# Effect of two-photon absorption on the picosecond pulse generation in a mode-locking laser

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Basing on the fluctuation model of the laser with mode-locking, an analysis of the influence of the parameters of two-photon absorbent (located in a resonator together with the amplifying medium and the nonlinear single-photon ansorbent) on the generated pulse characteristics has been carried out. It has been shown that — due to the application of the two-photon saturating absorbent the intensity and contrast may be considerably increased and the picosecond pulse duration reduced. Some approximate analytic criteria of the effective picosecond-pulse generation in laser with single- and two-photon absorbent are defined.

#### Introduction

The process of picosecond-pulse generation by the method of passive mode-locking in laser with single-photon absorbent was analysed an dexamined by many authors (see, for instance, [1-3] and the literature cited there), and the fundamental features of this process are pretty well known at the moment. One of the basic shortcomings of the mode-locking at the presence of the nonlinear absorbent is the statistical nature of the generation process resulting in the poorer repeativity of the pulse parameters than it is the case for the active mode-locking (for instance, acoustooptic modelocking). On the other hand, the passive mode-locking is simpler in the technical realization and allows to generate considerably shorther pulses. However, also in the case of the passive mode-locking, the minimal length of the generated pulses is few to several times greater than the limiting length (equal to the reciprocal of the luminescence line width of the amplifying medium), while their contrast not always meets the requirements needed in many applications of picosecond pulses. Therefore, the attention of many scientific workers is drawn to looking for methods allowing to obtain the extreme of these pulses (like power, contrast and length) parameters as well as for the methods of increasing the stability of the generation process. A possibility of using two-photon absorption for these purposes was indicated in the works [4-8].

In [4] an analysis of an ultra-short pulse evolution was given for the laser containing the autosaturating two-photon absorbent (in addition to the single-photon absorbent). It has been shown that there exists the possibility of an additional shortening of the pulse duration as well as diminishing, under some conditions, the influence of the initial distribution on its length. The case of two-photon absorbent with long relaxation-time was analysed in details starting with unjustified, in reality, premisses about the impossibility of pulse shortening when using the absorbent of short relaxation-time. The evolution of the single pulse in resonator was analysed putting

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aside the influence of two-photon absorption on the contrast of the generated pulse. Also in [6] an analysis of the influence of two-photon absorption on the picosecond pulse generation was given and a possibility of an improvement of its parameters indicated.

In that work no detailed results of calculations were presented. In [7, 8] the general criteria for pulse compression due to two-photon absorption were defined. It has been shown that the pulse compression may occur in the case of an absorbent with both long and short relaxation time. An experimental confirmation of the possibility of an additional shortening of the picosecond pulse as a result of the presence of the two-photon absorbent in the laser resonantor with the mode-locking was shown in [5, 6].

In the present paper the generation process in the laser with single- and two-photon absorbent is analysed with special attention being payed to the influence of absorbent parameters upon the generated pulse characteristics. It has been shown that there exists a possibility of considerable enhancement of intensity and contrast, while the picosecond-pulse duration may be shortened as a result of using the two-photon absorbent of both long and short relaxation time. Some approximate analytic criteria of effective picosecond pulse generation in the analysed system are proposed. The considerations were carried out on the basis of the fluctuation model of the laser with mode-locking by taking account of the statistic character of the initial distribution of the radiation.

## Laser model

The analysis of the effect of two-photon absorption on the picosecond pulse generation will be carried out on the basis of the fluctuation model of laser with the modelocking [1, 2], which at the present moment describes in the most correct way the process of generation of such pulses in the systems with solid amplifying medium. For shortening the description we will denote by  $G_1A_1$  the system containing an amplifying medium and a single-photon absorbent in the resonator, and by  $G_1A_1A_2$ —a system containing additionally the two-photon absorbent.

The laser action in the generator with the mode-locking begins at the moment of including the optical (or other) pump exciting the atoms of the active medium. The excited atoms positioned in the upper lasering level radiate spontaneously in the spectral range corresponding to their luminescence line. As long as the population inversion in the active material is less than the threshold inversion, the number of the photons emitted spontaneously in the generation channel is considerably greater than the number of photons created due to stimulated transitions and by the same means the time and spatial distribution of the photons in resonator is, at this stage, determined by the properties of the spontaneous emission. Since the spontaneous emission is, by nature, a statistical process, the radiation intensity fluctuates with the

characteristic correlation time  $\tau_0 = \frac{2\pi}{\Delta\omega_{\rm um}}$  ( $\Delta\omega_{\rm lum}$  – the luminescence line width).

