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Letters to the Editor

Simple relations and diagrams for antireflection thin-film coatings on absorbing substrates at oblique incidence

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Explicit relations are presented for double-layer and symmetrical triple-layer antireflection dielectric thin-film coatings on absorbing substrates at oblique incidence. Diagrams of admissible layer refractive indices are readily obtained. Examples are presented for absorbing silicon substrates.

Antireflection (AR) coatings that consist of stacks of dielectric thin films on transparent substrates at normal incidence are described in many works (see the reviews [1]–[3]). The simplest designs consist of layers with optical thicknesses which are integral multiples of quarter-wave. These designs normally make use of layers with refractive indices that do not occur when ordinary optical coating materials are applied. So, they have to be replaced by a combination of existing materials to simulate the missing refractive index [4]. Relations for simulating a quarter-wave layer by double-layer and symmetrical triple-layer stacks are well known [1], [4]. If these relations are adequately transformed [5], they are valid also at oblique incidence for *p*- or *s*-polarized light waves.

Single-layer AR coatings on absorbing substrates at oblique incidence have been analysed in [6]. Rather intricate relations for double-layer AR coatings on absorbing substrates at oblique incidence have been presented in [7] and [8].

In this paper, we present simple explicit relations for dielectric double-layer and symmetrical triple-layer thin-film AR coatings on absorbing substrates at oblique incidence for *p* or *s* polarization. Useful diagrams of refractive indices for optical coating materials are readily obtained. Examples are shown for absorbing silicon substrates.

Let us denote by n_a and $\hat{n}_g$ the refractive indices of the ambient medium of incidence and the absorbing substrate, respectively, where $\hat{n}_g = n_g - jk_g$. For the *i*-th layer of a stack, we denote by n_i the refractive index, d_i – the geometrical (physical) thickness, and $\beta_{i\Phi}$ – the phase thickness at the angle of incidence Φ

$$\beta_{i\Phi} = (2\pi/\lambda)D_{i\Phi} \tag{1}$$

with λ being the light wavelength in vacuum, and $D_{i\Phi}$ – the optical thickness at the angle of incidence Φ, $D_{i\Phi} = d_i S_{i\Phi}$, where

$$S_{i\Phi} = (n_i^2 - n_a^2 \sin^2\Phi)^{1/2}. \tag{2}$$

The complex reflection coefficients r_p and r_s for p and s polarizations of a double-layer film-substrate system are [7]:

$$r_v = (r_{01v} + r'_v X_1)/(1 + r_{01v} r'_v X_1), \quad v = p,\ s \tag{3}$$

where

$$r'_v = (r_{12v} + r_{2gv} X_2)/(1 + r_{12v} r_{2gv} X_2). \tag{4}$$

Thin layers are counted from the ambient side towards the substrate. r_{abv} are the complex (in general) Fresnel interface reflection coefficients [9] for the interface $a-b$ and v polarization. X_1 and X_2 are complex periodic functions of the layer phase thicknesses given by

$$X_K = \exp(-j2\beta_{i\Phi}),\ K = 1,\ 2. \tag{5}$$

In the case of a symmetrical triple-layer coating with the first and third layers of equal refractive indices ($n_1 = n_3$) and thicknesses ($d_1 = d_3$, $\beta_{1\Phi} = \beta_{3\Phi}$), the complex reflection coefficients r_v, $v = p, s$ of the film-substrate system are given by Eq. (3), where r'_v is determined from:

$$r'_v = (r_{12v} + r''_v X_2)/(1 + r_{12v} r''_v X_2) \tag{6}$$

and

$$r''_v = (-r_{12v} + r_{1gv} X_1)/(1 - r_{12v} r_{1gv} X_1). \tag{7}$$

The intensity (or power) of reflectance is given by $R_v = r_v r_v^*$, $v = p, s$, where r_v^* is the complex conjugate of r_v. Let us denote:

$$\hat{S}_{g\Phi} = (\hat{n}_g^2 - n_a^2 \sin^2\Phi)^{1/2} \tag{8}$$

and

$$\hat{\varkappa}_g = \eta_g + j\zeta_g. \tag{9}$$

At given refractive indices of the substrate and thin layers, the layer thicknesses can be adjusted to achieve zero reflectance of the film-substrate system [1]. The following relations are obtained from the AR condition in the case of double-layer coatings on absorbing substrates

