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Compensation of aberrations — wave approach

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The influence of aberrations and apodization of a complex optical system on the aberration compensation condition in the wave approach is considered. Both factors change the compensation condition as compared to the geometrical solution, however, for well corrected separate elements the differences appear not to be significant.

1. Introduction

The design of an optical system composed of two or more objectives leads usually to the aberration compensation problem. Even if each of these objectives fulfils the correction criterion separately, the whole system may not preserve the suitable correction level. In such a case, the sum of residual aberrations may exceed the admissible limiting value if the signs of the individual components are not opposite. A good example of such situation is the choice of the correction type of the objective and the eyepiece in either a microscope or a telescope. The undercorrection, for example, of the objective should require the overcorrection of the eyepiece. Usually, the compensation is accomplished comparing the geometrical characteristics of both systems or tracing light rays through the whole combined system. To this end, we propose to achieve it basing on the wave approach and analysing the propagation of an aberrated wave. For that purpose, the method of optical system synthesis will be used [1]. Accordingly, the analysis of the wave propagation through combined optical system can be reduced to the analysis of the propagation through a set of distorters. In the substitutional system of distorters, the influence of aberrations of all optical elments (objectives, eyepieces or others) is taken into account, neglecting only their focusing properties.

The wave approach proposed offers the possibility of the aberration compensation for apodized systems. The common use of combined laser systems justifies the studying of this problem.

2. General consideration

Let D_1 and D_2 be a system of two distorters (Fig. 1). They represent the influence of aberration of two lenses (in a particular case, the objective and the eyepiece) upon a wave propagating through an optical system combined of these lenses. The relation between a real optical system and the substitutional system composed of two

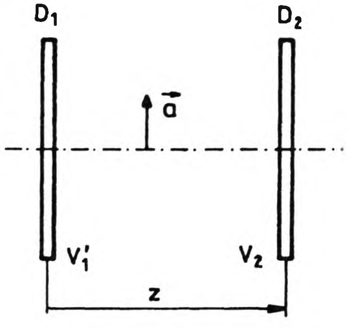


Fig. 1. System of two distorters

distorters will be given later. We start the analysis with a simplified model of the optical system reduced to two plates D_1 and D_2 shown in Fig. 1 and consider the propagation of the wave through this system which appears to be sufficient to approximately solve our problem. The transmittance of both plates is described by the expression $\exp[2\pi W_m(\mathbf{a})]$, where the quantity $W_m(\mathbf{a})$ is an aberration introduced by the plate D_m $(m = 1, 2)$. This quantity is expressed as a fraction of the wavelength λ. The vector $\mathbf{a}$ represents the radial co-ordinate.

Let us assume that a wave propagating through the system under consideration has at the plane D_1 (in front of the plate) a constant phase and a variable amplitude distribution $V(\mathbf{a})$. It means that immediately behind the plane D_1 the complex amplitude distribution takes the following form:

$$V'_1(\mathbf{a}) = V_1(\mathbf{a})\exp[2\pi i W_1(\mathbf{a})]. \quad (1)$$

When the distance z between the plates is negligibly small, then the phase distribution just behind the second plate D_2 is equal to the sum $W_1(\mathbf{a})+W_2(\mathbf{a})$, therefore, the mutual compensation of aberrations of both plates occurs if the relation

$$W_1(\mathbf{a})+W_2(\mathbf{a}) = 0 \quad (2)$$

is fulfilled for every vector $\mathbf{a}$.

Since the phase distribution of a propagating wave changes [2], the aberration compensation condition (2) can be replaced by the following one:

$$W_1(\mathbf{a})+\delta W_1(\mathbf{a})+W_2(\mathbf{a}) = 0 \quad (3)$$

where $\delta W_1(\mathbf{a})$ is a change of the phase distribution of the wave (the change of aberration) during the propagation between the two plates.

