{F}$ means the Fourier transform operator, λ represents the wavelength and a is the amplitude of plane wave incident on the transparency, i is the imaginary unit.

After substituting (1) into (2) and executing the $\mathscr{F}$ operator action the following intensity distribution $|A(\xi, \eta)|^2$ is obtained

$$|A(\xi, \eta)|^2 = \left(\frac{akl^2}{8f}\right)^2 \frac{4J_1^2\left(\frac{kl}{2f}\sqrt{\xi^2+\eta^2}\right)}{\left(\frac{kl}{2f}\sqrt{\xi^2+\eta^2}\right)^2} \frac{\sin^2\frac{M\Delta k\xi}{2f}\sin^2\frac{M\Delta k\eta}{2f}}{\sin^2\frac{\Delta k\xi}{2f}\sin^2\frac{\Delta k\eta}{2f}}. \tag{3}$$

Function J_1 is the Bessel function of first kind and first order and $k = 2\pi/\lambda$. The above formula is typical of diffraction by periodic structures and illustrates strong maxima; they are repeated regularly and modulated by Airy distribution which results from the circular shape of holes in light diffracting screen [6].

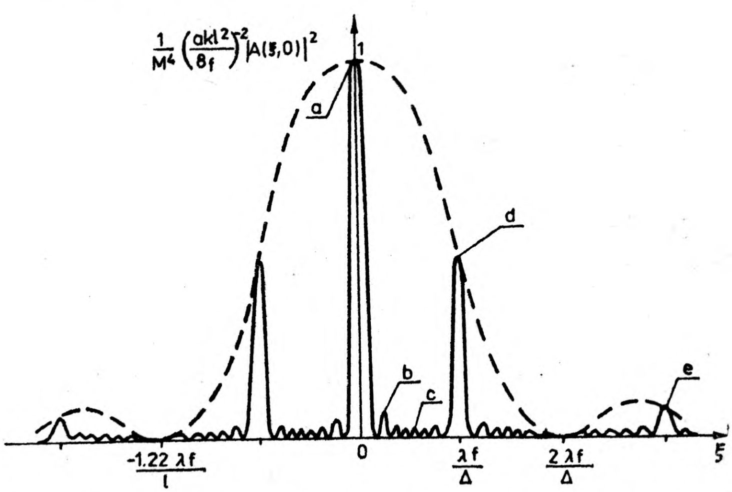


Fig. 4. Normalized dependence (3) in case when $l/\Delta = 0.61$ and $M = 8$. Linear scale is not preserved along the axis of ordinates

Graphical interpretation of the formula (3) is given in fig. 4. In case of the above given typical values of parameters M, Δ, and l, the ratios of intensity values in the maxima of distribution (3), denoted in fig. 4 by the letters o, b, c, e, g, are read as follows: $o:b:c:e:g = 1:4.5\times10^{-2}:10^{-4}:3.7\times10^{-1}:1.7\times10^{-2}$. The letter symbols refer to the particular maxima for any M, Δ and l as follows: 0 — zero order maximum, b — maximum the nearest to zero order one, c — the maximum placed at the nearest vicinity of the frequency $1/2\Delta$, e — the maximum corresponding to basic frequency of transparency, and g — the maximum corresponding to third harmonic of basic frequency.

The above data indicate that in practice it is possible to get a reasonable compromise between the recording linearity and diffraction efficiency as concerns the objects under consideration recorded in the Fourier system.

The problem of smoothing the spectra content in the Fourier plane without changing essentially the amplitude information coded in transparency may be solved by modifying the phase in the transparency plane (till now the transparency was assumed to be illuminated by a plane wave). It can be practically realised by bringing an additional transparency possessing a constant and possibly low photographic density into a direct contact with the object transparency; either its thickness or the refractive

