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## MATHEMATICAL MODEL AND ITS CALIBRATION FOR RZESZÓW WASTEWATER TREATMENT PLANT

A model of the activated sludge wastewater treatment plant in Rzeszów is presented. It consists of submodels of primary clarifiers, aeration basins and secondary clarifiers, including equations describing the physical and biochemical transformations taking place in the vessels: sedimentation and thickening in the clarifiers and biological transformations of the influent wastewater components in the aeration tanks. The model parameters were estimated in two steps. In the first step, the active volumes of the tanks were estimated from the results of the planned experiment carried out in the plant, and in the second one, kinetic and stoichiometric parameters were estimated based on the data collected during regular plant operation.

### 1. WASTEWATER TREATMENT PLANT IN RZESZÓW

The designed capacity of the Rzeszów wastewater was 75000 m<sup>3</sup>/d. The sewage comes from the part of the city of Rzeszów on the left bank of the Wisłok river. The sewerage system is partly of the mixed type. Apart from the domestic sanitary wastewater there are also industrial discharges from 112 sources including pharmaceutical, food processing and big metal industry works.

The plant wastewater treatment facilities consist of a raw sewage pumping station, screens, grit chambers, two primary clarifiers, three parallel activated sludge aeration basins and two secondary clarifiers. After mechanical treatment the wastewater flows to the activated sludge tanks equipped with fine pore aeration system where organic and nitrogenous matter is being decomposed biologically under oxic conditions. The mixed liquor from the tank outlet flows to the secondary clarifiers where the sludge is

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thickened and separated from the treated effluent. The layout of the plant is shown in figure 1, and table 1 presents the plant influent characteristics measured in 1994.

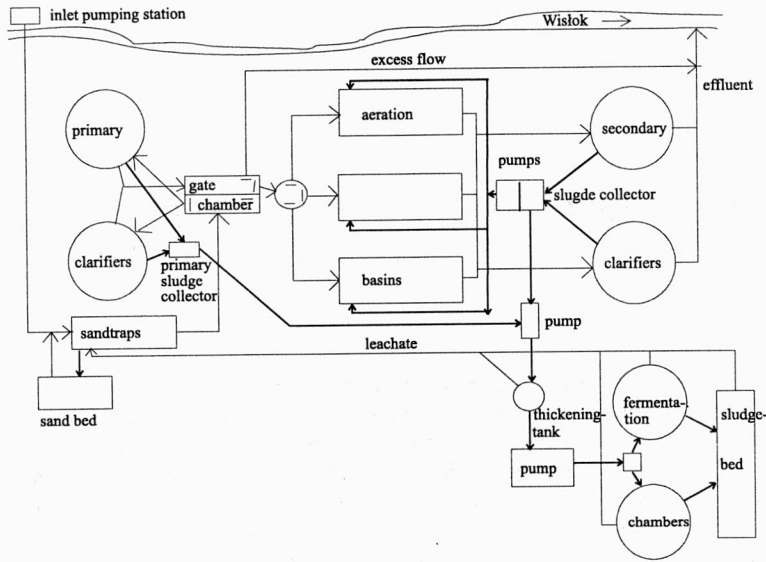


Fig. 1. Layout of the Rzeszów wastewater treatment plant

Table 1

Characteristics of the influent to the Rzeszów treatment plant

	Unit	Min.	Average	Max.
Flow rate	m <sup>3</sup> /d		35 000	
Suspended solids	g/m <sup>3</sup>	84.0	309.2	349.1
BOD <sub>5</sub>	g/m <sup>3</sup>	80.0	416.8	770.0
Dichromate COD	g/m <sup>3</sup>	325.0	507.6	753.0
Ammonia nitrogen	g N/m <sup>3</sup>	14.0	28.3	40.0
TKN	g N/m <sup>3</sup>	29.0	52.9	81.0
Total phosphorus	g P/m <sup>3</sup>		15.1	

Each of the main facilities of the Rzeszów treatment plant (primary clarifiers, activated sludge bioreactors and secondary settling tanks) can successfully be modelled. The submodels of the individual elements together with the model of inputs (influent flow rate, concentrations) constitute a complete model of the treatment plant. Screens and grit chambers do not contribute much to the inputs modifications, and can usually be neglected in modelling.

## 2. MODEL OF THE PLANT INPUTS

Concentrations of pollutants in wastewater are structured according to their nature (organics and nitrogen) and the transformations they undergo in treatment processes. The items of the structure are then state variables in the models of the processes.

COD is a superior measure of the wastewater organic material for the modelling purposes. It provides a link between electron equivalents in organic substrate, biomass and oxygen utilised by the biomass. The organic matter in wastewater can be subdivided into four categories [1]:

- inert soluble organics ( $s_i$ , mg COD/dm<sup>3</sup>),
- inert particulate organics ( $x_i$ , mg COD/dm<sup>3</sup>),
- readily biodegradable organics ( $s_s$ , mg COD/dm<sup>3</sup>),
- slowly biodegradable organics ( $x_s$ , mg COD/dm<sup>3</sup>).