According to [2], the field distribution $V_2(\mathbf{a})$ at the plane of the distorter D_2 induced by the field distribution $V'_1(\mathbf{a})$ can be found from the following approximate relation

$$V_2(\mathbf{a}_n) \approx V'_1(\mathbf{a}_n)+i\frac{G}{4\pi}\nabla^2 V'_1(\mathbf{a}_n) \quad (4)$$

where

$$G = \frac{\lambda z}{a_0^2}. \tag{5}$$

For the two-dimensional function

$$\nabla^2 V_1'(\mathbf{a}_n) = \frac{\partial^2 V_1'}{\partial a_{nx}^2} + \frac{\partial^2 V_1'}{\partial a_{ny}^2} \tag{6}$$

where $\mathbf{a}_n$ denotes the normalized vector position

$$\mathbf{a}_n = \frac{\mathbf{a}}{a_0} \tag{7}$$

where $2a_0$ is an arbitrary dimension at the plane D_1. The maximum dimension of the area with the field distribution may be taken as a value of $2a_0$. The quantities a_{nx} and a_{yn} are the Cartesian components of the vector $\mathbf{a}_n$. Relation (4) is valid in cases in which the coefficient $G/4\pi$ is sufficiently small as compared to 1, which means that $z \ll 4\pi a_0^2/\lambda$.

Taking into account relation (1), we can find that

$$V_2(\mathbf{a}_n) \approx V_1'(\mathbf{a}_n)[1 + \delta v_r + i\delta v_i] \tag{8}$$

where

$$\delta v_r = -0.5G\left[\frac{2}{V_1}\operatorname{grad} W_1 \operatorname{grad} V_1 + \nabla^2 W_1\right], \tag{9}$$

and

$$\delta v_i = -0.5G\left[\frac{\nabla^2 V_1}{V_1} - 4\pi^2(\operatorname{grad} W_1)^2\right]. \tag{10}$$

The quantities δv_r and δv_i are real and considerably smaller than 1. According to equations (1), (8)–(10) and the relation

$$V_2(\mathbf{a}_n) = [V_1'(\mathbf{a}_n) + \delta V(\mathbf{a}_n)]\exp\{2\pi i[W_1(\mathbf{a}_n) + \delta W(\mathbf{a}_n)]\} \tag{11}$$

we can write

$$\delta V(\mathbf{a}_n) = V_1\delta v_r = -0.5G[2\operatorname{grad} W_1 \operatorname{grad} V_1 + V_1\nabla^2 W_1] \tag{12}$$

and

$$\delta W(\mathbf{a}_n) = \frac{1}{2\pi}\delta v_i = -0.5G\left[(\operatorname{grad} W_1)^2 - \frac{1}{4\pi^2}\frac{\nabla^2 V_1}{V_1}\right]. \tag{13}$$

The quantity $\delta V(\mathbf{a}_n)$ describes the change of the amplitude distribution of the field $V_1'(\mathbf{a}_n)$ occurring during the propagation between two plates, and $\delta W(\mathbf{a}_n)$, as previously, represents a respective change of the phase distribution.

The aberration compensation problem requires that only the quantity $\delta W_1(\mathbf{a}_n)$ be considered (see Eq. (3)). The change $\delta W_1(\mathbf{a}_n)$ of aberration due to propagation depends on the aberrations of the plate D_1 (component with $(\operatorname{grad} W_1)^2$ in (13)) as well as on the apodization of the propagating wave (component with $\nabla^2 V$). Within the approximation assumption both effects are mutually independent.

3. Uniform amplitude illumination

Let the spherical aberration be taken as an example of the phase distortion introduced by the plate D_1. This means that according to the notation of HOPKINS [3] we have

$$W_1(a_n) = W_{20}a_n^2 + W_{40}a_n^4 + W_{60}a_n^6 \tag{14}$$

where besides the primary aberration W_{40} and the 5-th order spherical aberration W_{60} the coefficient of the defocusing W_{20} is needed. Because $(\mathrm{grad}\, W_1)^2 = 4W_{sf}^2$ [2], where

$$W_{sf}(a_n) = a_n(W_{20} + 2W_{40}a_n^2 + 3W_{60}a_n^4), \tag{15}$$

then according to (13) [$\nabla^2 V = 0$], the following equation holds:

$$\delta W(a_n) = -2GW_{sf}^2(a_n). \tag{16}$$

This is the required equation describing the changes of the spherical aberration introduced by the plate D_1 and related to the plane of the plate D_2 (Fig. 1). Consequently, the quantity δW can be significant for high aberration values or for the value of the quantity G not much less than 1. The last equation, according to (5), is related to great distances z as compared to the diameter $2a_0$. More precisely, the term δW ought to be taken into account if z is of order of a_0^2/λ. Thus, this quantity well corresponds to the light area [4].

As we have stated before, the approach used here is not valid for larger distances than mentioned above, because of limitation of the approximate Eq. (4).

