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Adjustment of image locations in zoom systems

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A simple method of determination of the derivatives of image locations with respect to both system parameters and the movements of the system components is proposed. A method of two adjusting displacements is suggested basing on selection of two adjusting components and removal of the deviation of the image position in the two extreme arrangements of the zoom system. The simulation process of a deviated zoom system in its most unfavourable form both in plus and in minus is described and the adjustment of the image position discussed. As an example the adjustment of a four-element zoom objective is given.

1. Introduction

A zoom system whose both optical powers Φ_i are determined, for $i = 1,\dots,n$, and kinetics of its components described by the due spaces $e_i(t)$, where t – the parameter of the working cycle $t = 1 \div 1$, suffers from executory deviations occurring during its production. These executory deviations for a thin-component model of a zoom system are composed of: the deviations of optical powers $\Delta\Phi_i$ the deviations of spaces between the components Δe for the starting arrangement and ΔE for the final arrangement, as well as the deviations of component movements Δm and ΔM for the same respective arrangements. A theoretical zoom system charged with hypothetical executory deviations is called deviated zoom system.

The problem of adjustment of image position in optical zoom systems has hardly been represented in publications so far. Here, papers by Stephansky [1], Shiue [2] and Ardashnikov [3] may by mentioned. These works are valuable attempts at explaining the complex problem of adjustment, which occurs in a particularly sharp form in the zoom systems. In the paters cited, some simplified methods of computational determination of the influences of the movement of particular components on the image position are presented. It is either all the components of a four-component zoom system or only two central ones that are usually taken a priori as the adjusting components. The lack of estimation method of the residual misadjustment remaining in spite of the attempts tof minimize it using the root mean square method is observed.

The best method of achieving the optimal kinetics of the components in the deviated zoom system would be to determine each time the kinetics of components taking account of the appearing executory deviations. Unfortunately, this would be rather expensive and difficult in realization. Therefore, instead of this, there exists a tendency to apply the kinetics determined for a theoretical zoom system to the real deviated zoom system.

Thus, the deviated zoom system requires some adjustment since in such systems some perturbations of the earlier achieved stability of the image position are observed. It is necessary to establish which components of the system will be treated as the adjusting ones and what range of their movement must be assured. The adjusting movements should be taken into account using the trial and error method as suitable extra spaces added to the spaces between the components.

The adjustment of the image position includes the correction of the positioning of the components in the deviated zoom system in order to achieve the image position stability which would be close to the theoretical one. Thus, the adjustment of the image position touches the most essential problems connected with the realization of a zoom system with mechanical compensation.

2. Identification of the transformation matrices

Optical imaging of a paraxial ray in a thin-component optical system (optical system composed of thin elements) may be expressed in terms of matrix calculus. The description of such an approach may be found in two known books by BROUWER [4], and GERRARD and BURCH [5].

In this work, the matrix notation is applied to determine the coordinates of the paraxial aperture ray (the height h and the angle α with the optical axis) in a zoom system. The single optical imaging of the aperture ray by each component of optical power Φ and the space e prescribed to the starting arrangement is composed of the transition of the height from one component denoted by the conventional index -1 to the next component and the refraction of the ray on this component according to the following formulae:

$$\begin{aligned} \mathrm{h} &= \mathrm{h}_{-1} - e_{-1}\alpha_{-1}, \\ \alpha &= \Phi \mathrm{h}_{-1} + (1 - \Phi e_{-1})\alpha_{-1}. \end{aligned} \tag{1}$$

Formulae (1) represent the transformation of linear coordinates of the aperture ray, which may be alternatively written in the matrix form

$$\begin{bmatrix} h \\ \alpha \end{bmatrix} = [M] \begin{bmatrix} h \\ \alpha \end{bmatrix}_{-1}, \tag{2}$$

where $[M]$ is an elementary matrix of transformation for a single component, the elements of which are expressed in the following way:

$$[M] = \begin{bmatrix} 1 & -e_{-1} \\ \Phi & -\Phi e_{-} \end{bmatrix}. \tag{3}$$

As it can be seen from formula (3), the elements of the matrix $[M]$ for a single component are expressed in terms of construction data. The optical imaging by a zoom system of k components is obtained by repeating the transformation (2) for the subsequent components of the system, which leads to the following matrix notation:

$$\begin{bmatrix} \mathrm{h} \\ \alpha \end{bmatrix}_k = [M]_{1,k} \begin{bmatrix} \mathrm{h} \\ \alpha \end{bmatrix}_{-1}, \tag{4}$$

where $[M]_{1,k}$ is the transformation matrix for the whole zoom system of elements denoted in the following way:

$$[M]_{1,k} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{1,k}. \tag{5}$$

The method of expressing the optical imaging in the matrix form is often called the *ABCD* method in accordance with the system of elements in formula (5). The transformation matrix $[M]_{1,k}$ for the whole system is a product of transformation matrices connected with the subsequent components taken in reverse sequence. The elements of the matrix $[M]_{1,k}$ for the whole system are also expressible by the construction data while the distance to the object (e_0) or the distance to the reference plane are included into considerations. The immediate expression of the elements of the transformation matrix $[M]_{1,k}$ by the construction parameters of the zoom system leads to very complicated formulae, therefore, here these elements will be expressed in an indirect way.