Inert organics ( $s_i$  and  $x_i$ ) do not undergo transformations and leave the treatment plant unchanged in form. Readily biodegradable material ( $s_s$ ) consists of relatively simple molecules that can be taken in directly by heterotrophic bacteria and used for their multiplication. The slowly biodegradable organics ( $x_s$ ) consist of relatively complex molecules, which must be hydrolysed extracellularly and converted into readily biodegradable substrate and thus available to heterotrophic bacteria. For the purposes of activated modelling, the  $x_s$  fraction is treated as if it was particulate.

Nitrogenous matter in wastewater can be divided into five categories:

- ammonia nitrogen ( $s_{NH}$ , mg N/dm<sup>3</sup>),
- biodegradable, soluble organic nitrogen ( $s_{ND}$ , mg N/dm<sup>3</sup>),
- biodegradable, particulate organic nitrogen ( $x_{ND}$ , mg N/dm<sup>3</sup>),
- inert, soluble organic nitrogen ( $s_{NI}$ , mg N/dm<sup>3</sup>),
- inert, particulate organic nitrogen ( $x_{NI}$ , mg N/dm<sup>3</sup>).

Table 2

Organic and nitrogenous fractions in raw wastewater

Symbol	Fraction of total COD	Fraction of total nitrogen
$s_i$	0.045	—
$x_i$	0.356	—
$s_s$	0.161	—
$x_s$	0.438	—
$s_{NH}$	—	0.784
$s_{ND}$	—	0.062
$x_{ND}$	—	0.154

A content of the inert nitrogen ( $s_{NI}$  and  $x_{NI}$ ) in wastewater is usually very low and may be excluded from modelling. Proportions of the COD and nitrogen fractions in a given wastewater are characteristic of the wastewater and supposed to be more or

less constant over a time. For the sewage from Rzeszów the fractions given in table 2 have been identified [2].

### 3. HYDRAULIC MODEL OF THE MAIN PLANT VESSELS

The hydraulic model consists mainly of submodels of the basic vessels which may be treated as the ideally mixed tanks. Thus each submodel contains only one unknown parameter, viz. the vessel volume. The volume, which takes an active part in the flow dynamics (called the active volume), may differ significantly from the geometric volume, which happens also in this case. The reason for this will be discussed in the sequel. The main difficulty in estimating experimentally the active volume arises from the unsteady inflow to the plant during the day (see figure 2). This gives the nonstationary model to describe the vessel dynamics.

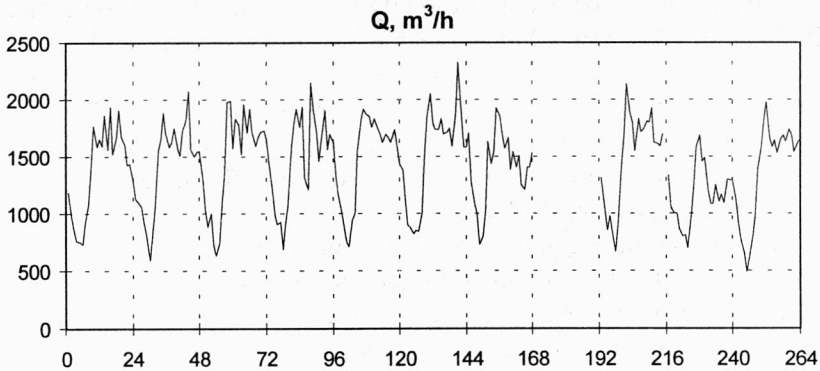


Fig. 2. The influent flow rate in a treatment plant

If a tracer is used in an experiment, the balance of the tracer mass yields the equation

$$V \frac{dc(t)}{dt} = Q(t)[c_i(t) - c(t)] \quad (1)$$

where  $V$  is the active volume,  $c(t)$  and  $c_i(t)$  are the concentrations of the tracer in the vessel and in the influent, respectively, and  $Q(t)$  is the influent flow rate. This is the first order ordinary differential equation with a varying parameter.

Let us introduce a new variable [3]

$$\xi(t) = \int_0^t Q(\tau) d\tau$$

which is the volume of wastewater which has passed through the vessel from the beginning of observations. As  $Q(t) > 0$  there exists the inverse differentiable function  $t = g(\xi)$ . Inserting it to (1) we have

$$V \frac{dc(\xi)}{d\xi} \frac{d\xi}{dt} = Q(\xi)[c_{in}(\xi) - c(\xi)]$$

where  $c(\xi)$  is equal to  $c(g(\xi))$  and similar to  $c_i(\xi)$  and  $Q(\xi)$ . But  $d\xi/dt = Q(t)$  and therefore

$$V \frac{dc(\xi)}{d\xi} = c_{in}(\xi) - c(\xi). \quad (2)$$

Thus, when the lapse of time is measured as the passed liquid flow instead of time, then the vessel is described by the equation with a constant parameter.

When planning the experiment it was found that the piecewise constant administration of the tracer would be most suitable because of technological reasons. In this case, equation (2) can be transformed to

$$c(\xi_n) = \frac{1}{V} \int_0^{\xi_n} e^{-\frac{\xi_n - \tau}{V}} c_{in}(\tau) d\tau = e^{-\frac{\Delta_n}{V}} c(\xi_{n-1}) + (1 - e^{-\frac{\Delta_n}{V}}) c_{in}(\xi_{n-1}) \quad (3)$$

where  $\xi_n = \xi(t_n)$ ,  $t_n$  is the time of  $n$ th observation, and  $\Delta_n = \xi_n - \xi_{n-1}$  is the observation interval.

