Environment Protection Engineering

Vol. 17 1991 No. 3–4

ANDRÁS TÓTH*, ISTVÁN GRESH*

COMPUTER OPTIMIZATION OF WATER MIXING

Mixing of natural waters being close to equilibrium state often results in their aggressivity or causes precipitation, sometimes together with unpleasant odour and taste. If storage volume is inadequate for satisfactory mixing, it is advisable to choose the components and their ratios properly in order to decrease the aggressivity of the mixture. For optimization by computer, the third-order equation relating the equilibrium CO_2 content (a) and the carbonate hardness (k) can be derived:

$$a = (0.67 + 0.03t)10^{-2}k^3$$

where t is temperature. The combined CO_2 content (b) can be calculated from the well-known formula:

$$b = \frac{k}{2.8}22.$$

The necessary calculations lead to an equation of the 3rd order. When solved by Cardano's formula, this equation gives values of aggressive and equilibrium CO_2 content of the mixture. Computer programs based on the above-mentioned method give a vast body of data in a short period of time, in graphics or in tabulated form, as desired. A complete investigation of the mixing ratios can be obtained in order to facilitate optimization. Correct mixing ratios satisfying the prescribed or permitted aggressive CO_2 levels can easily be selected. The method can also be applied to the mixing of waters which were originally aggressive.

1. INTRODUCTION

The problem of mixing waters of different origin and quality is frequently encountered in both communal and industrial water supplies. Although the majority of waters found in nature are in equilibrium (or more precisely close to equilibrium) state, their mixtures may become aggressive. The combination of two different spring waters entails no serious consequences, since the calcium or magnesium carbonates

* Research Centre for Water Resources Development, VITUKI, Kvassay Jenõ ut. 1, H-1095 Budapest, Hungary.

needed for their neutralization are dissolved from the bed material by aggressive carbon dioxide and any precipitate settles in the stream. The resulting water will again become "crystal clear" and will reach equilibrium conditions within a brief period of time.

In communal or regional distribution networks waters of different qualities are conveyed through pipelines and the mixing thereof creates a different situation, unless sufficient storage space and detention time are available for the balancing reactions to take place. However, no mixing problems will arise even in the absence of storage space, if we succeed in controlling the mixture proportions so as to keep the aggressivity of the resultant mixture at a low level.

It may be anticipated that mixing of different waters may give rise to two undesirable phenomena:

1. Lime–carbon dioxide equilibrium is unbalanced upset making the mixture aggressive.

2. Metal precipitates appear in the water.

Lowering of odour and taste standards may accompany these phenomena, especially in the case of surface waters. If chemical reactions did not proceed in the course of the mixing process, the resulting composition of the mixture could be estimated as the weighted average of ions present in the component waters. However, new compounds are formed in the mixture, some of them remain in solution, others precipitate, settle, remain suspended, float to the surface, dissolve in gaseous form, or escape through the water surface.

2. ESTIMATION OF PARAMETERS RESULTING FROM MIXING OF WATERS ACCORDING TO STANDARD SPECIFICATION

In order to determine the quality parameters of mixed waters, mixing experiments have been performed [1]. These are, however, difficult to carry out at the original site and at the original temperature. Fortunately, analytical methods are also available. A numerical example based on the Hungarian Standard Specifications MSZ 448-23 [2] will be presented subsequently for estimating the concentration of aggressive carbon dioxide originating in a water mixture. (The results obtained in this way will be irrealistic, unless the amount of magnesium hydrogen carbonate is relatively small in comparison to the calcium hydrogen carbonate content in the water.) The temperature and carbonate hardness of component waters are the only basic data needed. In the example, two waters will be mixed in 1:1 ratio, but the estimation could be performed equally for several components and without limitations on the mixing ratios (table 1).