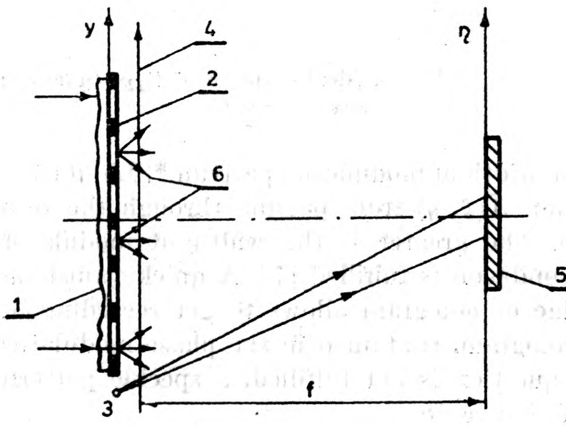


Fig. 5. Hologram recording system with a phase modulator:
1 — phase modulator, 2 — transparency, 3 — reference beam source, 4 — transforming lens, 5 — photoplate, 6 — diffracted (scattered) light by the object

index are functions of spatial co-ordinates. Amplitude transmittance — $\tilde{t}_a(x, y)$ referring to the object which consists of a modulator and transparency (fig. 5) reads as follows:

$$\tilde{t}_a(x,y) = t_a(x, y)t_{MF}(x, y), \tag{4}$$

where

$$t_{MF}(x, y) = \exp\{ik[n(x, y)-1]h(x, y)\},$$

t_{MF} standing for transmittance, h and n for thickness and refractive index of modulator, respectively. The amplitude distribution of the object beam field in hologram plane is now covered by the formula

$$\tilde{A}(\xi,\eta) = \frac{ia}{\lambda f}\exp\left[-\frac{i\pi}{\lambda f}(\xi^2+\eta^2)\right][\mathscr{F}(t_a)\otimes\mathscr{F}(t_{MF})]. \tag{5}$$

The wanted distribution of $\mathscr{F}(t_{MF})$ should assure the sufficient regularity of function $\tilde{A}(\xi,\eta)$ within the hologram plane and its tendency to a rapid vanishing at the edge of hologram. The fulfillment of the first of two above mentioned conditions improves the recording linearity and diffraction efficiency. In order to realize this condition in the best way $\mathscr{F}(t_{MF})$ should have the form of circ $(\sqrt{p^2+q^2}/d)$ (fig. 6) and d be greater than

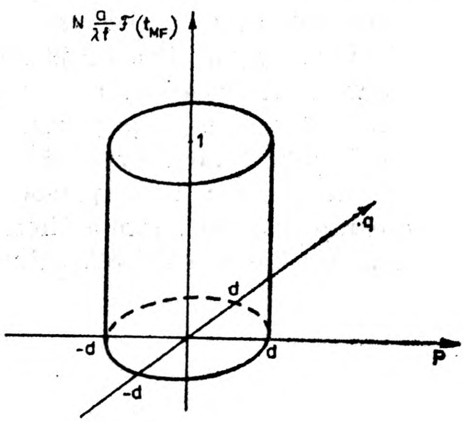


Fig. 6. Ideal shape of $\mathscr{F}(t_{MF})$ function: N – normalization factor

Δ^{-1} ($p = \xi/\lambda f$, $q = \eta/\lambda f$, d is the width of modulator spectrum*. Limitation of $\Delta^{-1} \leqslant d$ prevents the function $\tilde{A}(\xi,\eta)$ from passing through the zero values in the hologram plane. The greater is the width of modulator spectrum the better the first condition is fulfilled [7]. A quick vanishing of function $\tilde{A}(\xi,\eta)$ on the edge of hologram allows to get recording of all spatial frequencies in the hologram, contained in the phase modulator spectrum. If the condition in question is not fulfilled, a speckle pattern might appear in reconstructed image [8].

If $2a$ is the size of hologram foreseen in advance (in $\lambda f/\Delta$ units), then the following limitation can be put on the width of modulator spectrum

$$\frac{1}{\Delta} \leqslant d \leqslant \frac{a}{\Delta} - \frac{1}{\Delta}. \tag{6}$$

* The function circ $(\sqrt{p^2+q^2}/d)$, though it is ideal for our purposes, does not represent a spectrum of any complex function modulus of which is equal to one. Fortunately, there are t_{MF} functions the power spectra of which are good approximations of circ function.