Let two tentative runs of paraxial rays be determined: the one of coordinates b, β, and the other of coordinates c, γ (given in the sequence: incidence height, angle between the ray and the axis). It is assumed that the runs of these rays differ significantly, which means that the Lagrange–Helmholtz invariant for those rays is definitely different from zero. The optical imaging of the tentative rays by a four-component zoom system may be represented with the aid of the following matrix transformations:

$$\begin{bmatrix} \mathrm{b} \\ \beta \end{bmatrix}_k = [M]_{1,k} \begin{bmatrix} \mathrm{b} \\ \beta \end{bmatrix}_0, \quad \begin{bmatrix} \mathrm{b} \\ \beta \end{bmatrix}_k = [M]_{1,k} \begin{bmatrix} \mathrm{b} \\ \beta \end{bmatrix}_0. \tag{6}$$

In both the formulae (6) the transformation matrix is the same. The optical imaging (6) is determined when the transformation matrix $[M]_{1,k}$ is known. Below, a reverse problem is presented, namely, it is assumed that the transformation matrix is unknown, while the coordinates of both the tentative paraxial rays are determined at the rim of the zoom system. The reverse problem will be called the identification of the transformation matrix elements with the help of two tentative paraxial rays. Formulae (6) after transformation represent a double system of two linear equations with four unknowns, which are the elements of the transformation matrixc $[M]_{1,k}$. This set of equations is easy to solve, since the unknowns occur in pairs as it is given by the following formulae:

$$\begin{aligned} &\mathrm{b}_0 A_{1,k} + \beta_0 B_{1,k} = \mathrm{b}_k, \\ &\mathrm{c}_0 A_{1,k} + \gamma_0 B_{1,k} = \mathrm{c}_k, \\ &\mathrm{b}_0 C_{1,k} + \beta_0 D_{1,k} = \beta_k, \\ &\mathrm{c}_0 C_{1,k} + \gamma_0 D_{1,k} = \gamma_k. \end{aligned} \tag{7}$$

The solution of the system of equations (7) determines the sought elements of the transformation matrix $[M]_{1,k}$ and leads to the following formulae:

$$
\begin{aligned}
&A_{1,k} = (\beta_0 \mathrm{c}_k - \gamma_0 \mathrm{b}_k)/J,\\
&B_{1,k} = (\mathrm{c}_0 \mathrm{b}_k - \mathrm{b}_0 \mathrm{c}_k)/J,\\
&C_{1,k} = (\beta_0 \gamma_k - \gamma_0 \beta_k)/J,\\
&D_{1,k} = (\mathrm{c}_0 \beta_k - \mathrm{b}_k \gamma_k)/J
\end{aligned}
\tag{8}
$$

where J is a generalized form of the Lagrange–Helmholtz invariant for the tentative paraxial rays, which can be calculated in an arbitrary place of the zoom system from the following formula:

$$J = (\beta c - \gamma b)_i. \tag{9}$$

Formulae (8) are designed from the expressions of determinant type in which the coordinates of the tentative paraxial runs at the boundary of the zoom system as well as the Lagrange–Helmholtz invariant appear together.

The elements of the transformation matrix $[M]_{1,k}$ as expressed in formulae (8) and the generalized invariant J defined by formula (9) take a simple form when the coordinates of the tentative rays are normalized, *i.e.*, chosen in a special way. This normalization will consist in the following. The coordinates of the two tentative paraxial rays incident on the last component take some specially selected values in both the starting arrangement (lower case) and final arrangement (capital letters) according to the following situations:

$$
\begin{aligned}
&\mathrm{b}_k = 1, \quad \beta_k = 0, \quad \mathrm{c}_k = 0, \quad \gamma_k = -1, \quad J = 1,\\
&\mathrm{B}_k = 1, \quad B_k = 0, \quad \mathrm{C}_k = 0, \quad \Gamma_k = -1, \quad J = 1.
\end{aligned}
\tag{10}
$$

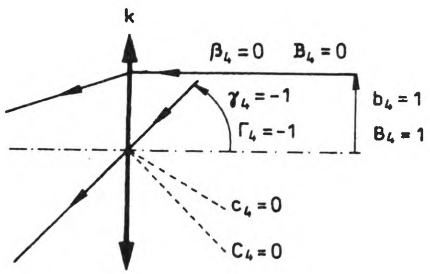

Fig. 1. Normalization of the tentative paraxial rays incident on the last component of the optical system

The normalization of the tentative paraxial rays incident on the last component accepted in this work is shown in Fig. 1. The other coordinates of both tentative paraxial rays incident on each component of the system are obtained by applying a recurrence scheme for paraxial backward run according to the following formulae:

$$
\begin{aligned}
&\beta_{i-1} = \beta_i - \mathrm{b}_i \Phi_i,\\
&\mathrm{b}_{i-1} = \mathrm{b}_i + \beta_{i-1} e_{i-1},
\end{aligned}
$$

$$\gamma_{i-1} = \gamma_i - c_i \Phi_i,$$
$$c_{i-1} = c_i + \gamma_{i-1} e_{i-1} \tag{11}$$

where the component index $i = k, k-1, \ldots, 1$.