As the population inversion approaches the threshold value, the number of stimulated photons increases and when it begins to exceed the number of spontaneous photons there appears a periodic correlation of fields at the moments t, and  $t+T_{\rm res}$ , where  $T_{\rm res}$  — the time of the complete round trip of the radiation in the reconator. When expressed in term of spectral quantities this corresponds to forming (due to the resonator operation) of an ensamble of longitudinal modes of frequencies

$$\omega_k = \omega_0 + k \frac{2\pi}{T_{\text{res}}}$$
, lying within the limits of the luminescence line.

The fluctuation structure of the radiation at this stage is characterized by two characteristic times:  $\tau_0$  and  $T_{\rm res}$ . The probability distribution for the field amplitude is described by the Rayleigh law (see below).

When the population inversion exceeds the threshold value the process of linear amplification begins in the resonator — the intensity and field energy being at this stage too small to evoke the nonlinear effects both in absorbent and the amplifying medium. Due to the dependence of the amplification coefficient upon the frequency, the modes lying close to the centrum luminescence line are amplified more strongly than the extreme modes, which results in narrowing of the radiation spectrum and, by the same means, in increasing of the characteristic correlation time. The process of linear amplification lasts until the radiation intensity reaches the value close to that needed for the absorption saturation. Since the radiation intensity at the generation threshold is less by many orders of magnitude than the saturation intensity, the process of linear amplification lasts long enough to allow an essential reduction of the generated spectrum width. Due to the linearity of the amplification, the time distribution of the radiation at this stage of the process has still a fluctuation structure.

Thus, at the moment  $t_1$  of the beginning of the third phase of generation, the phase of the nonlinear interaction with the medium, the radiation possesses a quasi-periodic noise structure with shorther correlation time  $\tau_1$  equal to [2]:

$$au_1 = rac{2\pi}{\Delta\omega_1} pprox rac{T_{
m res}}{m_0} \sqrt{v \mu(\varkappa + \varrho) \Delta t_l},$$

where  $m_0 = \frac{\Delta \omega_{\text{lum}} T_{\text{res}}}{2\pi}$  — initial number of modes,  $\nu$  — light velocity in the

resonator,  $\varkappa$ ,  $\varrho$  — initial coefficients of single-photon and the nonresonant losses for the length unity in the amplifying material,  $\Delta t_l = t_1 - t_{\rm th}$  — duration of the linear amplification phase,  $\mu$  — filling coefficient of the resonator.

The number of the modes at the moment  $t_1$  and the corresponding number of the intensity maxima in the period  $T_{res}$  is:

$$m_1 = \frac{T_{\rm res}}{\tau_1}.$$

The probability distribution for the radiation intensity at the beginning moment of nonlinear generation is given by the expression [2]:

$$w(I)dI = \frac{1}{\langle I \rangle} \exp \left[ -\frac{I}{\langle I \rangle} \right] dI,$$

where  $\langle I \rangle$  — average intensity. If the set  $m_1$  of nonoverlapping pulses of effective lengths  $\tau_1$  and randomly distributed amplitudes is chosen as an initial ditribution of intensity within the interval  $T_{\rm res}$ , then the probability distribution for a pulse of the highest amplitude has the form [2]:

$$w_1(x)dx = m_1 e^{-x} [1-e^{-x}]^{m_1-1} dx$$

where x — the pulse amplitude normed to the average values. The magnitude  $w_1(x)$  reaches its maximum at the point  $x_1 = \ln m_1$ , which means that  $x_1$  is the most probable value of the maximal pulse amplitude. For the second greatest pulse we have, in turn,

$$w_2(x)dx = m_1(m_1-1)e^{-2x}[1-e^{-x}]^{m_1-2}dx$$

with the maximum at the point  $x_2 = \ln \frac{m_1}{2}$ , and so on. We may, thus, determine

the most probable relative amplitudes and the durations of highest pulses within the interval  $T_{res}$ , which enables the exact analysis of nonlinear stage of generation.