While establishing a maximum value d, it had been assumed that a considerable insertion into the convolution (5) originating from the transparency spectrum is created only by those frequencies which are contained in the rectangle $\left\{|p| \leqslant \frac{1}{\Delta}, |q| \leqslant \frac{1}{\Delta}\right\}$. The said assumption is justified with respect to the values of l/Δ ratio applied in practice. The system of inequalities (6) is consistent in case when $a \geqslant 2$. It means that in case when a phase modulator is applied, the spectrum of which is similar to the ideal one, the holograms produced should not to be smaller than $4\lambda f/\Delta$. The value of parameter a depends upon the chosen resolution of image reconstructed from the hologram. When $a = 2$. and $l/\Delta = 0.78$ the images of two neighbouring bits are separated in accordance with the Rayleigh criterion.

Since in order to fulfil the first of two above mentioned conditions the greatest value d is needed and at the same time the right-hand side of (6) must be satisfied, we assume that

$$d = (a-1)\Delta^{-1}. \tag{7}$$

From the last equation it follows that there is a close relationship between the modulator spectral width, the linear information packing density in the transparency as well as the resolving power of hologram.

The least possible typical value of d is 1 mm^{-1}. Among spatial optical signals of the size of few centimeters, only high-frequency random signals possess a regular power spectrum of the above or greater width. The examples of such signals may be given by amplitude transmittance of ground-glass or distribution of light intensity in the speckle pattern, covering the image of light diffusing surface which has been achieved by applying coherent light illumination.

Equality $\langle|\mathscr{F}(t_{MF}t_a)|^2\rangle = \langle|\mathscr{F}(t_{MF})^2\rangle \otimes \mathscr{F}(t_a)|^2$, where $\langle\ \rangle$ denotes assembly average, is correct for a stationary random function t_{MF}. Thus, eq. (7) may be applied to determine the width of power spectrum and amplitude spectrum as well.

3. Method of producing modulator with the given spectrum of spatial frequencies

Neither ground-glass nor modulator in form of bleached photoplate, on which the speckle pattern had been registered, can be applied directly to the hologram recording setup, because of too great spectral width and a strong maximum for the zero frequency. The spectra of the above mentioned objects ought to be modified so that they resemble to the highest degree the spectrum shown in fig. 6. Let us consider the modulator performed by means of photographic technique, having in mind that a photographically registered image can be easily exposed under filtering process and that the difficulties lie in production of a ground-glass with the a priori assumed scattering indicatrix.

The process proposed to produce a modulator with the given spectral properties consists of the following phases:

— photographic recording of ground-glass image in optical system with coherent illumination,

— spatial filtering of ground-glass image covered by the speckle pattern,

— photographic recording of a ground-glass image with the speckle pattern featured by a new spectral distribution and chemical treatment with bleaching.

The power spectrum — $S(p, q)$ of speckle pattern coming into sight on the image plane of coherent circular aperture optical system [9] is described by the following function

$$BS(p, q) = \pi s_c^2 \delta(p, q) + 1 - \frac{2}{\pi} \arcsin\left(\frac{s}{2s_c}\right) - \frac{2}{\pi}\left(\frac{s}{2s_c}\right)\sqrt{1-\left(\frac{s}{2s_c}\right)^2}, \tag{8}$$

where B is a normalization constant, and

$$s = \sqrt{p^2+q^2},$$

s_c is the cut-off frequency of the optical system.

The graph of eq. (8) is shown in fig. 7. It is obvious that, in order to obtain a spectrum distribution approximate to that shown in the fig. 6, it is necessary:

— to use in the imaging process an optical systems possessing a possibly wide-band transfer function,

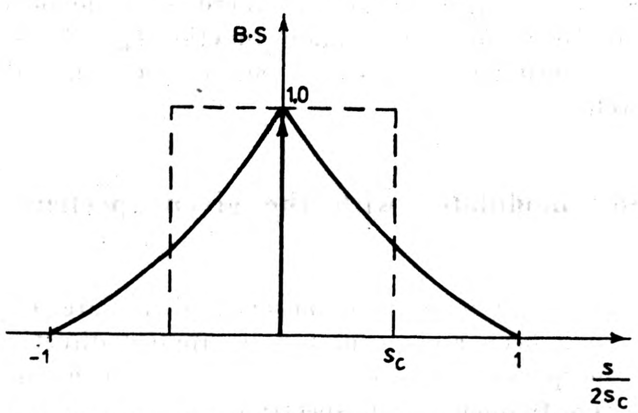


Fig. 7. Section of the spatial power spectral density for a uniformly illuminated diffuse surface imaged through a circular aperture optical system. A complete two dimensional spectrum is obtained by rotating the above curve about the vertical axis. Coherent transfer function of imaging system is represented by a broken line

— to filter off spatial frequencies contained in pulse response width of the lens owing to which the first Fourier transform in filtering system is performed,

— to filter off high frequencies in such a manner to get the effect of satisfied equation (7).