In accordance with our intention, the expressions for the elements of the transformation matrix $[M]_{1,k}$ for the tentative rays being normalized according to (10) are simplified to the following formulae:

$$A_{1,k} = -\gamma_0,$$
$$B_{1,k} = c_0,$$
$$C_{1,k} = -\beta_0,$$
$$D_{1,k} = b_0. \tag{12}$$

As it may be seen from formulae (12), the elements of the transformation matrix $[M]_{1,k}$ in the presence of the normalization (10) are identified with the aid of single coordinates of the tentative paraxial rays taken in the object plane ($i = 0$) and determined with the aid of the recurrence scheme (11). The simplifications resulting from normalization cause that the dimension analysis may not be applied to the elements $C_{1,k}$ and $D_{1,k}$ of the transformation matrix in formulae (12) (due to hidden division by J).

3. Method of determination of the partial derivatives

The optical imaging in the matrix notation is exploited in order to determine the partial derivatives of the aperture ray with respect to construction parameters of the system, such as: optical power of each component, space between the components and the component movement.

The deviation parameter of the index i divides the zoom system into parts, the most attractive of which is that positioned behind the deviation parameter. Thus, we need the separated partial transformation matrices $[M]_{i+1,k}$ which are constructed, analogically to (4), in the following way:

$$\begin{bmatrix} h \\ \alpha \end{bmatrix}_k = [M]_{i+1,k} \begin{bmatrix} h \\ \alpha \end{bmatrix}_i. \tag{13}$$

By applying the differential operator $\partial/\partial\Phi_i$ to the matrix formula (13), the following equations is obtained:

$$\frac{\partial}{\partial\Phi_i} \begin{bmatrix} h \\ \alpha \end{bmatrix}_i = [M]_{i+1,k} \frac{\partial}{\partial\Phi_i} \begin{bmatrix} h \\ \alpha \end{bmatrix}_i. \tag{14}$$

The separated transformation matrix $[M]_{i+1,k}$ occurring in Eq. (14) may be also identified with the aid of the coordinates of the tentative paraxial runs taking account of the normalization according to substitution (10) in a way similar to that used for the whole system according to formulae (12). However, in this case the change of the component index causes that the following formulae are obtained:

$$A_{i+1,k} = -\gamma_i,$$
$$B_{i+1,k} = c_i,$$
$$C_{i+1,k} = -\beta_{1i},$$
$$D_{i+1,k} = b_i. \qquad (15)$$

The elements of the separated transformation matrix $[M]_{i+1,k}$ preserve constant values during differentiation under the condition that the deviation parameter is positioned directly in front of the separated part. The elements of the separated transformation matrix $[M]_{i+1,k}$ are essentially dependent on the construction parameters of the system positioned behind the deviation parameter and therefore they preserve the constant value.

As it may be seen from formulae (15), the elements of the separated transformation matrix $[M]_{i+1,k}$ obtained after taking account of the normalization consistent with the substitution (10) are identified also with the aid of single coordinates of the tentative paraxial rays taken at the component of index i.

The partial derivatives of the coordinates of the aperture ray incident on a component with respect to the power of this component may be determined from the elementary imaging (1) which results in obtaining the following formulae:

$$\frac{\partial h_i}{\partial \Phi_i} = 0, \quad \frac{\partial \alpha_i}{\partial \Phi_i} = \mathrm{h}_i. \qquad (16)$$

The partial derivatives of the final coordinates of the aperture ray with respect to the optical power Φ_i are obtained by substituting the groups of formulae (16) and (15) to Eq. (14), which leads to the following formulae:

$$\frac{\partial \mathrm{h}_k}{\partial \Phi_i} = \mathrm{c}_i \mathrm{h}_i, \quad \frac{\partial \alpha_k}{\partial \Phi_i} = \mathrm{b}_i \mathrm{h}_i. \qquad (17)$$

As it may be seen from formulae (17), the partial derivatives of the final coordinates of the aperture ray with respect to the optical power of the component of index i are expressed by the product of the incidence height of the aperture ray and the incidence height of the corresponding tentative paraxial ray taken at the component of deviation power considered.