Next the value of the amplification coefficient at the starting moment of this stage has to be determined. If the characteristic time of pumping  $T_{\text{pump}}$  is much longer than the time of linear generation stage  $\Delta t_l$  (which usually occurs in the solid lasers), the value of the amplification coefficient at the moment  $t_1$  may be presented in the form [2]

$$a(t_1) = a_{th} + \Delta \alpha$$
,

where

$$a_{\rm th} = \varkappa + \varrho$$
,

$$\Delta a \sim \frac{\Delta t_l}{T_{\rm pump}} \; a_{\rm th} \sqrt{p(p-1)} \; , \label{eq:delta_a}$$

and p is the ratio of the maximal pump power to the threshold power. Since in the solid laser  $T_{\text{pump}} \sim 10^{-4} - 10^{-3} \text{s}$ ,  $\Delta t_l \sim 10^{-6} - 10^{-5} \text{s}$ , then usually  $\Delta a \sim 10^{-2} a_{\text{th}}$ .

The stages of the generation development described above are identical both in  $G_1A_1$  and in  $G_1A_1A_2$  systems, since the noticeable influence of two-photon absorption on the generation process may occur only for radiation intensities exceeding considerably those appearing near the threshold. The course of the nonlinear phase of generation depends essentially both on the macroscopic and molecular parameters of the system and may be, of course, highly different in the  $G_1A_1$  and  $G_1A_1A_2$  systems. Due to discriminating properties of a two-component systems with the sufficiently short relaxation time of the single-photon absorption  $T_{11}$  (shorter than  $\tau_1$ ) the most intensive fluctuations in this system became separated from the noisy structure, shortened and amplified. Consequently, the laser generates a periodic sequence of single pulses or groups of, at most, several pulses considerably exceeding the background intensity. The case when only one pulse occurs during the period  $T_{res}$  corresponds to the so-called complete mode-locking in the spectral terms. The forming of the described structure is the most essential moment in the laser operation with

the mode-locking. The influence of different parameters of the  $G_1A_1$  system on this structure was analyzed and verified experimentally by many authors (see [1-3]) and therefore we will examine below only the influence of parameters of the two-photon absorbent for the case of nonliner generation stage.

By accepting the model of the travelling wave generator and assuming that the changes in radiation parameters caused by resonator are not too great, and that  $T_{21} \ll \tau_p \ll T_{11}$  ( $T_{11}$  — the inversion relaxation time,  $\tau_p$  — pulse length), the changes in intensity after the successive passages through the resonator in the nonlinear stage of generation may be described by the equation (for instance, [9]):

$$R_{N+1}(\tau') = R_{N}(\tau') + R_{N}(\tau') \left\{ a \exp\left[ -\eta \sum_{i=0}^{N-1} \int_{\tau_{a}}^{\tau_{b}} R_{i}(\tau') d\tau' - \eta \int_{\tau_{a}}^{\tau_{b}} R_{N}(\tau'') d\tau'' \right] - \frac{\varkappa'}{1 + R_{N}(\tau)''} - \gamma'_{s} F[R_{N}(\tau')] - \varrho \right\}, \tag{1}$$

where  $\tau'=\frac{\tau}{\tau_1}$ ,  $R_N=\frac{I_N}{I_{21}^s}$ ,  $\alpha_1'=aL$ ,  $\varkappa'=\sigma_{21}N_{21}^eI_1$ ,  $\gamma_s'=2b\sigma_{12}N_{12}^eI_{21}^sI_2$ ,  $\varrho'=\varrho L$ , L— the amplifying layer thickness,  $l_1$ ,  $l_2$ — single-and two-photon absorbent layer thicknesses, respectively,  $\sigma_{21}$ ,  $\sigma_{12}$ — the cross-sections for the single-and two-photon absorptions, respectively,  $\varrho$ — coefficient of the total losses in the resonator  $N_{21}^e$ ,  $N_{12}^e$ — the initial differences in the population density in absorbents,  $F[R_N]$ — the function describing the dependence of the two-photon absorption upon the intensity,  $\eta=\frac{I_{21}^s\tau_1}{\varepsilon_{11}^s}$ ,  $\tau_b-\tau_a=\frac{T_{\rm res}}{\tau_1}$ ,  $I_{21}^s$ —intensity of single-photon absorption saturation,  $\varepsilon_{11}^s\hbar\omega$ —energy of amplification saturation.