The data used for estimation were gathered during a planned experiment with chlorine ions as the tracer. This was achieved by adding salt (NaCl) to the wastewater flowing into the vessel. An optimal (piecewise constant) input for the constant sampling time was found to be periodic with the period between  $4V$  and  $6V$  (depending on the value of  $\Delta$ ). The optimal sampling time was around  $V$  or  $1.5V$  (two optima) with a rather flat criterion function between and around them (see [4]).

If the tracer concentration is observed with a good accuracy, the volume  $V$  can be estimated from the expression

$$\frac{c(\xi_n) - c_{in}(\xi_{n-1})}{c(\xi_{n-1}) - c_{in}(\xi_{n-1})} = e^{-\frac{\Delta_n}{V}}, \quad n = 1, 2, \dots, N. \quad (4)$$

The non-linear least squares method can be used for this purpose.

As it was not possible to measure the tracer concentrations in the effluents and influents of the parallel vessels separately, the volumes of the two primary clarifiers, three aeration basins and two secondary clarifiers have been estimated together. Two estimation methods have been used: (i) linear least squares (LS) after interpolation of the signal values in equidistant flow points (constant  $\Delta$ ) and estimation of the constant

parameter  $a = \exp(-\Delta/V)$  in the difference equation (3), (ii) nonlinear least squares (NL) with (4). A selection of the results from [5] is presented in table 3.

Table 3

Estimates of the active volumes of the basic vessels [m<sup>3</sup>]

	Primary clarifier	Aeration basin	Secondary clarifier	
			Upper	Lower
LS	5676	11246	4723	6303
NL	6283	12087	4612	6514
Average	5980	11667	4668	6408
Geometric	7820	13500	11060	

In the secondary clarifiers, the tracer output concentrations were measured both in the effluent and in the recirculated sludge stream. Therefore, the total volume is divided into two zones: the first is attributed to the effluent (upper zone, or clarification zone) and the second – to the recirculated sludge stream (lower zone, or thickening zone). Both of them were estimated and the results obtained are shown in the right-hand part of table 3. In all cases presented there, the sum of the zone volumes is very close to the geometric volume of the clarifier.

In the primary clarifiers, only effluent tracer concentration was measured, as the rate of sludge discharge is much lower there. Thus the volume estimated is significantly smaller than the geometric volume. Few factors could also cause the active volume estimate of the aerated basins to be smaller than the geometric volumes. Among them the dead parts of the volume and the air bubbles are likely to be.

#### 4. PRIMARY CLARIFICATION MODEL

In the primary clarifier, settleable matter is removed by gravity. It causes lowering of the load into biological reactor. There is no doubt that the fundamental nature of particle settling in a primary settling tank is extremely complex and essentially stochastic. Fortunately, however, it has been proved that relatively simple mechanistic lumped-parameter models applied to real-world problems are reliable and give fairly good predictions of pollutant concentrations in the primary settler effluent [6], [7]. The model conceptually divides the primary settler into three separate functional elements: a solids-liquid separator, a sludge zone at the bottom, a series of imaginary completely-mixed compartments to account for the hydraulic mixing. Efficiency of removing the suspended solids is determined from an empirical equation relating the rate of removing suspended solids to the suspended solids concentration. It is assumed that only suspended solids are being removed, while the dissolved solids are subjected

only to mixing in the compartments. A model describing the mixing in an ideally mixed compartment is given by the equation

$$V_p \frac{ds}{dt} = Q(s_{in} - s)$$

where  $V_p$  is the active volume of clarifiers,  $Q$  is the flow rate,  $s_{in}$  is the concentration of the influent dissolved fraction and  $s$  is its concentration in the compartment.

The model for the suspended fraction includes, besides that of ideally mixed tank, also sedimentation of the suspended solids ( $x$ ). The sedimentation rate is expressed as  $Ax^B$ . The parameters  $A$  and  $B$  are estimated experimentally. Thus the model is

$$V_p \frac{dx}{dt} = Q(x_{in} - x) - V_p Ax^B.$$

## 5. MODEL OF BIOLOGICAL PROCESSES

Most designs of biological wastewater treatment incorporate long solids retention times. Because of that the differences in concentrations of soluble biodegradable organics in the effluent at various system configurations are small. Conversely, large differences in activated sludge concentrations and electron acceptor requirements are common. This means that models describing substrate removal should mostly be directed to their impact upon activated sludge concentrations and electron acceptor requirements. The first good example of such a model was IAWPRC Activated Sludge Model 1 [1].

The model incorporates carbon oxidation, nitrification and denitrification. The fate of the carbonaceous and nitrogenous components of sewage in bioreactors is schematised in the model as shown in figure 3. For purposes of modelling, the readily biodegradable material ( $s_s$ ) is treated as if it was soluble, whereas the slowly biodegradable material ( $x_s$ ) is treated as if it was particulate. A portion of the readily biodegradable organics taken in by heterotrophic bacteria ( $x_{bh}$ ) is oxidised either with oxygen (oxic process) or nitrates (anoxic process, denitrification) as terminal electron acceptors. The rest of organic molecules is incorporated into the biomass. The energy released during the oxidation covers the energetic expenditures of the biomass synthesis. The biomass synthesis is associated with assimilation of some nitrogen and phosphorus.