The optical system typefied by a wide-band transfer function is shown in fig. 8.

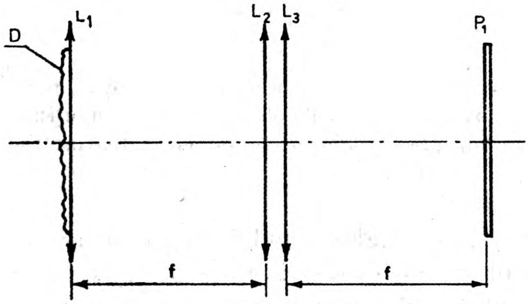


Fig. 8. Wide-band invariant optical system for recording the ground-glass D image in P_1 photoplate
L_1, L_2, L_3 — lenses

If, while recording the speckle pattern, the exposure value E corresponds to linear interval of the curve $t_a(E)$, then the distribution of field amplitude at the filtering system input (fig. 9) will reconstruct the speckle pattern intensity. Under the above assumption the intensity distribution in the spatial frequency plane can be well described by eq. (8).

While recording the coherent noise exposed to filtering in the system shown in fig. 9, the time of exposure should be chosen so as to minimize

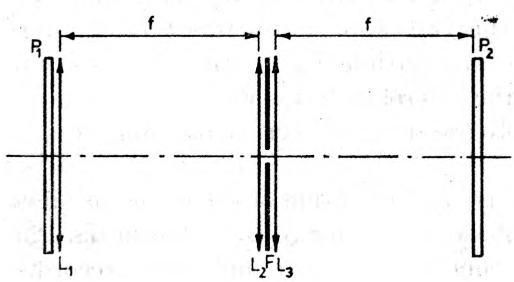


Fig. 9. System for filtering coherent noise recorded in P_1 plate. In the P_2 plate the coherent noise is recorded whose spectrum is modified by means of the filter F

the participation of higher order harmonics in the spectrum of bleached plate P_2 coming into sight due to the non-linear character of phase modulation process. In view of the considerations presented in chapter 2 and the analysis of distribution (8), it is likely to suggest a filter profile as shown in fig. 10a. In case when non-linear effects are neglected, the modulator power spectrum will be such as illustrated by fig. 10b.

The above presented mathematical expressions and diagrams which illustrate random modulator power spectrum are statistical averages over a great ensamble of modulators. The said expressions and diagrams

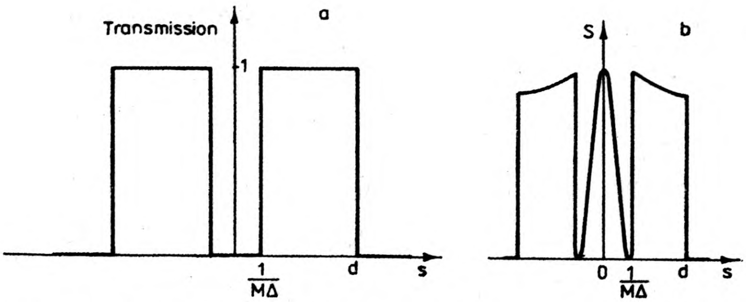


Fig. 10. a. Transmission shape of coherent noise filter. b. Power spectrum of P_2 plate transmittance exposed under conditions shown in fig. 9 and bleached. The magnitude of central maximum depends upon the amplitude of Fourier components creating the spectrum of P_2 plate transmittance