Below we will discuss two cases of the dependence  $F(R_N)$ : the case of two-photon absorbent with the short relaxation time  $(T_{12} \ll \tau_p)$  in which [7]:

$$F(R_N) = \frac{R_N}{1 + A^2 R_N^2},\tag{2}$$

and the case of two-photon absorbent with long relaxation time ( $\tau_p \ll T_{12} < T_{\rm res}$ ) for which [7]:

$$F(R_N) = R_N \exp\left[-B^2 \int_{\tau_0}^{\tau'} R_N^2 d\tau''\right]. \tag{3}$$

In these expressions:  $A=b\frac{I_{21}^s}{I_{12}^s}$ ,  $B=b\sqrt{s_{12}\sigma_{12}\tau_1}\,I_{21}^s\,I_{12}^s$  — two-photon absorption saturation intensity,  $1\leqslant s_{12}\leqslant 2$  [7], and b is the ratio of the beam cross-section areas in single-photon and two-photon absorbents, respectively.

The intensity distribution at the moment of nonlinear generation initiation will be assumed in the following form\*

$$R_0(\tau) = \sum_{k=1}^{m_1} R_k \exp\left(-2\frac{\tau - a_k}{\tau_1}\right)^2,$$
 (4)

where  $R_k = C \ln \frac{m_1}{k}$ ,  $C \le 1$ . For the analysis of the generator with the single-photon absorbent of short relaxation time it is possible to restrict the attention to the evolution of several highest level pulses in the distribution, since due to strongly discriminating properties of this absorbent the influence of the remaining pulses on the saturation of both the amplification and two-photon absorption is usually not very strong.

### The results of numerical calculation and discussion

We now start to discuss the results of numerical analysis for nonlinear stage of generation in the  $G_1A_1A_2$  system carried out on the base of the equation (1). In our calculations the following parameters typical of the neodymium-glass laser have been assumed as constants:  $\alpha'_{th} = 1.4$ ,  $\varrho' = 0.4$ ,  $\kappa' = 1$ ,  $\Delta \alpha' = 0.01 \alpha'_{th}$ ,  $\eta = 10^{-3}$ ,

$$T_{\text{res}} = 5 \cdot 10^{-9} \text{ s}, \ \Delta t_l = 10^{-5} \text{ s}, \ \nu = 2.5 \cdot 10^{10} \ \frac{\text{cm}}{\text{s}}, \ m_0 = 5 \cdot 10^3, \ \mu = 0.2, \ C = 0.01.$$

The parameters of the two-photon absorbent  $\gamma'_s$ , A, B have been changed. In the distribution (4) the discussion has been limited to the three highest pulses; the majority of calculations (figs. 1-7) were carried out for the configuration in which the maximum pulse is located at the mid-point between two others.

The influence of the parameter  $\gamma_s'$  on the parameters of the pulses generated in the  $G_1A_1A_2$  system in two-photon absorbent of short relaxation time is shown in figs. 1, and 2. In this figure (as well as in the others) the magnitudes  $I_2$ ,  $\tau_{p_2}$  denote top intensity and the half-width of the maximum pulse, respectively, in the train of pulses generated in the  $G_1A_1A_2$  system, while  $I_1$ ,  $\tau_{p1}$  — denote the same quantities obtained in the  $G_1A_1$  system. The quantities  $k_2$  and  $k_1$  determine the contrast of the maximal pulse in the  $G_1A_1A_2$ , and  $G_1A_1$  systems, respectively, understood as the ratio of the top pulse intensity in the interval  $T_{\rm res}$  to that of the second top pulse in this interval. From the figs. 1 and 2 it may be seen, first of all, that the presence of the two-photon absorbent in the system leads to an essential increase in the top intensity, a reduction of the pulse length and an increase of the contrast of the generated pulses. This positive influence of the two-photon absorption occurs at the values  $\gamma_s'$  less than certain critical value for which the generation process of the isolated picosecond pulse sequence is broken. There exists an optimal value  $\gamma_s'$  lying close to the critical value,

<sup>\*</sup> The concrete form of the function  $R_0(\tau)$  is not essential here. For our purposes the relation between the amplitude  $R_k$  of the highest pulses is important above all.

