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Considering location of stops in the Fourier transform lens system

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In an optical system, it is the aperture and proper location of stops that are of fundamental importance to the image quality. The stops limit the amount of light actually allowed to reach the image from any object point. Besides, another effect of the stop is the limitation of the field of view. The consequence of this limitation is that for all object points there is steady diminution of the light which reaches the image point. Therefore, the image is of approximately uniform brightness only up to the point of the field not limited by the field stop, and the other image points are only part that brightness, and the latter dies down to zero in the limiting case [1]. Diaphragms of this type are also met in optical systems that are applied to perform the Fourier transform of any amplitude transmittance [2].

In a coherently illuminated aberrationless optical system, there exists a Fourier transform relationship between light amplitude distribution in the front and the back focal planes of the lens. The complex amplitude of the Fraunhofer diffraction pattern of light waves diffracted by an object plane is the Fourier transform of the amplitude transmittance of the object, an is based upon the Fresnel diffraction formula with approximations that restrict to cases involving small diffraction angles, and is very useful, especially for large diffraction angles in a number of interesting applications. The Fourier transform of a spatially varying object determines the spatial frequency components of the object as long as the principal rays are close enough to the optical axis, so that nonliner terms of the field angles may be neglected. The restriction to small field angles imposes severe limitations upon the useful range of Fourier transform lens. But such a lens transforms the amplitude distribution of an object from the front focal plane to the back focal plane, and usually operates in a telecentric configuration on the image side [3]. For example, if one-dimensional amplitude transmittance of an object is expressed by the equation

$$t_O(x_O) = \cos\left(2\pi\frac{x_O}{a}\right)+\cos\left(2\pi\frac{x_O}{b}\right), \tag{1}$$

then the Fourier transform of the diffraction object transmittance is a sum of four delta functions. Therefore, in the back focal plane we obtain four spots located at points of coordinates:

$$x'_f = \pm\frac{\lambda f}{a}, \quad x''_f = \pm\frac{\lambda f}{b}, \tag{2}$$

that determine the position of the spatial frequency components of the object which is said to possess spatial periods *a* and *b*. The diffracting plane is usually inserted in the front focal plane, and if it is of axial symmetry, it simultaneously coincides with the entrance pupil of the system. Such a system is equivalent to the telecentric ray system configuration, and thus the field angles are equal to the diffraction angles. In this case, we do not have an ordinary imaging process, where object points are imaged into the image points, because the input transparency inserted in the front focal plane of the lens produces an angular spectrum of plane waves that are focused by the lens to suitable points in the back focal plane, forming the Fourier transform of the input transmittance. In the Fourier transform lens, the aperture stop is identical with the input transparency, being thus defined by the sizes of the latter. Therefore, it is all to the good, if the transparency is inserted in the front focal plane symmetrically with respect to the axis of the Fourier transform lens, then the principal rays of all pencils representing the respective spatial frequencies of the input are parallel to the axis in the image space. The amount of light which reaches the image points in the back focal plane of the lens depends on the sizes of transparent parts of the input. In each beam of rays which proceeds to the lens at fixed diffraction angle, one can find the principal ray passing through the centre of the aperture stop. Furthermore, the maximum diffraction angle at the input transparency defines the field angle of the system.

If the spatial frequency components of the input transparency are not of axial symmetry, then the entrance pupil of the plane wavefront diffracted at the off-axis part of the transparency by a fixed frequency component can be either in front of or