By acting with a differential operator $\partial/\partial e_{i-1}$ on the matrix formula (13), the following equation can be obtained:

$$\frac{\partial}{\partial e_{i-1}} \begin{bmatrix} \mathrm{h} \\ \alpha \end{bmatrix}_i = [M]_{i+1,k} \frac{\partial}{\partial e_{i-1}} \begin{bmatrix} \mathrm{h} \\ \alpha \end{bmatrix}_i. \qquad (18)$$

The elements of the separated transformation matrix $[M]_{i+1,k}$ are identified with the aid of substitution (15) in the same way as it was the case previously.

The partial derivatives of the coordinates of the aperture ray incident on the component of the index i with respect to the space of index $i-1$ can be determined from the elementary imaging (1), thus, obtaining the following formulae:

$$\frac{\partial h_i}{\partial e_{i-1}} = -\alpha_{i-1}, \quad \frac{\partial \alpha_i}{\partial e_{i-1}} = -\alpha_{i-1}\Phi_i. \tag{19}$$

The partial derivatives of the final coordinates of the aperture ray with respect to the space between the components e_i are obtained by substituting the group of formulae (19) and (15) to formula (18), shifting the index from $i-1$ to i and performing the necessary rearrangements. This procedure leads to the following formulae:

$$\frac{\partial h_k}{\partial e_i} = \gamma_i \alpha_i, \quad \frac{\partial \alpha_k}{\partial e_i} = \beta_i \alpha_i. \tag{20}$$

As it may be seen from formulae (20), the partial derivatives of the aperture ray coordinates with respect to the space of index i are expressed by a product of the angle made by the tentative ray with the axis and the angle of the suitable tentative paraxial ray with the axis both taken at the component located at the beginning of the deviated space.

Formulae (17) and (20) derived till now concern initial arrangement of the zoom system. For further considerations we need the coordinates of the tentative paraxial rays for the final arrangement of the zoom system which are denoted by capital Latin letters (heights) and italic capital Greek letters (angles) – B, B and C, Γ:

$$\frac{\partial \mathrm{H}_k}{\partial \Phi_i} = \mathrm{C}_i \mathrm{H}_i, \quad \frac{\partial \mathrm{H}_k}{\partial \mathrm{E}_i} = \Gamma_i A_i,$$

$$\frac{\partial A_k}{\partial \Phi_i} = \mathrm{B}_i \mathrm{H}_i, \quad \frac{\partial A_k}{\partial \mathrm{E}_i} = B_i A_i. \tag{21}$$

Formulae (21) are valid only when the runs of the tentative rays suitable for the final arrangement denoted by capital letters are normalized similarly: $\mathrm{B}_k = 1$, $B_k = 0$, $\mathrm{C}_k = 0$, $\Gamma_k = -1$ for $J = 1$, as it has been shown in Fig. 1.

The image position in the zoom system in both the extreme arrangements of the latter is denoted by the spaces s'_k and S'_k, respectively. The image positions can be determined from the final coordinates of the aperture ray at the last component according to the following formulae:

$$s'_k = \frac{\mathrm{h}_k}{\alpha_k}, \quad S'_k = \frac{\mathrm{H}_k}{\mathrm{A}_k}. \tag{22}$$

The partial derivatives of the image positions with respect to the construction parameter x are as follows:

$$\frac{\partial s'_k}{\partial x} = \left(\frac{\partial \mathrm{h}_k}{\partial \mathrm{x}}\alpha_k - \frac{\partial \alpha_k}{\partial \mathrm{x}}\mathrm{h}_k\right)\Big/\alpha_k^2,$$

$$\frac{\partial S'_k}{\partial x} = \left(\frac{\partial \mathrm{H}_k}{\partial \mathrm{x}}A_k - \frac{\partial A_k}{\partial x}\mathrm{H}_k\right)\Big/A_k^2. \tag{23}$$

The partial derivatives of the image positions with respect to the optical power of the component $x = \Phi_i$ are obtained after the substitution of the partial derivatives of the final coordinates of the aperture ray from the group of formulae (20) and (21) to formulae (23) in two extreme arrangements, which leads to the following derivatives:

$$\frac{\partial s'_k}{\partial \Phi_i} = (c_i h_i \alpha_k - b_i h_i h_k)/\alpha_k^2,$$

$$\frac{\partial S'_k}{\partial \Phi_i} = (C_i H_i A_k - B_i H_i H_k)/A_k^2. \tag{24}$$

The partial derivatives of the image positions with respect to the distance $x = e_i$ are obtained similarly after substituting the partial derivatives of the final coordinates of the aperture ray with respect to the space between the components from the groups of formulae (20) and (21) to formulae (23) in two extreme arrangements, which leads to the following derivatives:

$$\frac{\partial s'_k}{\partial e_i} = (\gamma_i \alpha_i \alpha_k - \beta_i \alpha_i h_k)/\alpha_k^2,$$

$$\frac{\partial S'_k}{\partial E_i} = (\Gamma_i A_i A_k - B_i A_i H_k)/A_k^2. \tag{25}$$

The movement of the system component consists in a change of two corresponding neighbouring spaces by the same values but of opposite signs. The movement of the component of the index i depending on the extreme arrangement will be denoted by Δm_i and ΔM_i, respectively.