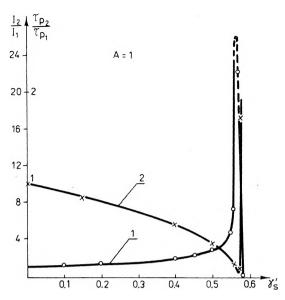


Fig. 1. The dependence of the relative intensity (1) and duration (2) of a picosecond pulse upon the effective coefficient of two-photon absorption

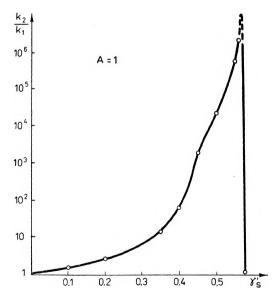


Fig. 2. The dependence of the relative contrast of the picosecond pulse upon the effective coefficient of two-photon absorption

for which the top intensity and the contrast of the generated pulses are the greatest, while their length is the least one. In the region of the values  $\gamma'_s$  "safe" from the viewpoint of the mode-locking process stability the intensity increases several times, while the length of the generated pulses is reduced also several times and their contrast increases by two or three orders of magnitude. An analogical influence of the

value  $\gamma'_s$  on the parameters of generated pulses takes place also in the case of two-photon absorbent with the long relaxation time.

The influence of the saturation parameter A on the response of the generated pulses in the system with short relaxation time for two-photon absorption is shown in figs. 3 and 4. Also there exists an optimal value A for which the pulse parameters

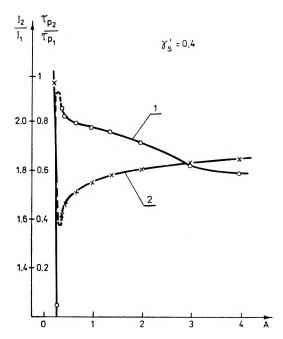


Fig. 3. The relative intensity (1) and duration (2) of a picosecond pulse vs. the saturation parameter of two-photon absorbent of short relaxation time

are the most advantageous. In the analysed example  $A_{\rm opt} < 1$ , which means that the process of single pulse separation from the fluctuation structure and its amplification and compression occur most effectively if the two-photon absorption saturation appears later than that for single-photon absorption. As it follows from the calculations the optimal value of A increases with the increase of  $\gamma_s'$ . The critical value of A exists below which the effective mode-locking is not possible either. The critical value of A is the greater the greater the parameter  $\gamma_s'$ .

The figures 5 and 6 illustrate the influence of the saturation parameter B on the generation characteristics in the system with two-photon absorbent of long relaxation time. One of the differences between this case and the case discussed above is that the value B, optimal for the length and intensity of the generated pulses, differs essentially from the optimal value due to their contrast. Besides, the contrast in this system is considerably less than that the contrast in the system with the two-photon absorbent of short relaxation time (but greater than in  $G_1 A_1$  system).

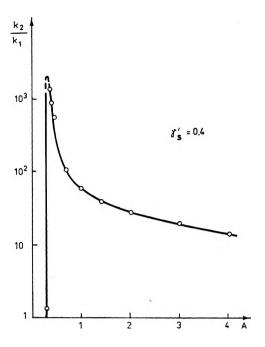


Fig. 4. The relative contrast of picosesond pulse vs. the saturation parameter A

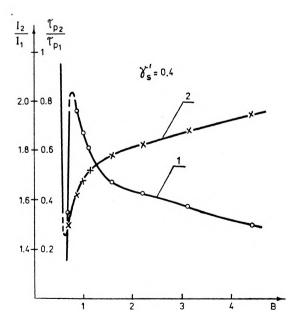


Fig. 5. The relation of the relative intensity (1) and duration (2) of the picosecond pulse upon the saturation parameter of the two-photon absorbent with long relaxation time

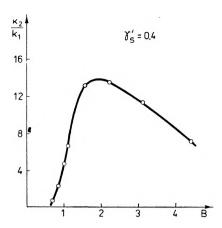


Fig. 6. Dependence of the relative contrast of the picosecond pulse upon the saturation parameters

The presence of the two-photon absorption in the system affects weakly the effective length and shape of the envelope of the train of the generated pulses. The figure 7 shows schematically the train of the generated pulses in the system  $G_1A_1$  (fig. 7a), and in the system  $G_1A_1A_2$  with short (fig. 7b) and long (fig. 7c) relaxation time  $T_{12}$ .