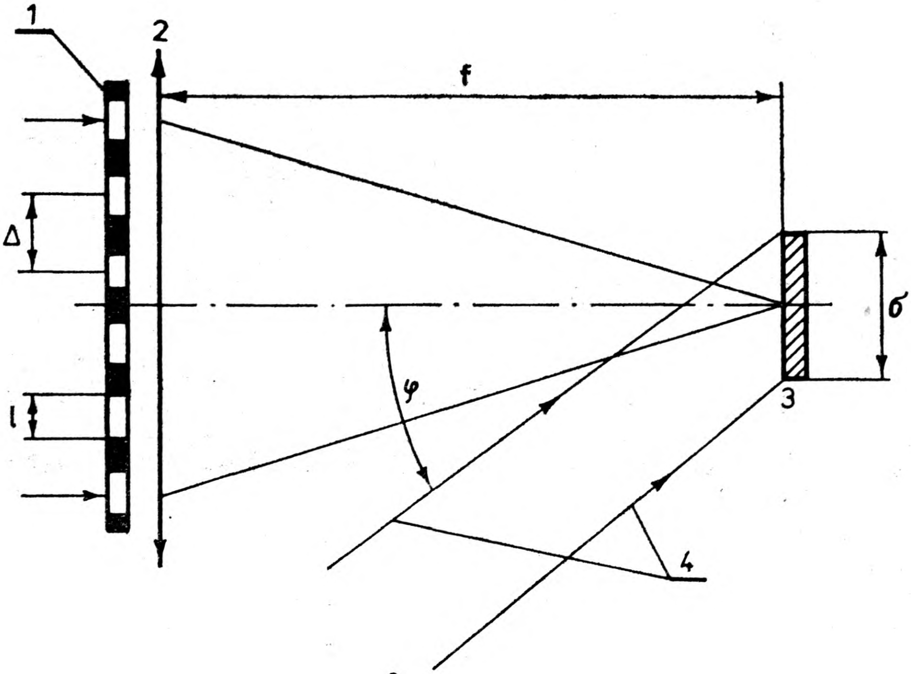
had been obtained on the basis of a ground-glass model presented in the paper [9]. For any chosen modulator, intensity distribution in the Fourier plane is featured by the appearance of speckle pattern due to finite aperture sizes of the Fourier transform system and modulator randomness.

4. Experimental results

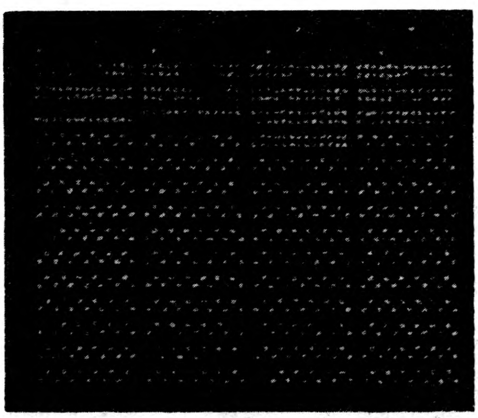
In order to supporting the presented considerations we have produced a hologram of binary transparence without any modulator and a set of holograms with phase modulators possessing different spectral characteristics. The holograms were performed as shown in fig. 11a. The sizes of transparency were 24×28 mm, the values of parameters l and Δ were 0.3 and 0.5 mm, respectively. The information distribution is shown in fig. 11b. A beam conjugated with the reference beam was used for reconstruction. The paper provides photographs of object beams and respective reconstructed image.

Photographic registration of intensity distribution of the beam obtained in the hologram without using the phase modulator is presented in fig. 12a. Figure 12b illustrates the reconstruction from the hologram recorded by using the same object beam as that used in fig. 12a, but at the presence of ground-glasses used as modulators. Figure 13 illustrates the intensity distribution in the object beam, while using a ground-glass transmitting a large amount of non-diffractive light and reconstruction adequate for the case like this. The spectrum of ground-glass applied for this purposes is relatively narrow (slightly wider than the hologram sizes) and possesses a very strong and narrow maximum for the zero frequency.