The change in the image position in both the extreme arrangements caused by a sufficiently small movement of the component is a total derivative of the image position evoked by the changes of the neighbouring spaces Δe and ΔE in accordance with the following formulae:

$$\frac{\partial s'_k}{\partial m_i}\Delta m_i = \frac{\partial s'_k}{\partial e_{i-1}}\Delta e_{i-1} + \frac{\partial s'_k}{\partial e_i}\Delta e_i,$$

$$\frac{\partial S'_k}{\partial M_i}\Delta M_i = \frac{\partial S'_k}{\partial E_{i-1}}\Delta E_{i-1} + \frac{\partial S'_k}{\partial E_i}\Delta E_i. \tag{26}$$

The changes of the neighbouring spaces depend on the movement of the component according to the substitutions:

$\Delta e_{i-1} = \Delta m_i$ and $\Delta e_i = -\Delta m_i$ — for the starting arrangement,
$\Delta E_{i-1} = \Delta M_i$ and $\Delta E_i = -\Delta M_i$ — for the final arrangement.

The partial derivatives of the image position with respect to the component of the index i in both the extreme arrangements are obtained by substituting the said changes of the spaces to formulae (26) which, after some simplifications, leads to the following derivatives:

$$\frac{\partial s'_k}{\partial m_i} = \frac{\partial s'_k}{\partial e_{i-1}} - \frac{\partial s'_k}{\partial e_i},$$

$$\frac{\partial S'_k}{\partial M_i} = \frac{\partial S'_k}{\partial E_{i-1}} - \frac{\partial S'_k}{\partial E_i}. \tag{27}$$

As it may be seen from formulae (27), the partial derivatives of the image position with respect to the component movement are expressed by the differences of derivatives with respect to the neighbouring spaces surrounding the moving component.

The derived formulae for the partial derivatives of the image positions (24), (25) and (27) play an important part while simulating the deviated zoom system and adjusting the image position.

4. Method of two adjusting movements

The adjustment of the image positions in the zoom systems is realized with the help of adjusting movements, which are the shifts of the selected components of the system. Two types of the adjusting movements may be distinguished – independent and dependent ones. The aim of the independent adjusting movement is to change one definite parameter of the imaging without simultaneous changing of the other imaging parameters. A typical example of independent adjusting movement is the adjustment of the image sharpness in the stationary optical systems. In the deviated zoom systems the dependent adjusting movements occur in most cases. The dependent adjusting movement influences several imaging parameters. The dependent adjusting movements are inconvenient in practice since the adjusting process is then more complicated and can be performed by stages in the consecutive steps called adjusting iterations. In this case, the imaging parameters are corrected successively with the aid of the chosen adjusting components.

It is assumed that the zoom system is charged with the following executory deviations: $\Delta\Phi$, Δe, ΔE, Δm and ΔM. In both the extreme arrangements of the zoom system the obtained image positions are different from theoretical ones. The one-sided extreme deviations Δ_{ext} of the image positions in both the extreme arrangements and in the most disadvantageous case when all the executory changes influence the image position in the only direction can achieve the following values:

$$\Delta_{ext} s'_k = \pm\left(\sum_{i=1}^{k}\left|\frac{\partial s'_k}{\partial \Phi_i}\Delta\Phi_i\right| + \sum_{i=0}^{k-1}\left|\frac{\partial s'_k}{\partial e_i}\Delta e_i\right| + \sum_{i=1}^{k}\left|\frac{\partial s'_k}{\partial m_i}\Delta m_i\right|\right),$$

$$\Delta_{ext} S'_k = \pm\left(\sum_{i=1}^{k}\left|\frac{\partial S'_k}{\partial \Phi_i}\Delta\Phi_i\right| + \sum_{i=0}^{k-1}\left|\frac{\partial S'_k}{\partial E_i}\Delta E_i\right| + \sum_{i=1}^{k}\left|\frac{\partial S'_k}{\partial M_i}\Delta M_i\right|\right). \tag{28}$$

As it may be seen from formula (28), the extreme deviations of the image position depend not only on the accepted executory deviations, in other words, on the execution standard, but also on the partial derivatives of the image position with respect to all the construction parameters of the system. This means that the construction of the theoretical zoom system influences the deviation values for the extremal image positions.

The most important task in the adjustment process of a deviated zoom system is the removal of the image position deviations appearing in both the extreme arrangements. The solution of this problem requires two adjusting movements of two chosen components of the system. In this form the method of two adjusting movements is sufficiently simple being simultaneously very effective. The choice of the adjusting components is made preliminarily by analysing the image position derivatives with respect to the movement of the components in both the extreme arrangements. The sufficient condition is that, at least, one of the adjusting components must influence, in a significantly different way, the image position in both the extreme cases. In the case when there exists a possibility of creating a number of combinations of two adjusting movements the choice of one of them is decided by an accurate check.