The slowly biodegradable material, after entering bioreactor, is supposed to be quickly enmeshed in the activated sludge flocs structure, and then to be acted upon extracellularly (hydrolysed). The hydrolysis is slower than utilisation of readily biodegradable substrate. The extent to which the fraction  $x_s$  in the sludge flocs is hydrolysed (before the flocs leave the system) depends mainly on the process temperature

and the time designed for the hydrolysis in the system (sludge age). The higher the temperature and the more advanced the sludge age, the more complete the hydrolysis.

Nonbiodegradable organic matter and mineral matter pass through the activated sludge system unchanged in forms. Inert soluble organic matter ( $s_i$ ) leaves the system at the same concentration that it enters. Inert suspended organic matter ( $x_i$ ) became enmeshed in the activated sludge and is removed from the system through sludge wastage. The same holds for mineral suspended solids ( $x_{min}$ ).

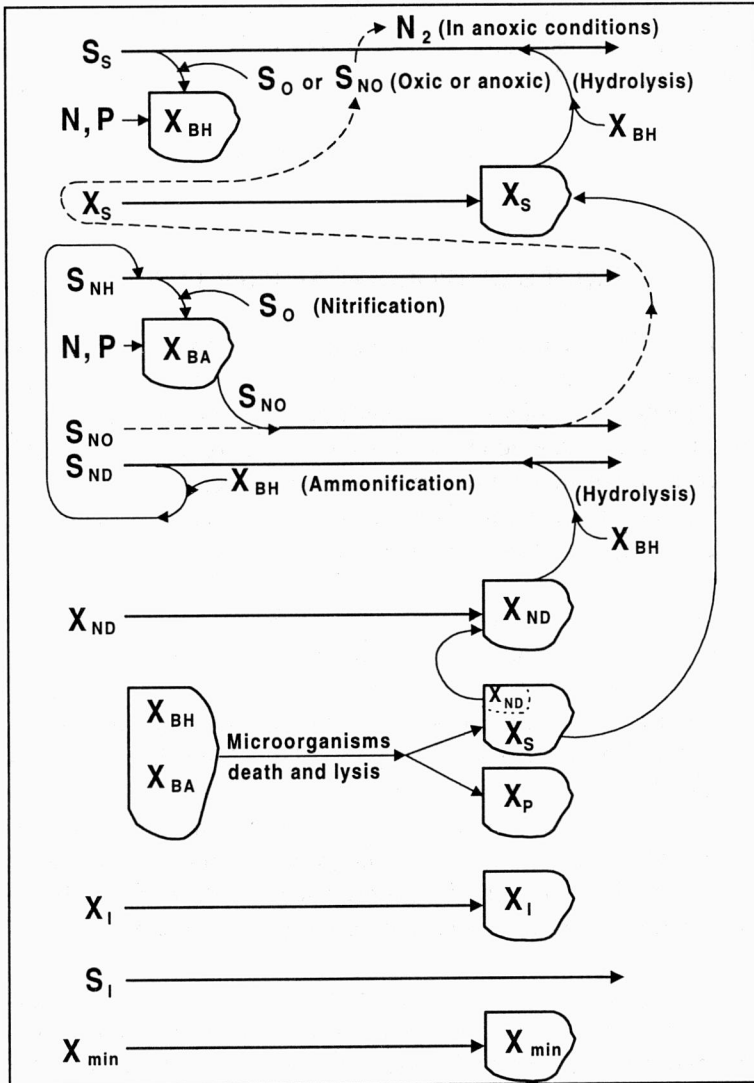


Fig. 3. Scheme of transformation of sewage pollutants in the activated sludge process



Under aerobic conditions ammonia nitrogen ( $s_{\text{NH}}$ ) serves as the energy source for autotrophic nitrifying bacteria ( $x_{\text{ba}}$ ). They oxidize ammonia nitrogen to nitrate nitrogen ( $s_{\text{NO}}$ ) (nitrification). The nitrate formed may serve as terminal electron acceptor for heterotrophic bacteria under anoxic conditions, yielding nitrogen gas. Usually nitrate nitrogen content in raw municipal wastewater is nearly zero.

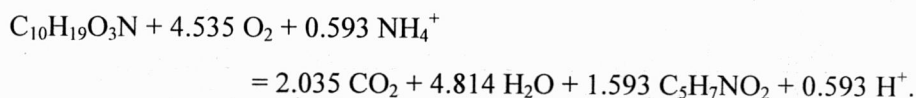
The soluble biodegradable organic nitrogen ( $s_{\text{ND}}$ ) is converted by heterotrophic bacteria to ammonia nitrogen (ammonification). The biodegradable particulate organic nitrogen ( $x_{\text{ND}}$ ) is enmeshed in the sludge flocs and hydrolysed by heterotrophic bacteria to soluble organic nitrogen ( $s_{\text{ND}}$ ) in parallel with hydrolysis of slowly biodegradable organics ( $x_{\text{S}}$ ).

Both heterotrophic and autotrophic bacteria decay, releasing slowly biodegradable matter ( $x_{\text{S}}$ ) which re-enters the cycle and is further hydrolysed; and unbiodegradable particulate organic matter ( $x_{\text{P}}$ ), which stays enmeshed in the sludge flocs, leaves the system with sludge wastage. The nitrogen present in the fraction  $x_{\text{S}}$  ( $x_{\text{ND}}$ ) undergoes hydrolysis in parallel. The more advanced the sludge age, the more decay products are released.