$$\tan^2\beta_{1\Phi} = \varkappa_1^2[(\varkappa_0 - \eta_g)(\varkappa_0\eta_g - \varkappa_2^2) - \varkappa_0\zeta_g^2]/[(\varkappa_1^2 - \varkappa_0\eta_g)(\varkappa_0\varkappa_2^2 - \varkappa_1^2\eta_g) + \varkappa_0\varkappa_1^2\zeta_g^2], \tag{10}$$

$$\tan\beta_{2\Phi} = \varkappa_2[\varkappa_1(\varkappa_0 - \eta_g) - \varkappa_0\zeta_g\tan\beta_{1\Phi}]/[(\varkappa_0\varkappa_2^2 - \varkappa_1^2\eta_g)\tan\beta_{1\Phi} + \varkappa_0\varkappa_1\zeta_g]. \tag{11}$$

At normal incidence, we have to insert

$$\Phi = 0, \quad \varkappa_i = n_i, \quad (i = 0, 1, 2), \text{ and } \hat{\varkappa}_g = \hat{n}_g. \tag{12}$$

At oblique incidence, we must insert

$$\varkappa_i = S_{i\Phi} \quad (i = 0, 1, 2), \quad \hat{\varkappa}_g = \hat{S}_{g\Phi} \text{ for } R_s = 0 \tag{13}$$

and

$$\varkappa_i = S_{i\Phi}/n_i^2 \quad (i = 0, 1, 2), \quad \hat{\varkappa}_g = \hat{S}_{g\Phi}/\hat{n}_g^2 \text{ for } R_p = 0 \tag{14}$$

where $S_{i\Phi}$ and $S_{g\Phi}$ are given by Eqs. (2) and (8), respectively.

Two sets of valid solutions for $\beta_{1\Phi}$ and $\beta_{2\Phi}$ are obtained, corresponding to the two real solutions of $\beta_{1\Phi}$ that result from Eq. (10).

In the case of symmetrical triple-layer coatings on absorbing substrates, one obtains a quadratic equation for $x = \tan^2\beta_{1\Phi}$ which has the form

$$ax^2 + bx + c = 0 \tag{15}$$

where

$$a = (\varkappa_0 - \eta_g)(\varkappa_0\varkappa_2^2\eta_g - \varkappa_1^4) - \varkappa_0\varkappa_2^2\zeta_g^2, \tag{16a}$$

$$b = 2\varkappa_0\varkappa_1\zeta_g(\varkappa_2^2 - \varkappa_1^2),$$

$$c = \varkappa_1^2[(\varkappa_0 - \eta_g)(\varkappa_0\eta_g - \varkappa_2^2) - \varkappa_0\zeta_g^2]. \tag{16b}$$

Solving Equation (15) gives

$$\tan^2\beta_{1\Phi} = [-b \pm (b^2 - 4ac)^{1/2}]/(2a). \tag{17}$$

Then one obtains

$$\tan\beta_{2\Phi} = \varkappa_1\varkappa_2[\varkappa_1(\varkappa_0 - \eta_g)(1 - \tan^2\beta_{1\Phi}) - 2\varkappa_0\zeta_g\tan\beta_{1\Phi}] \\ /[\varkappa_0\zeta_g(\varkappa_1^2 - \varkappa_2^2\tan^2\beta_{1\Phi}) + \varkappa_1(\varkappa_0 - \eta_g)(\varkappa_1^2 + \varkappa_2^2)\tan\beta_{1\Phi}]. \tag{18}$$

Equations (15)–(18) are valid at normal incidence with the insertion (12), and at oblique incidence for s or p polarization with insertion (13) or (14), respectively. Two sets of valid solutions for $\beta_{1\Phi}$ and $\beta_{2\Phi}$ are obtained. The two values of $\tan\beta_{2\Phi}$ that result from Eq. (18) have equal absolute values, that is, they have the form: $\tan\beta_{2\Phi} = \pm\alpha$. Useful diagrams of refractive indices for AR coatings on absorbing substrates are obtained. They result from the condition of real solutions, that is, the condition of positive right-hand side of Eq. (10) in the case of double-layer coatings and the condition of both $b^2 - 4ac \geqslant 0$ and positive right-hand side of Eq. (17) for symmetrical triple-layer coatings. Examples are shown in Figs. 1 and 2 at normal