Shiue proposed in paper [2] to exploit the attenuated method of least squares in order to determine the adjusting movements which are realized by all the components of the system. Under this principle the zoom system, after having been adjusted, shows some residual disadjustment in the extreme arrangements which is minimized in the whole working cycle. Such an approach to the adjustment deviates significantly from the praxis. Usually, the user sets the sharpness in one critical extreme position. The zoom system should be adjusted in such a way that the sharpness in the other extreme arrangement be assumed simultaneously and the deviation of the image position in the intermediate arrangements be acceptable.

The adjusting components will be denoted by the indices j_1 and j_2 while the adjusting movements – by Δm_{j1}, ΔM_{j1} and Δm_{j2}, ΔM_{j2}, respectively. The preliminary adjusting movements are determined in the nominal system from the conditions for compensation of the extreme deviations of the image positions determined according to formulae (28).

The adjusting equations of the zoom system are constructed in the form of total derivatives of values opposite to the extremal image positions as it is indicated in the following formulae:

$$\frac{\partial s'_4}{\partial m_{j2}}\Delta m_{j1}+\frac{\partial s'_4}{\partial m_{j2}}\Delta m_{j2} = \mp \Delta s'_4,$$

$$\frac{\partial S'_4}{\partial M_{j1}}\Delta M_{j1}+\frac{\partial S'_4}{\partial M_{j2}}\Delta M_{j2} = \mp \Delta S'_4. \tag{29}$$

In the adjusting equation (29), we took account of the fact that the adjusting movement of the components in both the extreme arrangements must be the same as that for the component of the index j, which can be written as follows:

$$\Delta m_j = \Delta M_j. \tag{30}$$

The condition for solving the set of Eqs. (29) is that the determinant of the matrix of the image position derivatives with respect to the movement of the adjusting components be definitely different from zero, which means that the derivatives of the image position with respect to, at least, one adjusting component should be sufficiently differentiated in both the extreme arrangements. The linear adjusting equations according to formulae (29) are valid only in the direct vicinity of the nominal zoom system. The deviations of the image positions in both the extreme arrangements of the deviated zoom system determined from the runs of the rays differ slightly from those determined from the adjusting equations, since both the executory deviations and the adjusting movements change the zoom system, thus making the derivatives and the extreme deviations of the image positions calculated for the nominal zoom system no more timely.

6. Residual disadjustment and its evaluation

An exact determination of the residual disadjustment in a complete working cycle as well as the needed ranges of adjusting movements requires performance of simulation for both the deviated zoom system and the adjusting process.

The simulation of the deviated zoom system consists in introducing the executory deviations to the nominal zoom system. The executory deviations may be preliminarily assumed as a certain standard of performance described by the extreme deviations. The aim of this analysis is to determine both the maximal residual disadjustment and the maximal ranges of adjusting movements. The obtaining of the most deviated zoom system in plus and in minus requires a suitable accounting of sings of extreme executory deviations depending on the signs of respective derivatives. In the preliminary analyses the standard values of executory deviations are assumed which often occur in the system realized. The standard executory deviations of optical powers, spaces and displacements of moving components (in other words, the accuracy of performance of cams) are assumed in accordance with the following specification:

$$\Delta\Phi = 0.075\Phi, \quad \Delta e = \Delta E = 0.15, \quad \Delta m = 0.015. \tag{31}$$

The accepted values of the standard deviations according to the specification (31) are greater than those given by ARDASHNIKOV [3] (0.005, 0.1, and 0.01, respectively). The deviations accepted in accordance with (31) may be treated either as the extreme deviations for high standard of performance or as the probable deviations for a lower standard.

The simulation of the adjusting process of the image position requires an iterative approach consisting in exploiting the actualized derivatives determined for the deviated system (executory deviations and successive approximations of the adjusting movements) and the image position deviations calculated from the paraxial runs to the adjusting equations according to formulae (29). The successively calculated corrections of the adjusting movements are less and less and hence the accuracy of linear adjusting equations is better and better.

In the intermediate working arrangements of an adjusted deviated zoom system when the theoretical kinematics of the nominal system is exploited, some residual disadjustment of the image position is observed which cannot be avoided but may be diminished either by selecting the suitable adjusting movements, and differentiating the executory deviations or more radically by changing the data conditioning determination of the zoom system its optical power and the kinetics of its components.

The residual disadjustment is determined in a simulated zoom system including the executory deviations by using the adjusting movements and theoretical kinematics of components with the aid of the paraxial runs in all the considered working points of the zoom system.

