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Theoretical and experimental treatments of discharge current modulation of output of a CO2 laser

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This paper describes experimental and theoretical studies of current modulation of a low-power CO_2 laser. The studies were carried out for CO_2+He gas mixture. It was found that modulation of the output was very effective up to a few hundred hertz. Above 200–400 Hz fast decreasing modulation depth of the output was mainly determined by resonant energy transfer from vibrationally excited levels of CO to (00^01) level of CO_2.

1. Introduction

Influence of discharge current modulation on the output power of a CO_2 gas laser has been investigated by a few workers [1–3]. Experimental results presented in the papers are rather fragmentary and differ from one another. Theoretical calculations presented in the paper [3] have been based on a simplified population inversion mechanism in a CO_2 laser, without taking into consideration the dissociation of CO_2 molecules.

The paper presents theoretical and experimental results of the output modulation by means of a discharge current modulation obtained for a two-component CO_2+He gas mixture. The theoretical calculations have been based on a physical model of the population inversion mechanism taking into consideration the presence of CO molecules inside a discharge tube filled with a CO_2 laser mixture.

For comparative reasons both theoretical calculations and measurements were carried out for the same CO_2 gas laser. The construction of the laser has been based on water-cooled discharge tube of 7 mm i.d. and 0.6 m long, directly connected on both ends with mirrors of a resonator. One of the mirrors was flat and made of Ge, the other, gold, was concave with a 1 m radius of curvature. The gas composition ratio was $CO_2:He = 1:2$. The pressure was varied over a range $p = 10$–22 hPa. The dc discharge current (I_d) was changed between 2–20 mA. Slow gas flow was used in the experiments. At the total gas pressure 21.2 hPa and $I_d = 16$ mA, the output power of the laser was maximum and equalled 4 W.

Both the theoretical and the experimental treatments have been performed for different total gas pressures and values of discharge current.

2. Physical model

In electric discharge containing the mixture of CO_2+He not only CO_2 molecules and He atoms are present, but also products of the CO_2 dissociation, i.e. CO molecules and O atoms, the latter recombining to form O_2. Electron cross-sections for vibrational excitation of O_2 are much smaller than that for either CO or CO_2 [4]. Therefore, the O_2 molecules have no significant effects on the population inversion in a CO_2 laser, and can be neglected in physical models describing laser action [5–8]. Hereafter it has been taken into account that the He atoms, present in the electric discharge, play the double role in the population inversion mechanism: on the one hand they reduce gas temperature by increasing thermal conductivity of the mixture, on the other one, they take part in depopulation of $CO_2(01^10)$

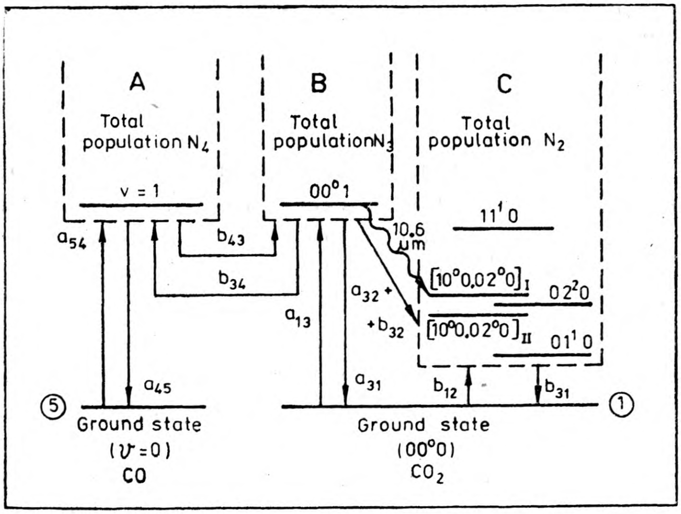

Fig. 1. The model used in the teoretical description of laser action in a CO_2+He gas mixture:

A – vibrationally excited CO, *B* – asymmetric stretching modes CO_2, *C* – symmetric stretching and bending modes CO_2

The figure 1 illustrates the physical model of population inversion mechanism to be adopted. Essentially, the model reduces the description of the laser action to a three level system. The vibrationally excited CO_2 molecules are divided into two groups: the symmetric stretching and bending modes and their combinations, and the asymmetric stretching modes; the upper vibrational levels of CO can also be regarded as one group. This division is reasonable because the levels within each group are linked together by relaxation processes which are faster by order of magnitude than the relaxation processes linking the groups [8]. More detailed description of physical models used in the theoretical analysis of CO_2 laser action can be found, among others, in the above mentioned papers [5–8].