The activated sludge flocs are comprised of all the insoluble sewage components and the particulate products of the biomass decay. The flocs, which are carried over from the final clarifier, contribute to the total organic and nitrogen pollutants in the secondary effluent.

### 5.1. KINETICS OF THE AEROBIC REMOVAL OF ORGANIC CARBON

Under aerobic conditions the transformation of a typical domestic wastewater organic substrate, i.e.  $\text{C}_{10}\text{H}_{19}\text{O}_3\text{N}$ , can be described by the following reaction



The rate of this transformation depends on the availability of the organic substrate ( $s_{\text{S}}$ ), oxygen ( $s_{\text{O}}$ ) and ammonia ( $s_{\text{NH}}$ ), so the growth rate of the heterotrophs can be given by the following equation

$$\frac{dx_{\text{bh}}}{dt} = \mu_{\text{h}} \frac{s_{\text{S}}}{k_{\text{S}} + s_{\text{S}}} \frac{s_{\text{O}}}{k_{\text{ob}} + s_{\text{O}}} \frac{s_{\text{NH}}}{k_{\text{NH}} + s_{\text{NH}}} x_{\text{bh}},$$

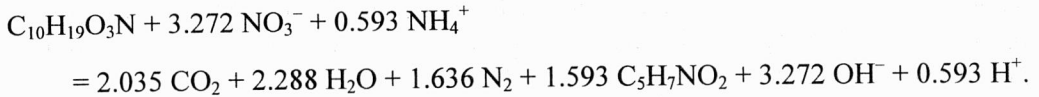
and the removal rate of the readily biodegradable substrate is expressed as

$$\frac{ds_{\text{S}}}{dt} = -\frac{\mu_{\text{h}}}{Y_{\text{h}}} \frac{s_{\text{S}}}{k_{\text{S}} + s_{\text{S}}} \frac{s_{\text{O}}}{k_{\text{ob}} + s_{\text{O}}} \frac{s_{\text{NH}}}{k_{\text{NH}} + s_{\text{NH}}} x_{\text{bh}}$$

where  $\mu_{\text{h}}$  is the maximum specific growth rate,  $Y_{\text{h}}$  is the growth yield coefficient, and  $k$  are the half-saturation constants for the respective substrates.

### 5.2. KINETICS OF THE ANOXIC REMOVAL OF ORGANIC CARBON (DENITRIFICATION)

Under anoxic conditions the heterotrophic bacteria transform nitrate into nitrogen gas. Anoxic conditions are obtained when high nitrate concentration coincide with lack of oxygen. Under these conditions heterotrophs use nitrate as the electron acceptor. For a typical wastewater composition the overall reaction is as follows



The growth rate of heterotrophic bacteria under these conditions can be described by the equation

$$\frac{dx_{\text{bh}}}{dt} = \eta_g \mu_h \frac{s_s}{k_s + s_s} \frac{s_{\text{NO}}}{k_{\text{N}} + s_{\text{NO}}} \frac{k_{\text{oh}}}{k_{\text{oh}} + s_o} x_{\text{bh}}$$

with the coefficient  $\eta_g < 1$  showing that the anoxic growth rate is lower than the aerobic one. The last limiting factor in the above equation is used to model the inhibitory effect of oxygen. The removal rate of readily biodegradable organics is

$$\frac{ds_s}{dt} = -\frac{1}{Y_h} \left( \frac{dx_{\text{bh}}}{dt} \right).$$

The removal rate of nitrates is given by

$$\frac{ds_{\text{NO}}}{dt} = -\frac{1 - Y_h}{2.86 Y_h} \mu_h \frac{s_s}{k_s + s_s} \frac{s_{\text{NO}}}{k_{\text{N}} + s_{\text{NO}}} \frac{s_{\text{NH}}}{k_{\text{N}} + s_{\text{NH}}} \frac{k_{\text{oh}}}{k_{\text{oh}} + s_o} x_{\text{bh}} \eta_g.$$

### 5.3. KINETICS OF THE HYDROLYSIS OF SLOWLY DEGRADABLE ORGANICS

Slowly degradable organics ( $x_s$ ) undergo hydrolysis both under aerobic and anoxic conditions. This process is rather slow. The rate of the hydrolysis is given by the Monod-like kinetic expression

$$\frac{dx_s}{dt} = -k_h \frac{x_s/x_{\text{bh}}}{k_x + x_s/x_{\text{bh}}} \left( \frac{s_o}{k_{\text{oh}} + s_o} + \eta_h \frac{k_{\text{oh}}}{k_{\text{oh}} + s_o} \frac{s_{\text{NO}}}{k_{\text{N}} + s_{\text{NO}}} \right) x_{\text{bh}}$$

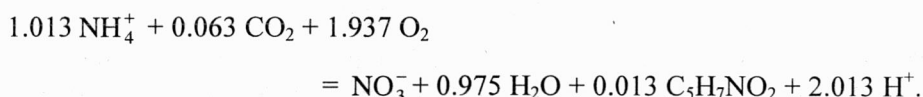
where  $k_h$  is the hydrolysis rate constant,  $k_x$  denotes the hydrolysis saturation coefficient, and  $\eta_h < 1$  indicates that the hydrolysis is slower under anoxic conditions.