As a general measure of the residual disadjustment, the following quantities may be taken: maximal and minimal values in a arithmetic average and root mean square deviations calculated from the following formulae:

$$\delta S'_{\max} = \max_j (ds'_j), \quad \delta s'_{\min} = \min_j (\delta s'_j),$$

$$\delta s'_{\mathrm{av}} = \frac{\sum\limits_j \delta s'_j}{N}, \quad \delta s'_{\mathrm{RMS}} = \sqrt{\frac{\sum\limits_j (\delta s'_j - \delta s'_{\mathrm{av}})^2}{N}} \tag{32}$$

The evaluation of the residual disadjustment in a deviated zoom system is connected with the depth of image position G_d which from the diffraction condition for the one-sided deviation is expressed by the following formula:

$$G_d = \pm \frac{\lambda}{2\mathrm{A}^2} \tag{33}$$

where λ is the light wavelength, A is the numerical aperture behind the system.

In the case when the imaging quality of the zoom system is far from the quality of the diffraction limited system dropping down below 1/10 of its resolving power ($s \leqslant 0.2$), the focusing depth is calculated from an empirical formula given by HOPKINS [6] in the following form:

$$G_{\mathrm{H}} = \pm \frac{0.2}{RA} \tag{34}$$

where R is the resolving power of the system in lp/mm.

The zoom system is suitably adjusted in the required resolution range when the residual disadjustment $\delta s'_j$ in the whole working cycle does not exceed the image depth

$$\delta s'_j \leqslant G. \tag{35}$$

In other cases, the zoom system is assumed to be adjusted when its residual disadjustment in its whole working cycle does not exceed the admissible depth matched to the given type of receiver.

7. Numerical example

The above method of image position adjustment will be illustrated by an example of four-component zoom objective of focal length $f = 10-100$ mm and the foreseen relative aperture $1{:}K = 1{:}1.6$. The optical powers of the components have been determined using an algorithm given in paper [7] and expressed in terms of focal lengths f given in Tab. 1.

In Table, 2 the coordinates of the aperture rays and tentative rays of a nominal objective $f10-100$ have been given for two extreme arrangements separated by a horizontal line.

In Table 3, the derivatives of the image position with respect to the executory parameters have been given for nominal variant of the objective $f10-100$ determined from the coordinates given in Tab. 2 in accordance with the method described in Sect. 3 of this work.

Table 1. Data of the objective $f = 10-100$

i	f	e	E
1	125.974240	15.0000	92.0000
2	−25.4206367	87.0000	19.0000
3	119.578418	59.0000	50.0000
4	32.0282993	32.0000	32.0000

Table 2. Coordinates of the aperture rays and the tentative rays of the objective $f = 10-100$

i	0	1	2	3	4
h	5.0000000	5.0000000	4.4046402	16.0260654	16.0000000
α	0.0000000	0.0396907	−0.1335796	0.0004418	0.5000000
b	−5.0466989	−5.0466989	−2.9457785	−0.8421209	1.0000000
β	−0.1000000	−0.1400614	−0.0241800	−0.0312224	0.0000000
c	−171.49436	−171.49436	−103.07419	−59.0000000	0.0000000
γ	−3.2000000	−4.5613447	0.5065999	−1.0000000	−1.0000000
H	5.0000000	5.0000000	1.3484598	1.6022089	1.6000291
A	0.0000000	0.0396907	−0.0133552	0.0000436	0.0500003
B	−7.3609671	−7.3609671	−1.0651877	−0.5611194	1.0000000
B	−0.0100001	−0.0684324	−0.0265299	−0.0312224	0.0000000
C	−335.55304	−335.55304	−61.0554226	−50.0000000	0.0000000
Γ	−0.3200058	−2.9836697	−0.5818643	1.0000000	−1.0000000

As it may be seen from Table 3, the derivatives of the moving components 1 and 2 are differentiated in both extreme arrangements while the components 3 and 4 are constant. The derivatives from the 3 together with the standard executory deviations

Table 3. Derivatives of the image position with respect to the executory parameters of the objective $f = 10-100$

$i =$	1	2	3	4
$ds'_4/d\Phi_i$	-100.00000	-77.603421	-1027.3391	-1024.0000
ds'_4/de	-0.0063014	-0.0713740	-0.0000008	-1.0000000
ds'_4/dm	0.0063014	0.0650726	-0.0713733	0.9999992
$dS'_4/d\Phi_i$	-9999.8694	-727.32805	-1026.8160	-1024.0239
dS'_4/dE	-0.6301310	-0.0713438	-0.0000008	-1.0000000
dS'_4/dM	0.6301310	-0.5587872	-0.0713430	0.9999992

according to (31) will be used to determine the extreme deviations of the image position from formula (28). The following estimations of deviations have been obtained:

$$\Delta_{ext}s'_4 = 0.3657, \quad \Delta_{ext}S'_4 = 1.1979.$$

From the analysis it follows that the objective $f = 10-100$ has rather rich adjusting possibilities. Five different pairs of adjusting movements may be chosen with the exception of the pair of indices 3 and 4 (constant derivatives). The differences in residual disadjustments for both pairs are small. The least ranges of adjustment movements and the best measures of residual disadjustment are obtained when the adjusting movements are the shifts of 4 and 5 components.