Let the groups of energy levels of interest be named as in fig. 1. Here, a_{ij} is transition rate (per second) for CO_2 or CO molecule from the vibrational level i to j by collisions with electrons, and b_{ij} is overall rate (per second) for $i \to j$ by molecular collisions. Both rates a_{ij} and b_{ij} depend on discharge parameters, namely, on composition of gas mixture, total gas pressure, gas flow velocity, discharge current and rate of dissociation. It makes the theoretical analysis far difficult.

The rate equations, governing the total occupation numbers per unit volume N_i of the three groups of the excited vibrational levels, and the concentration N_f of 10.6 μm photons inside cavity, are the following

$$\frac{dN_2}{dt} = b_{12}N_1 + (b_{32} + a_{32})N_3 + N_f(P_{32}f_3N_3 - P_{23}f_2N_2) - b_{21}N_2, \tag{1}$$

$$\frac{dN_3}{dt} = b_{43}N_4 + a_{13}N_1 - b_3N_3 - N_f(P_{32}f_3N_3 - P_{23}f_2N_2), \tag{2}$$

$$\frac{dN_4}{dt} = b_{34}N_3 + a_{54}N_5 - b_4N_4, \tag{3}$$

$$\frac{dN_f}{dt} = N_f(P_{32}f_3N_3 - P_{23}f_2N_2) - rN_f, \tag{4}$$

where r – rate of loss of resonator,

f_2, f_3 – the fraction of CO_2 molecules in groups 2 and 3 on the (10^00) and (00^01) levels, respectively,

P_{32}, P_{23} – induced emission and absorption rate coefficient respectively, and

$b_3 = b_{34} + b_{32} + a_{31} + a_{32}$, and $b_4 = b_{43} + a_{45}$.

It has been estimated by TYTE [5], that $P_{32} \approx P_{23} = P$, approximately. From definitions of the parameters b_3 and b_4 it results that b_3^{-1} and b_4^{-1} are life times of the level (00^01) of CO_2 and $(v = 1)$ of CO, respectively. In the present paper it is also assumed that the fraction of excited CO and CO_2 molecules is small, so that the number of the unexcited ones can be equalled to the total number in each case.

If the discharge current is sinusoidally modulated by a frequency ω, the concentration N_f of photons fluctuates in the form

$$N_f = N_{f0} + N_{fm}\exp(j\omega t), \tag{5}$$

where the subscript "0" denotes the quantity in steady state, and $N_{fm} \ll N_{f0}$. Similarly, b_{ij}, a_{ij} and N_i may also be written in the same form as eq. (5), except for the parameters b_{43} and b_{34} that are independent of frequency ω. On the ground of the eqs. (1)–(4) it is possible to calculate first the modulation depth of the output defined as $m_p = N_{fm}/N_{f0}$, and further the modulation index M defined as

$$M = \frac{m_p}{m_i}, \tag{6}$$

where m_i denotes the depth of modulation of discharge current.

3. Theoretical calculations

Supposing the discharge current to be sinusoidally modulated, linearizing the eqs. (1)–(4), and reasonably assuming $a_{450} \ll b_{43}$, $a_{310} \ll (b_{320}+b_{340})$, and $a_{310}, b_{340}, b_{320} \ll \frac{1}{f_2} b_{210}$, we can obtain

$$M(\omega) = M_0 \frac{1+j\omega R b_{43}^{-1}}{1+j\omega b_{43}^{-1}}, \tag{7}$$

where M_0 is the value of the index $M(\omega)$ for frequencies ω near zero (practically for $\omega \leqslant 100$ Hz). The coefficient R in eq. (7) means a discharge parameter, and usually is less than, and even much less than one. Reasonably supposing $R \ll 1$, we can simplify eq. (7) and obtain for absolute value of M

$$|M(\omega)| = \frac{M_0}{\sqrt{1+\left(\frac{\omega}{b_{43}}\right)^2}}. \tag{8}$$

The eqs. (7) and (8) are valid for the frequencies $\omega \ll b_{210}$, i.e. practically below 5 kHz.