4. Variable amplitude illumination

Since according to (13) the influences of phase and amplitude distributions on the wave propagation are independent, for the sake of simplicity, a constant phase distribution is assumed.

Let

$$V_1'(a_n) = V_0 \exp(-a_n^{2m}) \tag{17}$$

be an example of the amplitude distribution in the plane of the plate D_1 (Fig. 1). The circular symmetry is taken, then according to (6) we have

$$\nabla^2 V_1'(a_n) = \frac{\partial^2 V_1'}{\partial a_n^2} + \frac{1}{a_n}\frac{\partial V_1'}{\partial a_n}. \tag{18}$$

After substituting (17) into (18), and taking account of (13), we can write

$$\delta W(a_n) = \frac{G}{2\pi} W_{am}(a_n), \tag{19}$$

where

$$W_{am}(a_n) = m a_n^{2(m-1)}(m a_n^{2m} - 1), \tag{20}$$

because grad $W = 0$.

For Gaussian beam ($m = 1$) we have

$$W_{a1}(a_n) = a_n^2 - 1. \tag{21}$$

The Gaussian apodization, in addition to the axial phase changes, introduces spherical deformations of the beam wavefronts only [5].

The apodization with higher coefficient m introduces some aberrational phase changes. In particular, for cases of $m = 2$ and 3, the following equations hold:

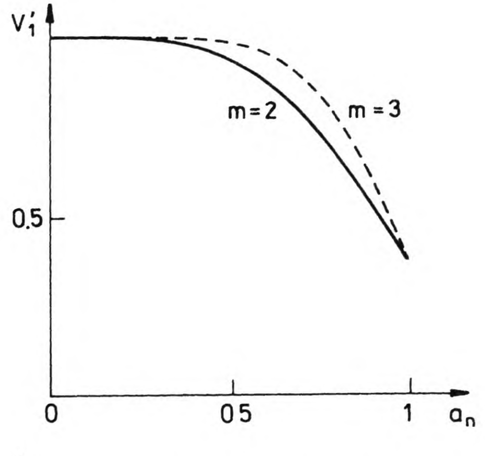
$$W_{a2}(a_n) = 4a_n^6 - 2a_n^2, \tag{22}$$

$$W_{a2}(a_n) = 9a_n^{10} - 3a_n^4. \tag{23}$$

The normalized amplitude distribution

$$V'_{1n} = \frac{V'_1}{V_0} = \exp(-a_a^{2m}) \tag{24}$$

and the relevant phase functions $W_{am}(a_n)$ ($m = 2, 3$) are shown in Fig. 2 and Fig. 3, respectively. We observe the edge effect. Strong amplitude changes (increase of m) bring an increase of the value of aberrational phase changes, while the area of significant changes decreases.



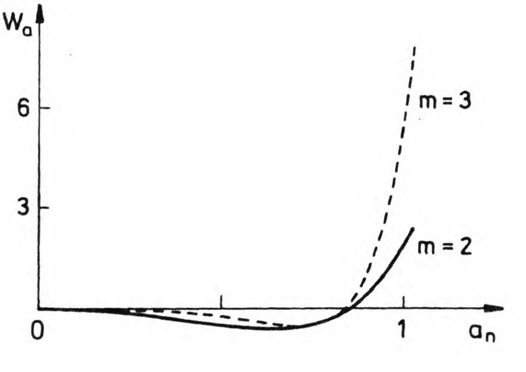


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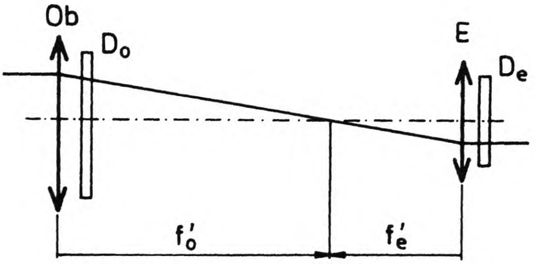
Fig. 2. Normalized amplitude distribution at the first distorter plane

Fig. 3. Phase function W_a defined at the second distorter plane and introduced by the amplitude distribution at the first distorter plane (see Fig. 2, for comparison)

It is worth emphasizing once more that Equation (4), being basic to our considerations, is approximate and any extrapolation of conclusions for a high value of m requires the study of higher terms of the expression (4) [2].

