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The third order aberrations of a narrow oblique beam focussed by a conflection lens*

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In the paper the formulae for the third order aberrations of a narrow oblique collimated beam focussed by a geodesic lens composed of two conical surfaces have been introduced. The measures of asymmetry of the beam with respect to the central ray as the function of the slope and the aperture of the beams have been proposed. The position of the stigmatic image for the lens R-2R has been examined.

ntroduction

Toraldo di Francia derived (in [1]) the formula for the longitudinal aberration of third order of a ray suffering from conflection on the border of thin film waveguides having the shape of conical surfaces. He also introduced the notation of conflection doublet of corrected aberration of third order. Such a doublet has been constructed and presented in paper [2]. In [1] the ray aberration has been calculated as a difference between the image distances of an arbitrary ray and paraxial ray and expressed as a function of the both paraxial distances of the given ray and the angle created by this ray with the "optical axis" of the lens (in other words, with the straight line along which the cones have been developed).

In the present paper the formulae allowing to calculate the quantities of the longitudinal and transversal spread of the focus of a single conflection lens have been derived under assumption that a narrow beam of parallel rays falls on the lens slightly obliqly with respect to the axis. The magnitude of confusion has been expressed as a function of aperture of the incident beam and its slope with respect to the axis.

Formulae derivation

Let a collimated beam of width $D = 2y$ fall upon the conflection lens (see figure) of the curvature radii R and R', respectively, and let the central ray fall on the line *1* under the angle ω at the point lying on the straight line along which the conic surfaces have been developed. In order to determine the aberrations of this beam it is necessary to find the point intersection of the rays travelling at a given distance y from the central ray with the latter, all in the image space. The x-axis of the coordinates system is directed along the central ray.

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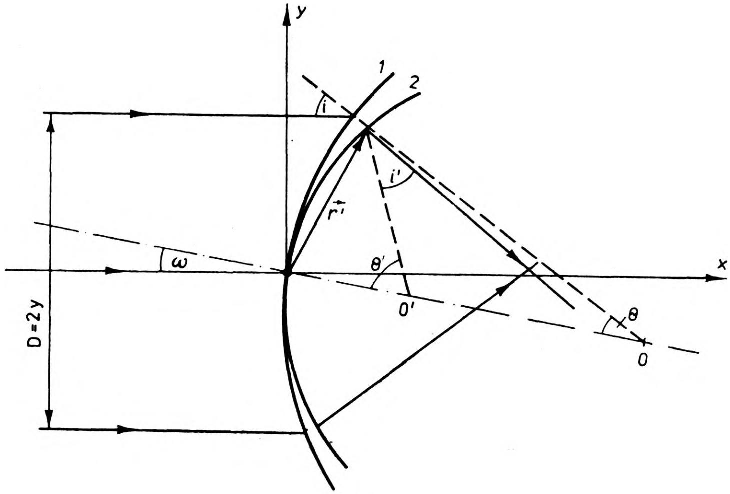

Fig.

The sine of the incidence angle of the ray on the first line of the lens is defined by the formula

$$\sin i = \frac{y}{R} - \sin \omega . \tag{1}$$

In accordance with the conflection law we have:

$$i' = i.$$

The central ray does not change its direction after passing through the lens. From the figure and the conflection lens geometry it follows that the angles Θ and Θ' created by the normals to the lines 1 and 2 with the axis at the intersection points of these lines with the ray considered are given by the formulae

$$\Theta = i + \omega , \tag{2}$$

$$\Theta' = \frac{R}{R'} \Theta . \tag{3}$$

The vector $\vec{r}'$ drawn from the origin of the coordinate system to the intersection point of the line 2 with the ray has the components

$$x' = r' \cos \gamma , \tag{4a}$$

$$y' = r' \sin \gamma , \tag{4b}$$

where r' is the value of the vector $\vec{r}'$, where

$$r' = 2R' \sin \frac{\Theta'}{2} . \tag{5}$$

Taking into account that $\gamma = \frac{\pi}{2} - \left(\frac{\Theta'}{2} - \omega\right)$ we obtain

$$x' = R'[\cos\omega - \cos(\Theta' - \omega)], \tag{6a}$$

$$y' = R'[\sin\omega + \sin(\Theta' - \omega)]. \tag{6b}$$

By expanding into series the functions of angles Θ' and ω in the formulae (6a) and (6b) with the accuracy up to III order terms we get the following expression

$$x' = R'\Theta'\left[\frac{\Theta'}{2} - \omega\right], \tag{7a}$$

$$y' = R'\Theta'\left[1 - \frac{\omega^2}{2} - \frac{\Theta'^2}{6} + \frac{\Theta'\omega}{2}\right]. \tag{7b}$$

The points of intersections of the ray incident on the lens at the height y with the central ray and with the straight line perpendicular to the central ray at the distance $x = f'$ may be found by solving the systems of equations:

$$\begin{cases} \dfrac{y - y'}{x - x'} = \tan(\Theta - \Theta') \\ y = 0, \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{y - y'}{x - x'} = \tan(\Theta - \Theta') \\ x = f'. \end{cases} \tag{9}$$