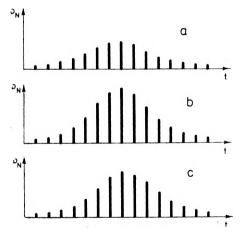


Fig. 7. The train of picosecond pulses: a - in the  $G_1A_1$  system, b - in the  $G_1A_1A_2$  system with a short relaxation time  $T_{12}$ , c - in the  $G_1A_1A_2$  system with long relaxation time  $T_{12}$ 

The generation of radiation in the system  $G_1 A_1 A_2$  is, in general, of nonstationary nature. Therefore, not only the ratio of fluctuation amplitudes at the starting moment of the nonlinear stage but also their ordering have the influence on the final parameters of the generated pulses.

The parameters of the maximal pulse obtained for different configurations of the "initial" distribution and related to the parameters of the same pulse in the configuration "a" in the system  $G_1A_1$  are presented in fig. 8. In the configuration "a" the highest fluctuation is the middle point of the distributions (between two others).

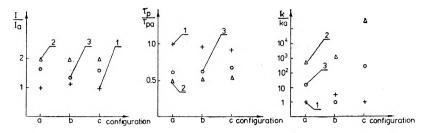


Fig. 8. Dependence of the picosecond pulse parameters upon the configuration of the initial intensity distribution:  $1 - \text{in the } G_1 A_1 \text{ system}, 2 - \text{in the } G_1 A_1 A_2 \text{ system with short relaxation time } T_{12},$   $3 - \text{in the } G_1 A_1 A_2 \text{ system with long relaxation time}$ 

in the configuration "b" — at the beginning of the distribution, and in the configuration "c" — at the end of the distribution. It may be seen that the effect of ordering on the top pulse intensity is the least in the case of  $G_1A_1A_2$  with the two-photon absorbent of short relaxation time (the points denoted by the number 2). The relative changes in pulse length caused by the change in configuration are close to each other and relatively small. On the other hand, the contrast of the generated pulses suffers from strong changes, especially in the  $G_1A_1A_2$  system (the points denoted by numbers 2, and 3). It has to be emphasized that, in spite of such strong dependence of the contrast upon the configuration in the three-component system, the conditions to obtain the complete mode-locking in this system are much more advantageous than in the  $G_1A_2$  system due to the higher average value of the contrast. The probability of obtaining a train of pulses with no side satellites is particularly high in the system  $G_1A_1A_2$  with short time  $T_{12}$ , since the minimal contrast in this system of parameters close to optimal one is by two or three orders of magnitude greater than in the  $G_1A_1$  system.

The final parameters of the generated pulses are also influenced by the time duration of the highest fluctuation at the moment of initiating the nonlinear stage of generation. The dispersion of the average length may be caused by many factors and especially by the time speed  $\Delta t_l$  resulting, for instance, from instability of the pumping system. In case of  $G_1A_1A_2$  system with the short  $T_{12}$  time the influence of the dispersion of the "initial" fuctuation lengths on the relative pulse parameters (referred to the parameters in the  $G_1A_1$  system) should — for the values of  $\gamma'_s$  and A not too close to the critical values — be small due to the smallness of the parameter  $\eta$ . However, in the three-component system of long time  $T_{12}$  the influence of this spread is more essential, because the value of  $\tau_1$  decides about the value of parameter B, which affects the final parameter of pulses stronger than the parameter  $\eta$ .

Summing up the results of the calculations carried out, it may be stated that:

- a. In the system  $G_1 A_1 A_2$  it is possible to obtain the pulses of higher top intensity and contrast and smaller duration than those in the  $G_1 A_1$  system.
- b. There exist optimal values of parameters  $\gamma'_s$ , A, and B for which the top intensity and contrast of the generated pulses are the greatest and their lengths the smallest. In the three-component system with long time  $T_{12}$  the values, which opti-

mize the contrast, are not identical with the values optimal with respect to the length and the top intensity of pulses.