The value of M_0, in first approximation, equals one. More correctly M_0 can be evaluated from the eq. (9)

$$M_0 = 1 - \frac{a_{320} N_{30}}{a_{130} N_{CO_2} + a_{450} N_{CO}}, \tag{9}$$

where a_{320} is a parameter describing the following elementary process $CO_2(00^01) + e^- \rightarrow CO_2(n, m^l, 0) + e^-$.

The eq. (8) may be easily presented in a normalized form as the dependence of the dimensionless quantity $\varkappa = M/M_0$ on $\gamma = \omega/b_{43}$,

$$\varkappa = \frac{1}{\sqrt{1+\gamma^2}}. \tag{10}$$

The last dependence is normalized function of the modulation of the output power of a CO_2+He gas laser by means of discharge current modulation. It has been shown in fig. 2.

From the equation (10) it follows that $\varkappa = 1$ for $\gamma \ll 1$. It means that for the frequency of modulation $\omega \ll b_{43}$, the discharge current modulation of the CO_2 laser emission is very effective (approximately $m_i = m_p$). At the frequency $\omega = \omega_0 = b_{43}$ ($\gamma = 1$) the 3 dB drop of the quantity $\varkappa$ appears. When $\gamma \gg 1$ then $\varkappa \approx 1/\gamma$, what means $m_p \sim 1/\omega$, as $\omega \gg b_{43}$.

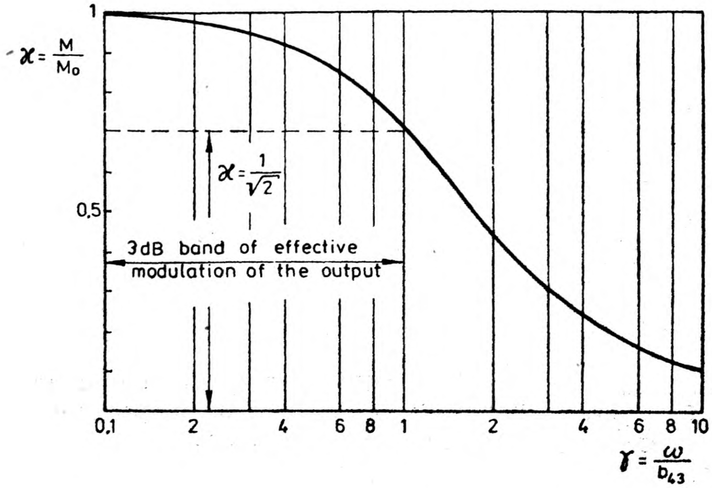

Fig. 2. The normalized function of the modulation of the output of a CO_2+He gas laser through modulation of discharge current

From the above presented results it follows that the parameter b_{43} plays an important role in the mechanisms of modulation of the output by means of discharge current modulation. The parameter b_{43} equals

$$b_{43} = k_{43} p_{CO}, \tag{11}$$

where k_{43} denotes the rate constant for the processes $CO^*(v = 1) + CO_2 = CO + CO_2^*(00^01)$. Taking into consideration that partial pressure of CO_2 depends both on gas temperature T_g and on rate of dissociation D, and that k_{43} increases linearly with temperature [9], we can note

$$b_{43}^*(T_g) = \frac{b_{43}(T_g)}{p_{CO_2}^*} = 0.0592\, T_g(T_g - 80)\cdot(1 - D), \tag{12}$$

where $p_{CO_2}^*$ — partial pressure of CO_2 at $D = 0$ and $T_g = 300$ K,
T_g — gas temperature in K,
$[b_{43}^*(T_g)] = s^{-1}Pa^{-1}$.

The dependence of the parameter b_{43}^* on gas temperature T_g for variety of dissociation rates is shown in fig. 3. Curves in this figure illustrate strong dependence of b_{43}^* on the parameters T_g and D, in other words, a strong dependence of the frequency range of effective discharge current modulation of the output (defined as the range $0-b_{43}$ Hz) on the discharge parameters such as total gas pressure, value of discharge current and gas flow rate.