By taking account of the formulae (3), (6a) and (6b) and expanding $\tan(\Theta - \Theta')$ into series with the accuracy up to the III order we obtain from (8) and (9) the following expressions

$$x_{s'} = -\frac{f'}{1 + \frac{1}{3}\left(1 - \frac{R}{R'}\right)^2 \Theta^2}\left[\frac{1}{2}\left(\frac{R}{R'}\right)\Theta^2 - \frac{1}{3}\left(\frac{R}{R'}\right)^2\Theta^2 - \Theta\omega + \frac{1}{2}\left(\frac{R}{R'}\right)\Theta\omega - 1 + \frac{\omega^2}{2}\right], \tag{10}$$

$$y_{s'} = \frac{R\Theta}{2}\left[-\omega^2 + \left(2 - \frac{R}{R'}\right)\left(\omega - \frac{1}{3}\Theta\right)\Theta\right], \tag{11}$$

where

$$\frac{1}{f'} = \frac{1}{R'} - \frac{1}{R}. \tag{12}$$

After substituting the formulae (2) and (1) to (10) and (11), respectively, with the accuracy to the III order terms, the expressions for the longitudinal aberration

$x_s - f'$ and the transversal aberration $y_{s'}$ take the forms:

$$x_{s'} - f' = -\frac{f'}{2}\left[\omega^2 + \frac{1}{3}(2-\varkappa)\left(\frac{y}{R} - 3\omega\right)\frac{y}{R}\right], \tag{13}$$

$$y_{s'} = -\frac{y}{2}\left[\omega^2 + \frac{1}{3}(2-\varkappa)\left(\frac{y}{R} - 3\omega\right)\left(\frac{y}{R}\right)\right], \tag{14}$$

where

$$\varkappa = \frac{R}{R'}.$$

In addition to aberrations of a single ray falling on the lens at the distance y from the central ray, it is possible (by taking advantage of formulae (13) and (14)) to calculated the magnitude of both longitudinal $\Delta x_s'$, and transversal $\Delta y_s'$ spreads of the beam of given width $D = 2y$, caused by the spherical aberration and its asymmetry with respect to the central ray.

Let $x_{s'}^{(+)}$ denote the intersection coordinate x of the ray falling on the lens at the distance $+y$ from the central ray and let $x_{s'}^{(-)}$ denote the intersection coordinate x of the ray falling on the lens at the distance $-y$ from the central ray. Analogically, let $y_{s'}^{(+)}$ and $y_{s'}^{(-)}$ be the coordinates y of the intersection of these rays with the straight line $x = f'$. Then, on the basis of (13), the value of longitudinal confusion of the beam $\Delta x_{s'} = x_{s'}^{(+)} - x_{s'}^{(-)}$, will be expressed as follows

$$\Delta x_{s'} = (2-\varkappa)\omega\frac{y}{R}f', \tag{15}$$

and by virtue of (14) the value of transversal confusion $\Delta y_{s'} = y_{s'}^{(+)} - y_{s'}^{(-)}$ is given by the formula

$$\Delta y_{s'} = y\left[\omega^2 + \frac{1}{3}(2-\varkappa)\left(\frac{y}{R}\right)^2\right], \tag{16}$$

where y is half total width of incident beam.

By taking advantage of the definition of the relative aperture $N = 2y/f'$, and the formula (12) for the focal distance, the values y/R and y appearing in the formulae (15) and (16) may be expressed by N, $\varkappa$ and f'.

Finally, the formulae for the longitudinal and transversal confusion of the conflex lens focus take the following forms:

$$\Delta x_{s'} = \frac{1}{2}\frac{2-\varkappa}{\varkappa-1}\omega N f', \tag{17}$$

$$\Delta y_{s'} = \frac{1}{2}f'N\left[\omega^2 + \frac{1}{12}\frac{2-\varkappa}{(\varkappa-1)^2}N^2\right], \tag{18}$$

and the relative value of the longitudinal $\Delta\bar{x}_{s'}$ and transversal $\Delta\bar{y}_{s'}$ aberrations are given by the formulae:

$$\Delta \bar{x}_{s'} = \frac{1}{2} \frac{2-\varkappa}{\varkappa-1} \omega N, \tag{19}$$

$$\Delta \bar{y}_{s'} = \frac{1}{2} N \left[\omega^2 + \frac{1}{12} \frac{2-\varkappa}{(\varkappa-1)^2} N^2 \right]. \tag{20}$$

The value $\Delta \bar{x}_{s'}$ is the measure of the longitudinal spread of the beam caused only by its asymmetry, while the term dependent only on N and expressing the pure spherical aberration appears in (20). In order to obtain the transversal measure of the beam asymmetry, independent of the spherical aberration, the quantity $\delta \bar{y}_{s'}$ has been calculated from (14) as a sum of $y_{s'}^{(+)}$ and $y_{s'}^{(-)}$.

$$\delta \bar{y}_{s'} = \frac{1}{8} \frac{2-\varkappa}{(\varkappa-1)^2} \omega N^2. \tag{21}$$

Discussion of the results

The aberrations of III order of the ray belonging to the beam incident under the given angle ω dissappear, if the expression in the square brackets in (13) and (14) are equal to zero. The aberrations of any other ray are then different from zero and the beam is asymmetric with respect to the central ray. The axial beam ($\omega = 0$) is evidently symmetric. The longitudinal and transversal spherical aberrations of such a beam may be evaluated from (13) nad (14). From the expressions (19) and (21) being the measures of beams asymmetry it follows that the beam preservs the symmetry when $\varkappa = 2$ (i.e. the lens R—2R [1]). The imaging is the stigmatic for an arbitrary ω and N, whereby the focus lies on the central ray at the point $x = f' \left(1 - \frac{1}{2} \omega^2\right)$.

References

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Аберрация III порядка узкого косого луча, фокусированного конфлекционной линзой

В статье даны выведенные формулы для определения аберрации третьего порядка узкого параллельного косого пучка, фокусированного геодезической линзой, состоящей из двух конических поверхностей. Предложены меры асимметрии пучка по отношению к центральному лучу в функции угла наклона пучка и его аппаратуры. Было исследовано положение стигматического изображения для линзы R-2R.