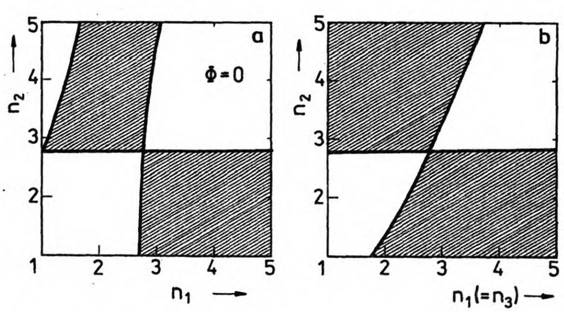

Fig. 1. Solution zones (hatched areas) of refractive-index sets in the case of AR double-layer coatings (a), and AR symmetrical triple-layer coatings (b), on absorbing Si substrate with the complex refractive index $\hat{n}_g = 5.063 - j3.218$, $\lambda = 0.325$ μm at normal incidence. Solution zones are delimited by lines and curves that result numerically in each case from the respective condition of real solutions

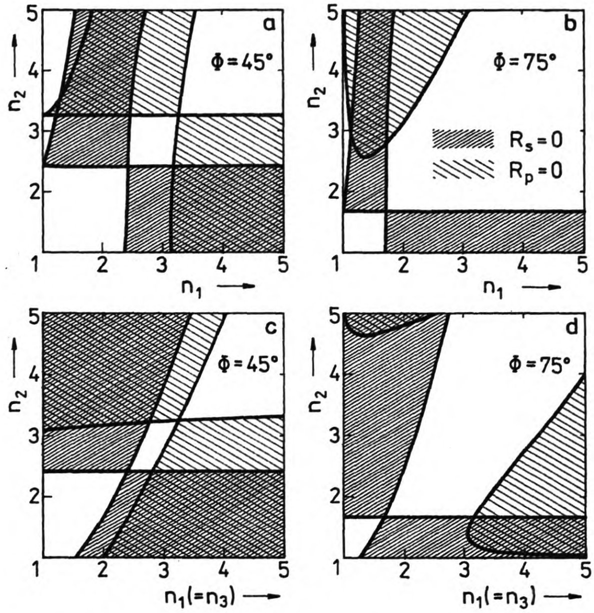

Fig. 2. Solution zones (hatched and cross-hatched areas) of refractive-index sets in the case of AR double-layer coatings at $\Phi = 45°$ (a), and $\Phi = 75°$ (b), and in the case of symmetrical triple-layer coatings with $n_1 = n_3$ at $\Phi = 45°$ (c) and $\Phi = 75°$ (d) on absorbing Si substrate with the complex refractive index $\hat{n}_s = 5.063 - j3.218$ at $\lambda = 0.325$ µm for $R_s = 0$ or $R_p = 0$

and oblique incidence for double- and symmetrical triple-layer AR coatings on absorbing silicon substrates at $\lambda = 0.325$ µm. These figures show that at a given angle of incidence, the triple-layer coatings have greater shaded areas of admissible refractive index values than double-layer coatings. At given refractive indices of the substrate and thin films, the values of the angle of incidence which obey the AR condition can be readily obtained. As for example, for Si substrate [8] at $\lambda = 0.325$ µm with $\hat{n}_s = 5.063 - j3.218$, and thin films of Si_3N_4 ($n_1 = 2.01$) and SiO_2 ($n_2 = 1.482$), the admissible values of Φ in the case of double-layer coating are in the range $63.27° \leqslant \Phi \leqslant 80.20°$ for $R_s = 0$, and $81.90° \leqslant \Phi \leqslant 83.95°$ for $R_p = 0$, and in the case of symmetrical triple-layer coating in the range $0.81° \leqslant \Phi \leqslant 80.24°$ for $R_s = 0$, and $81.72° \leqslant \Phi \leqslant 82.73°$ for $R_p = 0$.