In the convolution (5) the said maximum acts as $\delta(p, q)$ function and reproduces the distribution $\mathscr{F}(t_a)$, this is quite obvious while comparing figures 12a and 13a.



a

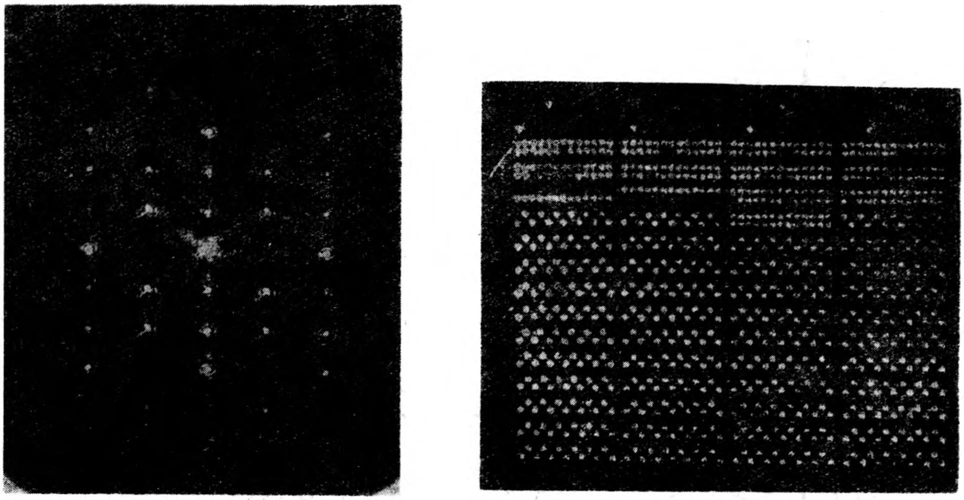


b

Fig. 11.
a. Experimental setup used for recording the holograms:
f = 220 mm, Δ = 0.5 mm, l = 0.3 mm, φ = 17°, σ = 2 mm, 1 – transparency, 2 – transforming lens, 3 – photoplate, 4 – reference beam
b. Transparency

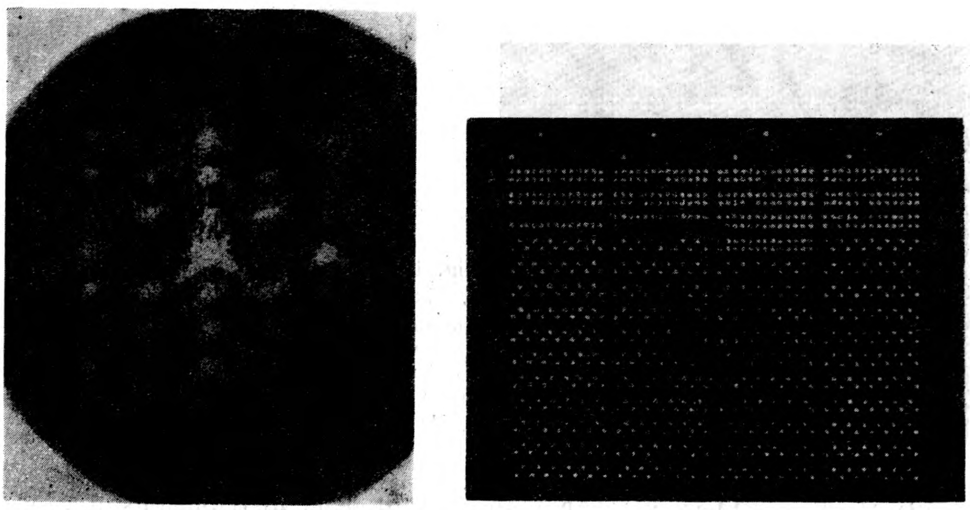
In the figure 14 we have an analogical intensity distributions obtained by using a ground-glass transmitting directly a very small stream of light. The spectrum of ground-glass in question is much wider than the hologram sizes and its maximum for the zero frequency is very small.