The adjusting equations according to formulae (29) in the case of deviated objective (without adjusting movements) are presented as follows:

$$0.0064697\Delta m_1 + 0.9999828\Delta m_4 = -0.3657,$$
$$0.7144341\Delta m_1 \quad 0.9989181\Delta m_4 = -1.1997.$$

When solving these equations, the first approximation of the adjusting movements ranges: $\Delta m_1 = -1.176$ and $\Delta m_4 = -0.358$ is obtained.

Further simulation of the deviated objective $f = 10-100$ most disadvantageous in the version in plus and in minus and the simulation of the adjustments during which the following magnitudes are actualized: deviation system after taking account of the adjusting movements, derivatives of the adjusting components displacements and the secondary deviations of the image positions from the paraxial runs, give after four iterations the final results of calculations of the adjusting movements ranges:

$\Delta m_1 = -1.407$ and $\Delta m_4 = -0.339$ in plus deviated objective,
$\Delta m_1 = 1.397$ and $\Delta m_4 = 0.333$ in minus deviated objective.

As a result of adjustment the zero values of disadjustment in both extreme arrangements of the objective were achieved and the following measures of residual disadjustment determined from formule (32) obtained:

– for the in plus deviated objective

$$\delta s'_{max} = 0.01534, \quad \delta s'_{min} = 0.00000,$$
$$\delta s'_{av} = 0.00717, \quad \delta s'_{RMS} = 0.00557,$$

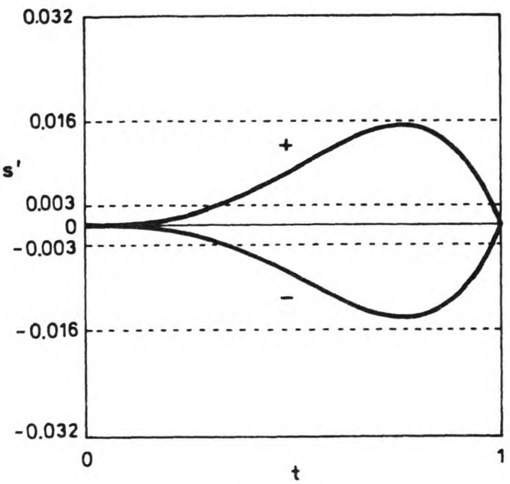

Fig. 2. Plot of the residual disadjustment for the objective $f = 10-100$

– for the in minus deviated objective
$\delta s'_{max} = 0.00000$, $\delta s'_{min} = -0.01405$,
$\delta s'_{av} = -0.00645$, $\delta s'_{RMS} = 0.00512$.

The plot of residual disadjustment for the objective $f10-100$ is presented in Fig. 2. The upper curve corresponds to the in plus deviated objective while the lower one – to the in minus deviated objective. In Figure 2, the depths following from the numerical aperture NA $= 0.3125$ (relative aperture 1:1:6) are marked. The diffraction depth for the aperture is equal to $G_d = \pm 0.003$, which corresponds to the resolution $R_d = 1063$ lp/mm. This depth is visualized in Fig. 2 in the form of broken lines located most closely to $\delta s' = 0$. The resolution $R = 40$ lp/mm ($s = 0.075$) foreseen for the objective differs significantly from the diffraction limited case. Therefore, more essential is the depth due to Hopkins which in this case is equal to $G_H = \pm 0.016$. Also this depth is presented in Fig. 2 in the form of a broken line. As may be seen from Fig. 2, the residual disadjustment is included in the Hopkins depth G_H in accordance with the condition (35).

8. Concluding remarks

The adjustment of the image position in a real zoom system differs slightly from the simulation method described above. In this case, the only sure information are deviations of the image position in both the extreme arrangements which may by measured on a universal or specialized apparatus. From the executory deviations the optical power deviations of the components are the easiest to measure. The estimation of these deviations of spaces (between the principal planes of the components) is very troublesome since it cannot be done with a sufficient accuracy. Lack of possibility of estimation of a real deviated zoom system results in the fact that the adjusting process cannot be aided with an immediately elaborated mathematical method. The adjusting process of a real zoom system consists in an iterative removal of the image position deviation in the following sequence: in the final arrangement by moving a single adjusting component and next in the starting arrangement by moving the second adjusting component. The value of the adjusting components displacement is empirically fitted by sharpening the image in a given extreme arrangement. Connection of the adjusting components with extreme arrangements, as well as ranges and directions of adjusting movements result from simulation of the image position adjustment. After adjustment of the real zoom system we should obtain the residual disadjustments lower than those which have been achieved during the adjustment simulation.

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