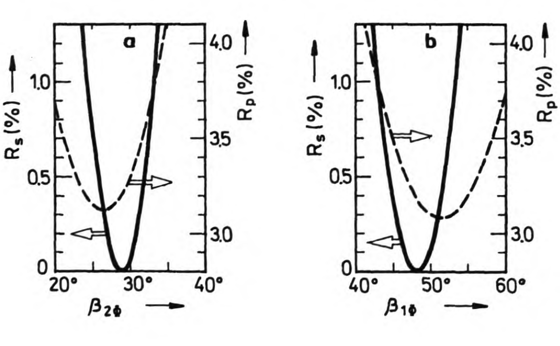

Fig. 3. a, b

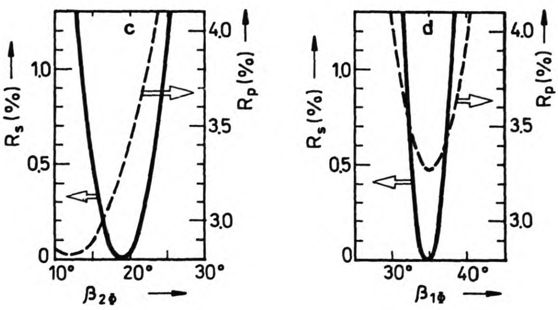

Fig. 3. Variations of R_s (solid) and R_p (dashed) as a function of $\beta_{2\Phi}$ at given $\beta_{1\Phi} = 48.223°$ (a), and as a function of $\beta_{1\Phi}$ at given $\beta_{2\Phi} = 28.730°$ in the case of a double-layer thin-film coating (b); variations of R_s (solid) and R_p (dashed) as a function of $\beta_{2\Phi}$ at given $\beta_{1\Phi} = 34.834°$ (c), and as a function of $\beta_{1\Phi}$ at given $\beta_{2\Phi} = 18.957°$ in the case of a symmetrical triple-layer coating (d). Thin films of refractive indices $n_1 = 1.98$ and $n_2 = 1.46$ are coated on an absorbing Si substrate with the complex refractive index $\tilde{n}_s = 3.85 - j0.02$ at $\lambda = 0.633$ μm and $\Phi = 45°$

In the case of absorbing substrates, total antireflection (TAR) coatings at oblique incidence for both s and p polarizations can be designed only by numerical iteration. The TAR condition is accomplished when both R_s and R_p at the end of iteration are small enough, for example, smaller than 10^{-12} [10].

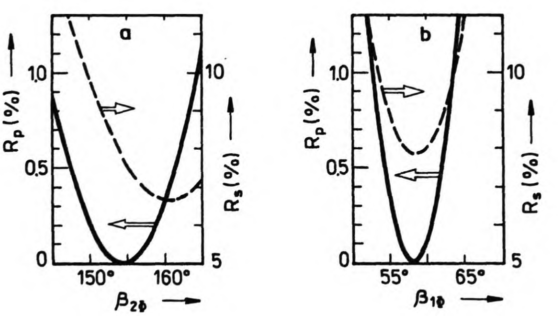

Fig. 4. Variations of R_p (solid) and R_s (dashed) as a function of $\beta_{2\Phi}$ at given $\beta_{1\Phi} = 58.044°$ (a), and as a function of $\beta_{1\Phi}$ at given $\beta_{2\Phi} = 154.283°$ in the case of a symmetrical triple-layer coating (b). The given values of $\beta_{1\Phi}$ and $\beta_{2\Phi}$ correspond to $R_p = 0$. The same thin films and angle of incidence as in Fig. 3 are chosen

Variations of R_s and R_p against $\beta_{2\Phi}$ at given $\beta_{1\Phi}$, and against $\beta_{1\Phi}$ at given $\beta_{2\Phi}$ are shown in Fig. 3. The given values of $\beta_{1\Phi}$ and $\beta_{2\Phi}$ are determined from the AR condition for s polarization ($R_s = 0$). The antireflection of s-polarized light is accompanied by an unextinguished p reflectance $R_p \simeq 3\%$. The same variations of R_s and R_p are shown in Fig. 4 for symmetrical triple-layer AR coating corresponding to $R_p = 0$. The antireflection of p-polarized light is accompanied by a greater unextinguished s reflectance, $R_s \simeq 8\%$.

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