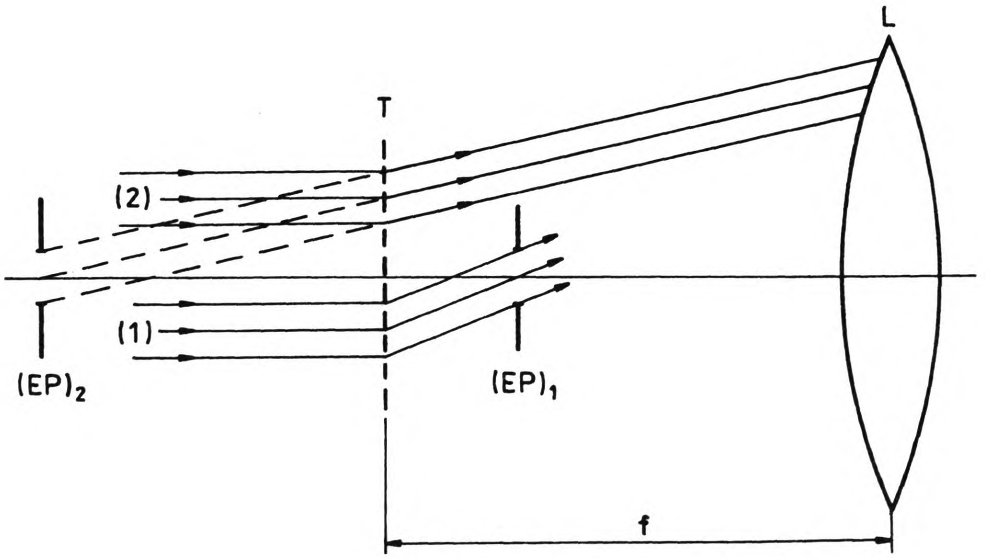

Fig. 1. Diffraction of a plane wave at two different spatial frequency components of the input transparency. L – lens, T – transparency, $(EP)_1$, $(EP)_2$ – entrance pupils of the first and the second beams, respectively

behind the front focal plane of the lens (Fig. 1). Position of the entrance pupil for a given beam depends on the location and the value of the spatial frequency component of the object transparency. In this method, the spatial frequency components of the transparency can be specified by the intersection points of principal rays with the Gaussian image plane (Fourier plane) of the lens. In general, location of these intersection points is expressed by the coordinates

$$x_f = \lambda f\xi, \quad y_f = \lambda f\eta \tag{3}$$

where: ξ, η — spatial frequency components, λ — wavelength of the light used, f — focal length of the lens.

On the other hand, the intersection points of the meridional and corresponding paraxial principal rays do not coincide with each other, and are given by the equations:

$$x'_f = (z'-\delta s)\tan\alpha', \quad x_f^0 = Mx_O, \tag{4}$$

respectively, where δs is the longitudinal spherical aberration in the exit pupil (Fig. 2), and M is the lateral magnification for the fixed object and image planes

$$M\left(\frac{dx'}{dx_O}\right)_{z\,=\,\mathrm{const}} = \frac{f}{z} = \frac{z'}{f'},$$

and the coordinate of an object point is $x_O = z_O\tan\alpha$. Therefore, for a Fourier transform lens, or for an object at infinity, we have: $z_O = \infty$, $M = 0$, and the inaccuracy of locations of spatial frequencies in the Fourier plane is defined by distortion

$$\Delta_D = \left(\frac{z'-\delta s}{f}\,\frac{\tan\alpha'}{\tan\alpha}-1\right)100\%. \tag{5}$$

Figure 1 shows two plane wavefronts diffracted at the input in two different directions, by two different parts of the transparency. In this case, we have two

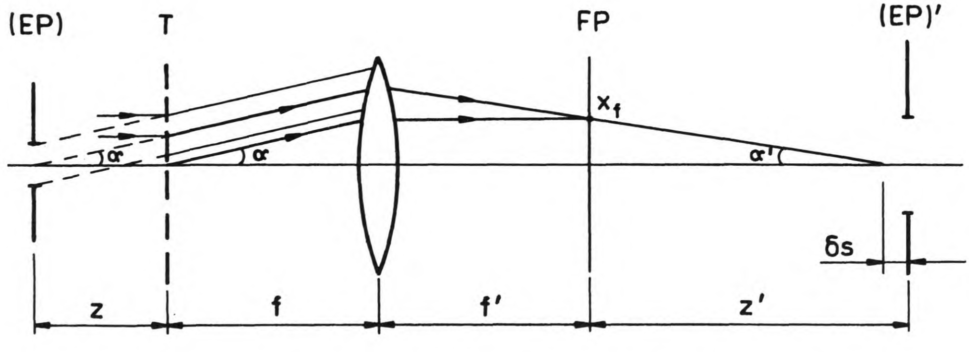

Fig. 2. Diffraction of an off-axis beam at the input transparency (T), and principal ray tracing through the lens. (EP) — entrance pupil, (EP)' — exit pupil, FP — Fourier plane, α, α' — field angles of the beam in the object and image spaces, respectively

different entrance pupils and, consequently, two different exit pupils of different aberrations and therefore different inaccuracies of frequency position in the Fourier plane. In the absence of aberrations, the principal ray passes through the centre of the entrance pupil and through the centre of the exit pupil. One can say that the Fourier transform lens possesses not only one but a number of aperture stops depending on the number of frequency components and their location in the input transparency, and the diffraction image in the Fourier plane is not of uniform brightness but changes from point to point since each frequency component has different aperture. In addition to aperture stops, the Fourier transform lenses possess field stops that determine the maximum value of the spatial frequency components imaged. Usually, in this case the field stop coincides with the lens and is determined by its diameter.

References

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