The resultant temperature of the mixture will evidently be 15°C. The weighted average of carbonate hardness is

$$\frac{1 \times 5 + 1 \times 25}{1 + 1} = 15°G.$$

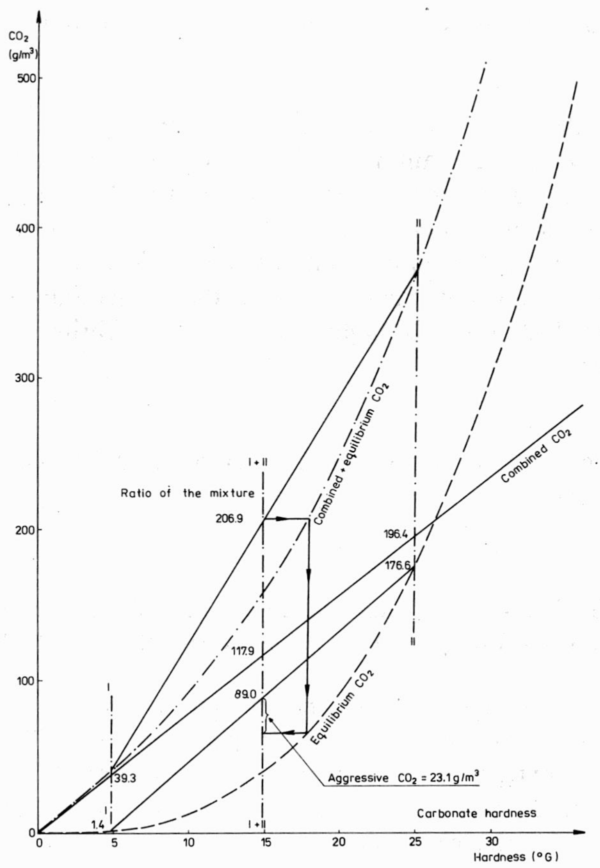
The chemically combined CO_2 of waters is found from the carbonate hardness:

In water 1: $\frac{5 \times 22}{2.8} = 39.3 \text{ g/m}^3$,

Table 1

Basic data for standard calculations

Component water	Water temperature (°C)	Carbonate hardness (°G)	Mxing proportion
water 1	15	5	1
water 2	15	25	1



Determination of aggressive CO_2 content in mixed waters

In water: $\dfrac{25 \times 22}{2.8} = 196.4$ g/m^3.

The weighted average of combined CO_2 in the mixture is

$$\frac{1 \times 39.3 + 1 \times 196.4}{1 + 1} = 117.9 \text{ g/m}^3.$$

The equilibrium carbonic acid content of the waters is not so simple to estimate, but can be found with the help of tabulations (Papp) attached to the standard specifications mentioned earlier. The equilibrium carbonic acid content is thus 1.4 and 176.6 g/m^3 in waters 1 and 2, respectively. The weighted average thereof will be the total free CO_2 content in the mixture:

$$\frac{1 \times 1.4 + 1 \times 176.6}{1 + 1} = 89.0 \text{ g/m}^3,$$

but aggressive CO_2 is also included.

The combined and free CO_2 content of the mixture is thus

$$117.9 + 89.0 = 206.9 \text{ g/m}^3.$$

Finding from Papp's tabulation of the corresponding balanced, combined and equilibrium CO_2 values (pertaining to the same carbonate hardness) the sum of which equals most closely the above values, it will be seen that in the band approaching 15°G this amounts to a total of 207.3 g/m^3, the combined CO_2 content being 141.4 g/m^3, while the equilibrium CO_2 is 65.9 g/m^3.

The mixture is, however, not balanced since the free (equilibrium and aggressive) CO_2 content, viz. 89.0 g/m^3 is higher than the equilibrium CO_2 content. The difference between the two is

$$89.0 - 65.9 = 23.1 \text{ g/m}^3$$

which is the aggressive CO_2 content in the mixed water. The calculation process is presented in the figure.