5. Practical remarks

The results of the aberration compensation obtained for two separate plates will be shown for the case of a telescope, as an example (Fig. 4). In the design of this type the compensation operation is frequently used in practice and, moreover, the telescopic system is typical of the laser systems. For the sake of simplicity, both the objective (Ob) and the eyepiece (E) are assumed to be thin and only spherical aberration will be taken into consideration.



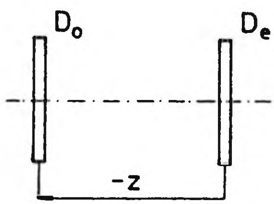


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Fig. 4. Optical system of a telescope

Fig. 5. Substitutional system of the telescope from Fig. 4 consisting of two distorters

Aberrated elements, such as the objective or the eyepiece, can be treated as combined systems consisting of a focusing element and a distorter [1]. The focusing element is such a part of any aberrated element which images objective field distributions without any distortion. On the other hand, distorters represent such factors like aberrations and apodization, which disturb the theoretical imaging relation. Since the distorter can be transferred by focusing elements to an arbitrarily chosen space of the optical system, the imaging of a real optical system can be consequently divided into two parts: the imaging by a set of distorters and the imaging by the corresponding focusing elements. However, the imaging by a system of focusing elements is perfect, which means that from the point of view of aberration analysis the imaging by the focusing elements can be omitted.

The optical system of a telescope contains two distorters D_0 and D_e (Fig. 4). Their transfer to the image space of the whole telescope is most convenient, because in this space the image plane is located at infinity, and, moreover, the diameter of the exit pupil in visual instruments is approximately constant.

The substitutional system of the distorters (without any focusing element) in the image space of the microscope is shown in Fig. 5. The distorter D'_0 is the image of the distorter D_0 given by the focusing element of the eyepiece U_e, which means that the distance between the distorters can be found from the relation $z = -f'_e(1+f'_e/f'_o)$, where f'_o and f'_e are the focal lengths of the objective and the eyepiece, respectively (see Fig. 4). Moreover, the transmittance of the distorter D'_o equals the transmittance of the distorter D_o at all conjugate points. The sign of z is negative because the image D'_o is located at right hand side of the eyepiece. The configuration of Fig. 5 with $z < 0$ is more convenient, because it emphasizes the real sequence of the elements in the optical system (the first one — objective, and the second — eyepiece).

In the case of uniform amplitude illumination the change of the mutual compensation condition (2) for the aberrations of the objective and the eyepiece is given by (16), where according to (5)

$$G = -\frac{2\lambda F}{a_0}\left(1-\frac{1}{\gamma}\right). \tag{25}$$

The quantity $2a_0$ is the diameter of the exit pupil of the telescope, $\gamma = -f'_o/f'_e$ is its angular magnification, $F = 0.5f'_e/a_0$ is its nominal focal ratio. For a typical construction of the telescope we have $2a_0 = 5$ mm, $F = 4$, $\gamma = -6$, $\lambda = 550$ nm, and consequently we have $G = -0.00205$. According to (16) for the objective and the eyepiece independently corrected (small values of W_{sf}) the change of the aberration compensation condition, as compared to the geometrical one, is not significant. The role of the wave approach increases for optical systems of small diameter $2a_0$ of their exit pupils and the high value of their nominal focal ratio F. The biological microscope belongs to such systems, however, in this case the objective is perfectly corrected.

Any nonuniform distribution introduces phase changes into the propagating wave. In the case of a Gaussian beam transmitted through a telescope, since the corresponding phase changes are small (see Eq. (19)), they can be compensated by the defocusing only. In other words, the optical laser system can be corrected as a system without apodization, and the proper shape of the wavefront of the image space can be reached by a proper adjustment of the distance between the elements of the system.

An apodization different from the Gaussian one introduces some aberrational changes, which are larger for stronger amplitude changes, however, in this case the area of the significant phase changes decreases and it occurs nearby the amplitude edge.

6. Conclusions

1. The change of the aberration compensation condition for two distanced objectives (or plates) related to the second objective is introduced by both the aberration and the apodization of the first objective. The influence of both factors mentioned has an additive character.

2. Gaussian beam does not require any special correction. It is sufficient to design a system without any apodization and next to set-up a proper defocusing.

3. For an optical system composed of well corrected objectives, usually the geometrical compensation is sufficient. The phase changes of the wavefront acquired upon the propagation between two objectives may be significant in the case of the objectives with large aberrations. However, such a case concerns systems of poor quality and in practice, the ray tracing through the whole system is used.

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