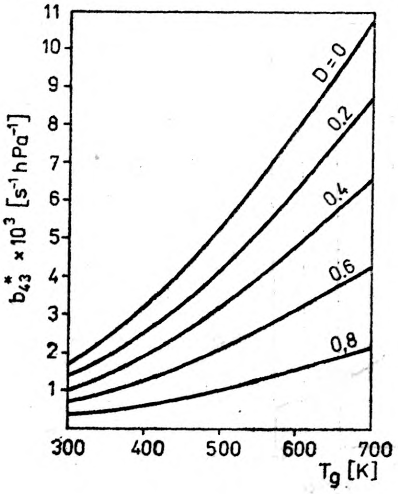

Fig. 3. Theoretical curves illustrating the dependence of b_{43}^{*} on the gas temperature at fixed rate of the dissociation

4. Experimental

The above theoretical results were tested using a set-up shown in fig. 4. In order to check the effect of discharge current modulation of the output, the output power *AC* component was measured when the AC component was superimposed on the discharge current at the modulation depth not exceeding 10%. It has been experimentally tested that for $m_i < 15\%$ discharge current is sinusoidally modulated without deformations. When modulation frequency was given, the output light *AC* component was measured by means of phase sensitive detection technique, what made the measurements more precise.

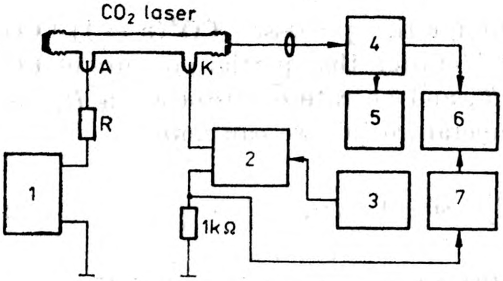

Fig. 4. Block diagram of the measuring system:
1 – power supply, 2 – discharge current modulator, 3 – sine wave generator, 4 – (Hg, Cd)Te detector, 5 – dc voltometer, 6 – lock-in nanovoltometer, 7 – selective nanovoltometer

The index M (eq. (6)) vs. modulation frequency has been investigated for the pressure of 10.6 hPa and the dc discharge current of 2.5, 7, and 13 mA as well as for pressure of 21.2 hPa and the dc discharge current of 4.5, 10, and 18 mA. The *dc* discharge currents of 7 and 18 mA refer to the maximum output power at pressures of 10.6 and 21.2 hPa, respectively. All modulation curves (the index M vs. ω) have been obtained for the frequency range 0.01–5 kHz.

All the measurements of the index M vs. frequency were repeated three times and the values of M obtained at fixed frequency were averaged. The measured values of M differed no more than by $\pm 20\%$ from the mean value of M. In each measurement the index M could be determined with accuracy of $\pm 4\%$.

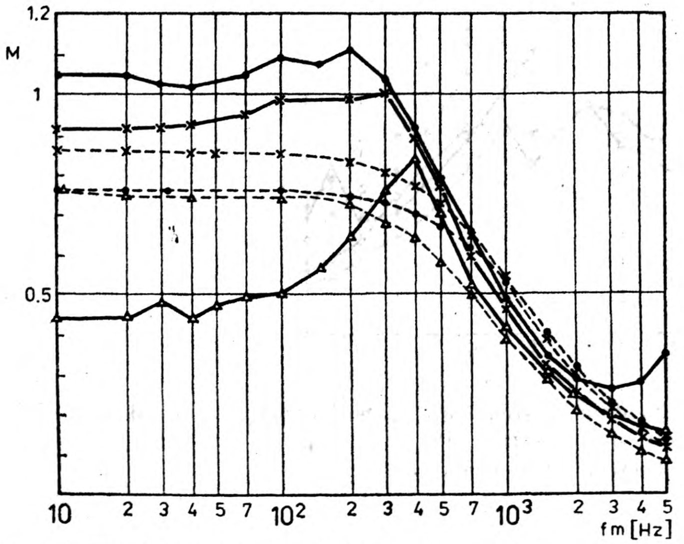

Fig. 5. The index M vs. modulation frequency f_m, experimental results are compared with theoretical (dashed) lines. Experimental conditions: p = 10.6 hPa
● – I_d = 2.5 mA, × – I_d = 7 mA, △ – I_d = 13 mA