- c. There exist critical values of parameters  $\gamma'_s$ , A, and B above which the effectivities of separation, shortening and amplification of the most intensive fluctuations from the noisy structure are radically reduced (breaking of the mode-locking process).
- d. The most stable generation of a train of single isolated pulses may be obtained in a  $G_1 A_1 A_2$  system with the short time  $T_{12}$ .
- e. For the values of parameters  $\gamma'_s$ , A, and B being not two close to the critical value the presence of the two-photon absorption weakly affects the effective duration and the shape of the envelope of the generated pulses train.

# The criteria of mode-locking in the $G_1A_1A_2$ system

As shown above the generation of the picosecond pulse (the realization of the mode-locking regime of the laser) in the  $G_1A_1A_2$  system is possible only in certain changeability regions of its parameters. The determination of these regions is an important problem. In the general case the derivation of the exact analytic expressions determining the admissible values of system parameters is not possible. In some cases the approximate relations, allowing an experimentator to choose properly the laser parameters, can be obtained from the analysis of the system amplification function [7]. Let us consider the case of a system with short relaxation time and both single-and two-photon absorption.

If the amplification saturation in the system occurs considerably later than the absorption saturation, which takes place usually in solid lasers, then the sufficient condition for realization of the regime of mode-locking is that the inequality

$$\alpha'_{th} + \Delta \alpha' - \frac{\kappa'}{1+R} - \gamma'_s R - \varrho' > 0$$
 (5)

be fulfilled within the intensity range  $0 < R \le \frac{1}{A}$ . This condition is satisfied when

the positive root of the polynomials (5) is greater than  $\frac{1}{A}$ ,  $R_2 > \frac{1}{A}$ . From (5) and the last inequality and the definition of  $a'_{th}$  we obtain

$$\kappa' + \Delta \alpha' - \gamma'_s + \sqrt{(\kappa' + \Delta \alpha - \gamma'_s)^2 + 4\gamma'_s \Delta \alpha} > \frac{2\gamma'_s}{4}.$$
 (6)

The condition (6) determines in the system  $G_1 A_1 A_2$  the region of parameter changeability in which the regime of the mode-locking will be realized for a certainly (under the accepted model conditions). This condition is not a necessary condition and by the same means the values of  $\gamma'_s$  are too low, while the values of A are too high. Due to the smallness of  $\Delta \alpha'$  the conditions (6) may be replaced by a simpler condition

$$\gamma_s < \frac{A}{1+A} \varkappa'. \tag{7}$$

From (7) it follows directly that with the increase of A the critical value of  $\gamma_s'$  increases, and that the critical value of A increases also with the increase of  $\gamma_s'$ , as mentioned earlier. The formula (6) or (7) allows also to estimate roughly the optimum parameter values  $\gamma_s'$  and A. The optimum values of  $\gamma_s'$  will be close to the maximal values of  $\gamma_s'$ , while the optimal value of A will be close to the minimum value of A obtained from the conditions (6) and (7). By assuming the values accepted for numerical calculations:  $\kappa' = 1$ , A = 1 we obtain from (7)  $\gamma_s' < 0.5$ . Taking, in turn,  $\kappa' = 1$ ,  $\gamma_s' = 0.4$  we have A > 0.66. By comparing the obtained values with the values  $\gamma_s'$  and A from figs. 1-4, for which the regime of mode-locking was realized, it may be seen that the condition (7) determines the "stability" regions in this process in the system  $G_1 A_1 A_2$  with the good accuracy and may be useful for the experimentator.