Two crossed phase diffraction gratings with a grating constant equal to 2 mm formed a modulator meeting approximately the conditions described in chapter 2 and used for recording of consecutive holograms. Fringes of these grating were directed parallelly with respect to the sides of transparency. The spectrum of single grating is shown in fig. 15. The



a b

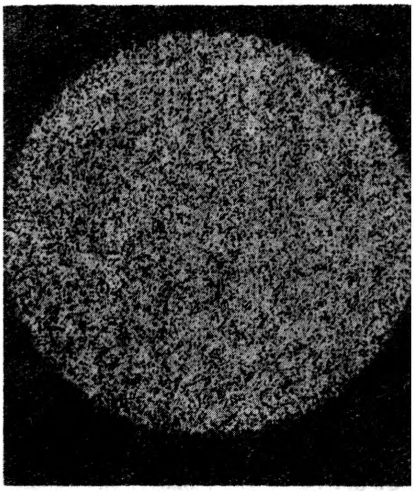
Fig. 12. a. Photographically registered power spectrum of transparency transmittance. b. Reconstruction from the hologram recorded with the object beam shown in sector *a* of this figure. Intermodulation deformations (false bits, differentiation of bits outlines) are visible



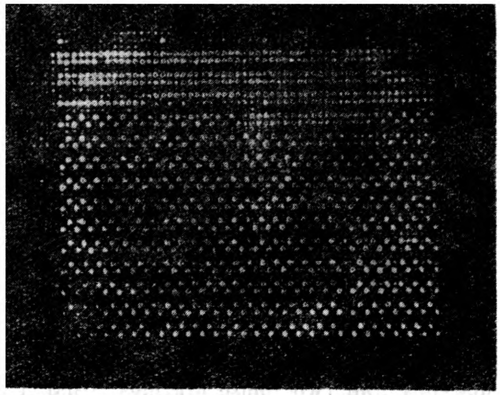
a b

Fig. 13. a. Intensity distribution within the spectrum of the object consisting of transparency and weakly scattering ground-glass. b. Reconstruction from the hologram recorded with the object beam shown in the sector *a* of this figure. Intermodulation deformation and weak coherent noise are visible

phase modulation factor involved by the said grating amounted to about 2.5 rad. This allowed to obtain the fourth diffraction order in the spectrum image.



a



b

Fig. 14. a. Intensity distribution within the spectrum of the object consisting of transparency and strongly scattering ground-glass. b. Reconstruction from the hologram recorded with the object beam shown in the sector *a* of this figure. Coherent noise is visible only

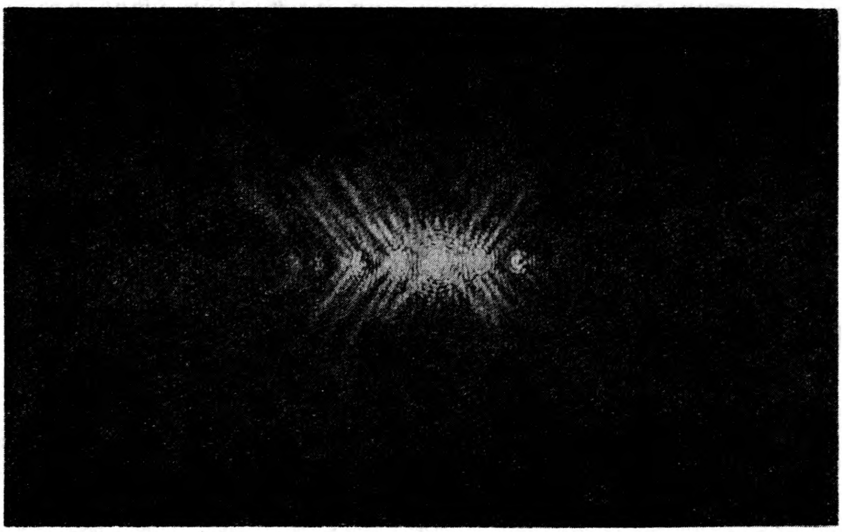
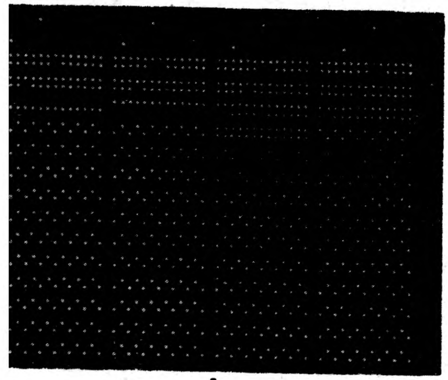


Fig. 15. Power spectrum of sinusoidal phase diffraction grating. Constant of the grating is equal to 2 mm and its sizes are 24×28 mm