## 5.4. KINETICS OF THE NITRIFICATION

Nitrification is a two-step process in which ammonia is transformed into nitrite and subsequently into nitrate



The yield coefficient for nitrifying (autotrophic) bacteria is significantly smaller compared to that of heterotrophic bacteria. Taking into account the assimilation process (building up the nitrogen in the biomass structure) and the yield coefficient, the following equation for both phases of nitrification is obtained



All three components on the left-hand side may be rate limiting, but in practice only the ammonia and oxygen concentrations impose limitations. Then the growth rate of the nitrifying bacteria is given by

$$\frac{dx_{\text{ba}}}{dt} = \mu_a \frac{s_{\text{NH}}}{k_{\text{NH}} + s_{\text{NH}}} \frac{s_{\text{o}}}{k_{\text{oa}} + s_{\text{o}}} x_{\text{ba}}$$

where  $x_{\text{ba}}$  is the concentration of autotrophic bacteria mass, and  $Y_a$  is the yield coefficient. The formation of nitrates is expressed by the rate

$$\frac{dx_{\text{ba}}}{dt} = -\frac{\mu_a}{Y_a} \frac{s_{\text{NH}}}{k_{\text{NH}} + s_{\text{NH}}} \frac{s_{\text{o}}}{k_{\text{oa}} + s_{\text{o}}} x_{\text{ba}},$$

and the ammonia nitrogen consumption by

$$\frac{ds_{\text{NH}}}{dt} = -\left(\frac{1}{Y_a} + i_{\text{xb}}\right) \mu_a \frac{s_{\text{NH}}}{k_{\text{NH}} + s_{\text{NH}}} \frac{s_{\text{o}}}{k_{\text{oa}} + s_{\text{o}}} x_{\text{ba}}$$

where  $i_{\text{xb}}$  is the proportion of nitrogen in the biomass.

## 5.5. KINETICS OF THE BIOMASS DECAY

Biomass (both of heterotrophic bacteria  $x_{\text{h}}$  and autotrophic bacteria  $x_{\text{a}}$ ) is lost by decay due to endogenous metabolism, death, predation and lysis. This way the active biomass is transformed into slowly biodegradable substrate ( $x_{\text{s}}$ ) and inert fraction ( $x_{\text{p}}$ ). The decay of biomass may be described as a first order equation

$$\frac{dx_{\text{b}}}{dt} = -bx_{\text{b}}.$$

The resulting rates of slowly biodegradable organics and the inert fraction production are given respectively by

$$\frac{dx_s}{dt} = (1 - f_p)bx_b$$

and

$$\frac{dx_p}{dt} = f_p bx_b$$

where  $b$  is the decay rate coefficient, and  $f_p$  the inert fraction of the decay products.

#### 5.6. MODEL OF AERATION BASINS

The aeration basins were modelled as ideally mixed reactors. The complete model incorporates the balance equations for each component undergoing transformations in the aeration tanks, with inputs from the outlet of the primary clarifiers and the recirculation stream of the secondary clarifiers. The model includes the following processes:

- aerobic growth of heterotrophs with assimilation of nitrogen from  $\text{NH}_4^+$ ,
- aerobic growth of heterotrophs with assimilation of nitrogen from  $\text{NO}_3^-$ ,
- anoxic growth of heterotrophs with assimilation of nitrogen from  $\text{NH}_4^+$ ,
- anoxic growth of heterotrophs with assimilation of nitrogen from  $\text{NO}_3^-$ ,
- aerobic growth of autotrophs,
- decay of heterotrophs,
- decay of autotrophs,
- ammonification of dissolved organic nitrogen,
- hydrolysis of slowly degradable organics

and the following components (state variables):

- readily biodegradable organics,
- slowly biodegradable organics,
- dissolved inert organics,
- particulate inert organics,
- heterotrophic bacteria,
- autotrophic bacteria,
- inert organics from biomass decay,
- ammonia,
- nitrates,
- dissolved organic nitrogen,
- particulate organic nitrogen,
- dissolved oxygen,

- pH,
- mineral suspended solids.

## 6. MODEL OF SECONDARY CLARIFIERS

The influent entering the secondary clarifiers consists basically of flocculated bacteria, nonviable particulate components (both organic and mineral) and water. The most important phenomenon in the secondary clarifiers is that of separation of these two components in the clarification and thickening processes. Only these processes are taken into account in the model. It is assumed that no biological activity takes place in the clarifier. The secondary clarifier consists of two zones shown in figure 4, i.e., the clarification zone and the thickening zone. There are two outlets from the clarifier. The effluent overflows the upper edge of the clarifier, while the sludge leaves the clarifier at the bottom and is being recirculated to the bioreactors.