An analogical procedure carried out for the system with a long relaxation time for two-photon absorption ( $T_{21} \ll \tau_p \ll T_{11}$ ,  $T_{12}$ ,  $T_{12} \ll T_{res}$ ) gives the next approximate "stability" condition:

$$B > \frac{\gamma_s'}{\varkappa' - \gamma_s'} \sqrt{\frac{\tau_1}{\tau_{\rm ef}}}, \tag{8}$$

where  $\tau_{\rm ef}$  — effective duration of the maximal pulse in the train understood as a ratio of energy generated in the interval  $T_{\rm res}$  to its top power. Due to uncertainity of  $\tau_{\rm ef}$ , which depends on the whole "history" of the process, the condition (8) allows only to estimate the orders of magnitude of some parameters of the system  $G_1A_1A_2$  on the base of typical experimental values  $\tau_{\rm ef}$ . In particular, it enables to estimate the values of cross-section fro two-photon absorption, for which the realization of the mode-locking regime is possible. From (8) and the definition of B we have

$$\sigma_{12} > \left(\frac{\gamma_s'}{\kappa' - \gamma_s'}\right)^2 \frac{1}{s_{12} b^2(I_{21}^s) \tau_{\text{ef}}}.$$
 (9)

By assuming for instance:  $\kappa' = 1$ ,  $\gamma'_s = 0.2$ ,  $s_{12} = 2$ ,  $b = 10^2$ ,  $I_{21}^s = 5 \cdot 10^{26} \frac{1}{\text{cm}^2 \text{s}}$ ,  $\tau_{\text{ef}} = 10^{-11} - 10^{-10} \text{ s}$  from (9) we have  $\sigma_{12} > (1.4 - 14) \cdot 10^{-49} \text{ cm}^4 \text{s}$ . This condition is fulfilled by the cross-section of molecules of a number of organic dyes [10, 11].

# Concluding remarks

The analysis carried out in this work confirms the possibilities of considerable improvement of the picosecond pulses by introducing a two-photon saturation absorbent to the resonator of the laser with mode-locking. The effect of the particular parameters of this absorbent on the generated pulses has been analysed. The analytic criteria obtained have enabled a proper choice of the experimental system parameters.

The effects analysed in this work were the basic ones as they appear in any system of the considered type. In addition to the phenomena discussed there exist also a number of other effects that may have a stronger or weaker influence on the characteristics of the generated radiation. Here, the medium dispersion, phase modulation

of the radiation induced by the refractive index nonlinearity of the medium, additional nonlinear losses in system elements, effects of the coherent interaction in system may be mentioned. The evaluation of the influence caused by these phenomena on the parameters of the generated pulses has been carried out in [2]. In the laser systems based on neodymium glass or rubi this influence, though sometimes essential, is usually small compared to the effect of the phenomena analysed in this work.

#### References

- [1] KRYUKOV P. G., LETOKHOV V. S., IEEE J. Quant. Electron. (New York) QE 8 (1972), 766.
- [2] ZELDOVICH B. Ya., KUZNETSOVA T. I., Usp. Fiz. Nauk 106 (1972), 47.
- [3] LAUBEREAU A., KAISER W., J. Optoelectronics 6 (1974), 1.
- [4] BRUNNER W., KLOSE E., PAUL H., Ann. Phys. (Leipzig) 30 (1973), 279.
- [5] Brunner W., Dürr H., Klose E., Paul H., Kvant. Elektr. 2 (1975), 823.
- [6] WILHELMI B., HEUMAN E., TRIEBEL W., Kvant. Elektr. 3 (1976), 732.
- [7] BADZIAK J., JANKIEWICZ Z., Zh. Tech. Fiz. 17 (1976), 85.
- [8] BADZIAK J., Zh. Tech. Fiz. 18 (1977) 325.
- [9] BADZIAK J., PATRON Z., Zh. Tech. Fiz. 19 (1978), 103.
- [10] TOPP M. R., RENTZEPIS P. M., Phys. Rev. 3A (1971), 358.
- [11] HERMAN J. P., DUCUING J., Optics Commun. 6 (1972), 101.

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# Влияние двуфотонной абсорбции на генерацию пикосекундных импульсов в лазере с самосинхронизацией модов

На основе флуктуационной модели лазера с самосинхронизацией модов был проведен анализ влияния параметров двуфотонного абсорбента, помещённого в резонаторе вместе с усиливающей средой и нелинейным однофотонным абсорбентом, на характеристики генерированных импульсов. Показана возможность значительного повышения интенсивности и контраста, а также уменьшения длительности пикосекундных импульсов в результате применения двуфотонного насыщающего абсорбента. Определены приближённые аналитические критерии эффективности генерации пикосекундных импульсов в лазере с одно- и двуфотонным абсорбентом.