The modulation curves obtained for various discharge conditions as well as the theoretical curves obtained from the eqs. (8) and (9) have been presented in figs. 5 and 6. In order to obtain the latter curves for the above given values of total pressure and the dc discharge current it was necessary to calculate gas temperature by the methods described in papers [10, 11]. Next, using experimental data included in various papers, values of the coefficients b_{ij}, a_{ij} and P of the eqs. (1)–(4), have been calculated. So the values of coefficient P have been taken from the papers [12–13], and the required values of the coefficient a_{ij} from the paper [14]. Basic data needed for calculation of b_{ij} have been taken from the paper [15]. The rate of dissociation D, indispensable for a precise calculation of b_{ij}, has been chosen theoretically, so that the calculated value of the output power be equal to its measured value, for all the values of pressure and discharge current to be used in the experiments.

Experimental results (figs. 5 and 6) show relatively great variations of the index *M*, when the modulation frequency is changed. Under all discharge conditions used in the experiments, and at the modulation frequency below 500 Hz, the values of the index *M* range within 0.3–1.1 to decrease relatively fast when modulation frequency increases above 500 Hz. At the frequency near 1 kHz the value of *M* is almost independent of the discharge conditions being equal to 0.4–0.5.

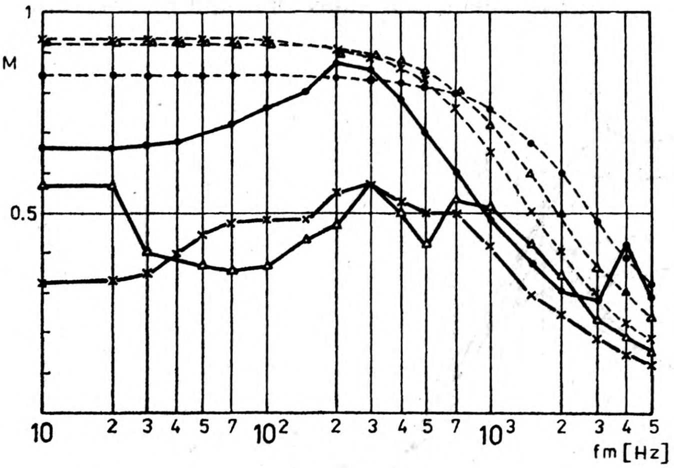

Fig. 6. The index *M* vs. the modulation frequency f_m, both experimental and theoretical curves. Experimental conditions: $p = 21.2$ hPa
● – $I_d = 4.5$ mA, × – $I_d = 12$ mA, △ – $I_d = 18$ mA

5. Conclusions

The agreement between the theoretical calculations and the experimental results is satisfactory over the entire frequency modulation range, considering that the values of the coefficients of the eqs. (1)–(4) as well as of gas temperature were calculated only approximately. It may be said that the assumed physical model describes relatively correctly the modulation of the output by means of the discharge current modulation.

The relatively great changes in the values of *M* accompanying the changes in the modulation frequency, particularly below 1 kHz, may be due to resonant properties of discharge tube impedance [16].

Furthermore, from these experimental and theoretical studies it may be concluded that the resonant energy transfer from $CO^*(v = 1)$ to (00^01) level of CO_2 mainly determines the modulated laser output in the CO_2+He laser gas mixture.

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Теоретические и экспериментальные исследования модуляции излучения CO_2 лазера разрядным током

Проведено теоретическое и экспериментальное исследование модуляции излучения CO_2 лазера, работающего со смесью CO_2+He. Исследование выполнено при различных токах разряда и различных полных давлениях газов. При всех условиях разряда подтверждается возможность модулирования разряда до частот, ограниченных константой скорости реакции $CO\,(\nu=1) + CO_2^*\,(00^00) \rightarrow CO\,(\nu=0) + CO_2\,(00^01)$.