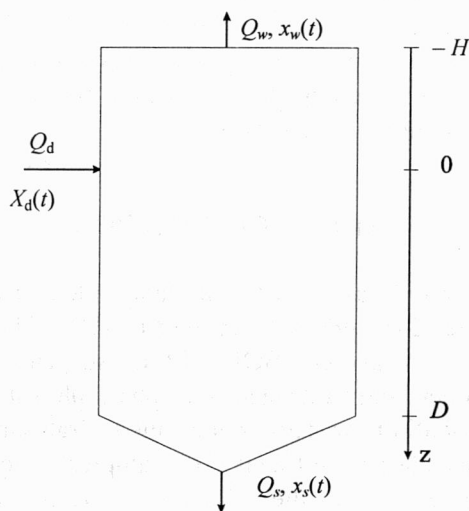


Fig. 4. Secondary clarifier

There are two currents connected with the outflows: the effluent current with the velocity  $w$ , predominant in the clarification (upper) zone, and the sludge current with the velocity  $v$ , predominant in the thickening (lower) zone. Additionally, the gravitational force acting on solids enforces their movement downwards. Its velocity  $u$  depends exponentially on the concentration of suspended solids  $x$

$$u(x) = u_0 e^{-bx},$$

see [8]. Note that  $x$  is a function of both depth  $z$  and time  $t$ . The flux  $q(x)$  of the suspended solids is then described by the following function (compare figure 4)

$$q(x) = \begin{cases} x[u(x) + v] & \text{for } 0 < z \leq D, \\ x[u(x) - w] & \text{for } -H \leq z < 0. \end{cases}$$

Now, the continuity equation gives

$$\frac{\partial x}{\partial t} + \frac{\partial q}{\partial z} = 0$$

valid with the initial condition

$$x(z, 0) = x_0(z), \quad -H \leq z \leq D$$

and the boundary (mass conservation at the inlet) condition

$$Q_d x_d(t) = P_0 x(0, t) [2u(x(0, t)) + v - w]$$

where  $x_d$  is the concentration of suspended solids in the influent,  $Q_d$  is the influent flow rate,  $P_0$  is the cross-section of the clarifier at  $z = 0$ . To solve the above equations we have to divide the clarifier into 12 conceptual horizontal layers.

The model for the dissolved components lacks the flux  $q(x)$  and actually reduces to an ideally mixed tank model for each layer.

## 7. MODEL CALIBRATION

The model was calibrated based on to the data collected during two-week sampling period from October 23, 1995 to November 6, 1995. The concentrations of the following pollutants were measured: BOD<sub>5</sub>, COD, suspended solids (organic and mineral), ammonia nitrogen, total nitrogen, phosphate phosphorus, total phosphorus and dissolved oxygen. Alkalinity was also determined. Grab samples were taken every 2 hours from the primary clarifier influent (168 samples), and the flow proportional 24-hour composite samples – from the effluents of the primary clarifiers, aeration basins and secondary clarifiers. Flow rates were recorded every 2 minutes.

The influent concentrations and flow rates were averaged for every sampling hour over all the sampling period to get an averaged diurnal input variations to the model. The averaged diurnal input variations were normalised by dividing them by their respective daily means. As an illustrative example, the results for the flow rate, organics and nitrogen are shown in figures 5 and 6.

The model calibration was started with "typical" values of the model unknown parameters taken from the literature [9]–[12]. The model was run when fed with the averaged diurnal inputs. The model predicted 24-hour concentrations of composite