A zero order maximum and four side maxima are separated from one another if, however, $a = 2$ they are equidistantly distributed in the area of spatial frequency determined by (7). Owing to the application of crossed gratings, the relation (7) is satisfied along the axis ξ and η as well.

a



b

Fig. 16. a. Intensity distribution within the spectrum of the object consisting of transparency and two phase gratings situated perpendicularly to each other. b. Reconstruction from the hologram recorded with the object beam shown in the sector *a* of this figure. Weak intermodulation deformations and coherent noise are visible

The gratings were performed by employing the method of bleaching the plates 10E75 on which interferencial fringes have been registered. The said fringes originate from two coherent and intersecting plane waves featured by equal intensities. The average value of exposure corresponded to the photographic density equal to 3. In compliance with the factory instruction of Gevaert Co., the above condition maximizes the phase modulation factor achieved after bleaching process is over.

The intensity distribution in the object beam with the modulator described above is shown in fig. 16a. Figure 16b presents the reconstruction from the hologram recorded with an object beam of such kind.

5. Conclusions

The enclosed photographs of the object beam indicate that, for any modulator, chosen amongst other phase modulators used, there exists a phenomenon of lowering the dynamic range of light intensity distribution in the hologram. At te same time photographs in question serve as experimental proofs that the above phenomenon can be described for random modulators by using the convolution. This is a natural attitude in the situation when a spatial frequency spectrum of two sticking transparencies is under examination.

The above conception allows to utilize the earlier explored mathematical expressions describing the power spectra of optical random signals [10]. Precisely, it is possible to estimate the average value of square modulus of (5), i.e. $\langle|\mathscr{F}(t_{MF})\otimes\mathscr{F}(t_a)|^2\rangle$ on the basis of $\langle|\mathscr{F}(t_{MF})|^2\rangle$ and $|\mathscr{F}(t_a)|^2$ *m*.

The expression $\langle \mathscr{F}(t_{MF}) \rangle$ cannot be used, since it is identically equal to zero.

The photographs of images reconstructed from holograms indicate that together with the lowering of the dynamic range of object beam intermodulation deformations decrease. If, however, the lowering of dynamic range is accompanied by such an increase of spectral width of the modulator-transparence complex that only a part of spectrum is recorded in the hologram, the reconstructed transparency image begins to be covered by the speckle pattern.

Thus, the assumptions given in chapters 2 and 3 of the present paper are proved. At the same it is understood that for production of the modulator with shaped spectrum, the use of a material featured by a great optical homogeneity is absolutely needed in order to minimize the quantity of light scattered in its volume. At the same time the said material should enable to obtain a great modulation factor in such a manner that the value of maximum situated in the origin of spatial frequency plane (fig. 10b) and non-linear effects could be minimized. Hence, it follows that thick dichromated gelatine emulsion may be considered as the material suitable for this purpose.

Of all inserted reconstructions presented the best is the quality of that shown in fig. 16b. It justifies the suggested spectrum shape of modulator used for holographic recording of binary periodic transparencies, as mentioned in chapter 2.

Good quality of reconstruction as shown in fig. 16b encourages also further investigations in the field of periodic phase modulators, when the period is greater than Δ and the transmittance contains high harmonics of its basic frequency. It may be also expected that it would be possible to obtain reconstruction which do not contain non-linear deformations and possess the coherent noise the power of which does not exceed the level tolerated in practice.

Acknowledgements — The author expresses his thanks to Dr. Katarzyna Chałasińska-Macukow, Dr. Ryszard Kulewski, and M.Sc. Jerzy Słaby, for their helpful remarks during drawing up the paper.

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Received November 27, 1979
in revised form March 16, 1980

Улучшение линейной записи голограмм Фурье путём применения случайного фазового модулятора со спектром, формированным методом пространственной фильтрации

В работе описаны проблемы, связанные с голографической записью в системе Фурье бинарных транспарантов, содержащих большое количество информации ($\simeq 10^4$ битов). Предложен метод, позволяющий достигнуть линейной записи посредством введения в предметный пучок фазового модулятора, выполненного на основе узкополосной фильтрации когерентного шума.

Помещены фотографии предметных пучков и восстановленных изображений, подтверждающие теоретические рассуждедния.