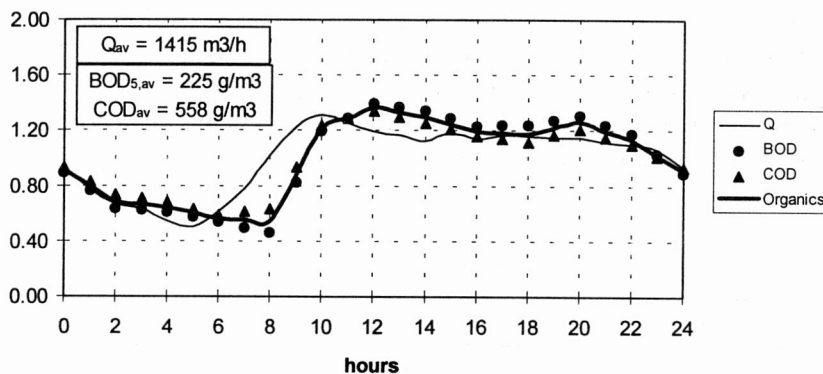


Fig. 5. Influent normalised diurnal variations of organics and flow rate

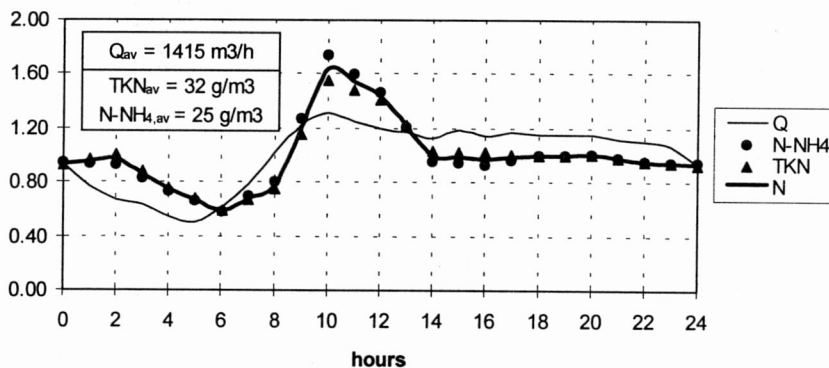


Fig. 6. Influent normalised diurnal variations of nitrogen and flow rate

Table 4

Calibration results and model fit for the primary clarifiers

Model parameter	Unit	Calibrated	Literature [7]	
<i>A</i>	$\text{g/m}^3\text{d}$	$1.2 \cdot 10^{-6}$	$1.2 \cdot 10^{-6}$	
<i>B</i>		3.76	4.05	
Pollutant	Unit	Measured	Predicted	Error (%)
SS	$\text{g/m}^3$	163	162	0.6
Mineral SS	$\text{g/m}^3$	60	47	22
BOD <sub>5</sub>	$\text{g/m}^3$	186	188	1
COD	$\text{g/m}^3$	448	451	0.7
N-NH <sub>4</sub>	$\text{g N/m}^3$	29.3	26.3	10
TKN	$\text{g N/m}^3$	39.3	32.0	19
Alkalinity	$\text{val/m}^3$	8.3	7.9	5

sample which were compared with the respective averages of the measured values. Some or all parameter values were manipulated to get a satisfactory fit. The results of the calibration are given in table 4 for the model of primary clarifiers, in tables 5 and 6 for the model of activated sludge and in table 7 for the model of secondary clarifier. Full details can be found elsewhere [2].

Table 5

## Calibration results for the model of activated sludge

Model parameter	Unit	Calibrated	Literature [9]–[11]
$\mu_h$	$d^{-1}$	2.20	1.5–8
$Y_h$	g COD/g COD	0.67	0.67
$k_s$	g COD/m <sup>3</sup>	10	5–30
$k_{oh}$	g O <sub>2</sub> /m <sup>3</sup>	0.1	0.1
$k_x$	g COD/g COD	0.03	0.02–0.05
$k_h$	g COD/g COD d	3.0	0.6–2.2
$\mu_a$	$d^{-1}$	0.5	0.2–0.8
$Y_a$	g COD/ g N	0.15	0.15
$k_{NH}$	g N/m <sup>3</sup>	1.0	1.0
$k_{oa}$	g O <sub>2</sub> /m <sup>3</sup>	0.5	0.5–1.0
$f_p$	–	0.08	0.08
$b_h$	$d^{-1}$	0.62	0.62
$b_a$	$d^{-1}$	0.05	0.05
$i_{xb}$	g N/g COD	0.086	0.086
$i_{xp}$	g N/g COD	0.06	0.06

Table 6

## Fit of the model of activated sludge

Parameter	Unit	Measured	Predicted	Error (%)
Biomass	g dw/m <sup>3</sup>	3815	3890	2
Sludge age	d	10.2	10.0	2
BOD <sub>5</sub>	g O <sub>2</sub> /m <sup>3</sup>	18	18.3	2
COD	g O <sub>2</sub> /m <sup>3</sup>	53	53.6	1
N-NH <sub>4</sub>	g N/m <sup>3</sup>	26.3	24.2	8
TKN	g N/m <sup>3</sup>	32.1	26.1	12
Alkalinity	val/m <sup>3</sup>	7.9	7.7	3

Table 7

## Calibration results and model fit for the secondary clarifiers

Model parameter	Unit	Calibrated	Literature [9]
$u_0$	m/d	187.2	187.2
$b$	m <sup>3</sup> /g	$6.23 \cdot 10^{-4}$	$6.23 \cdot 10^{-4}$
$p_f$	h/m	0.00322	0.01088



Continuation of table 7

Parameter	Unit	Measured	Predicted	Error (%)
Effluent SS	g/m <sup>3</sup>	19	19	0
Effluent mineral SS	g/m <sup>3</sup>	14	7	50
BOD <sub>5</sub>	g/m <sup>3</sup>	19	23	21
COD	g/m <sup>3</sup>	55	81	47
N-NH <sub>4</sub>	g N/m <sup>3</sup>	24.9	24.2	3
TKN	g N/m <sup>3</sup>	28.1	26.9	4
Alkalinity	val/m <sup>3</sup>	8.1	7.7	5
Recirculated SS	g/m <sup>3</sup>	5719	6090	6

The fit of the model is satisfactory, in many cases very good, of the order of few percents. Only in some cases the errors are bigger. The errors in nitrogen fit seem to be caused by bad operation of the primary clarifiers during the measurement period. The worst fit of the effluent suspended solids and organics for the secondary clarifiers is attributed to an unknown error source which is believed to be related to the measurement procedures.

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### MODEL MATEMATYCZNY I JEGO ESTYMACJA DLA OCZYSZCZALNI ŚCIEKÓW W RZESZOWIE

Przedstawiono model matematyczny oczyszczalni z osadem czynnym dla Rzeszowa. Model składa się z submodeli osadników wstępnych, komór napowietrzania i osadników wtórnych. Uwzględniono w nim odpowiednie równania opisujące fizyczne i biochemiczne transformacje: sedymentację i zagęszczanie w osadnikach oraz biologiczne transformacje składników dopływających ścieków w komorach napowietrzania. Przeprowadzono dwustopniową estymację parametrów modelu. Najpierw, na podstawie wyników czynnego eksperymentu znacznikowego, określono objętości czynne obiektów. Następnie oszacowano wartości parametrów kinetycznych i stechiometrycznych na podstawie danych zebranych podczas normalnej pracy oczyszczalni.