3. ANALYTICAL DETERMINATION OF THE EQUILIBRIUM AND AGGRESSIVE CO_2 CONTENTS IN MIXED WATERS

For estimating the temperature-dependent equilibrium CO_2 content described by the family of curves corresponding to Papp's tabulation, TÓTH [3] suggested a formula which in extreme cases differs by no more than 3% from the tabulated values, while in normal cases this difference is less than 1%. The equilibrium CO_2 content a is given by the expression

$$a = (0.67 + 0.03t)10^{-2}k^3 \tag{1}$$

where t is temperature of water (°C) and k denotes carbonate hardness (°G – German degrees).

As will be remembered from the foregoing, the combined CO_2 content b is simply found as

$$b = \frac{k}{2.8}22. \tag{2}$$

In the case of mixtures, the resultant carbonate hardness is

$$k = \frac{\sum k_i\alpha_i}{\sum \alpha_i} \tag{3}$$

where α_i is the proportion of the individual components.

The resultant equilibrium CO_2 content is

$$a = \frac{\sum a_i\alpha_i}{\sum \alpha_i}, \tag{4}$$

and the resultant combined CO_2 content is

$$b = \frac{\sum b_i\alpha_i}{\sum \alpha_i}. \tag{5}$$

From these the balanced carbonate hardness x must be estimated first. We may write

$$(0.67 + 0.03t)10^{-2}x^3 + \frac{22}{2.8}x = a + b,$$

or in canonic form

$$x^3 + \frac{22 \times 10^2}{(0.67 + 0.03t)2.8}x - \frac{(a + b)10^2}{0.67 + 0.03t} = 0.$$

Comparing the above equation with $Ax^3+Bx^2+Cx+D = 0$, we obtain:

$$A = 1,$$

$$B = 0, \tag{6}$$

$$C = \frac{22 \cdot 10^2}{(0.67 + 0.03t)2.8},$$

$$D = -\frac{(a + b)10^2}{0.67 + 0.03t} \tag{7}$$

and introducing the variable $y = x + B/3A$, we obtain the form

$$y^3 + 3pq + 2q = 0. \tag{8}$$

The Cardano formula is used to solve the third-order equation:

$B=0$, so that $y=x$ (substituting into eq. (8)).
$2q=D/A$, but since $A=1$, we have $q=D/2$.
Introducing the value of D from eq. (7) we get

$$q = -\frac{(a+b)50}{0.67+0.03t}, \tag{9}$$

$3p = C/A$, but since $A = 1$, we have $p = C/3$.
Combining with eq. (6)

$$p = \frac{22 \cdot 10^2}{3(0.67+0.03t)}. \tag{10}$$

The values obtained from eqs. (9) and (10) are now introduced into the formulae

$$u = \sqrt[3]{-q + \sqrt{q^2 + p^3}} \tag{11}$$

and

$$v = \sqrt[3]{-q - \sqrt{q^2 + p^3}}. \tag{12}$$

The summation of the results yields the balanced carbonate hardness:

$$x = u + v. \tag{13}$$

The equilibrium CO_2 content can now be calculated from eq. (1) as

$$a_x = (0.67 + 0.03t)10^{-2}x^3. \tag{14}$$

Taking into account the resultant equilibrium CO_2 content of the mixture obtained from eq. (4), we have the aggressive CO_2 content of the mixture:

$$\text{aggressive } CO_2 \text{ content} = a - a_x. \tag{15}$$

With data of the numerical example given in the first section, this algorithm yielded results which are summerized in table 2. Results of the calculation made using Papp's tabulation are given in brackets. The differences are less than 1%.

Table 2

Analytical results

Component water	Equilibrium CO_2 content (g/m^3)	Aggressive CO_2 content (g/m^3)
water 1	1.4 (1.4)	–
water 2	175 (176.6)	–
mixture	88.2 (89.0)	23.3 (23.1)

Note: The results of the calculations performed using Papp's tabulation are given in brackets.

4. COMPUTER PROGRAMS FOR WATER MIXING CALCULATIONS

For the mixing of any number of waters, two computer programs were compiled at VITUKI by

László Koncsos whose program involves successive approximation [3],

István Gresz whose program was worked out according to the algorithm described in section 2.

The results obtained basing on these programs were in fair agreement with data of the mixing experiments carried out by Licskó [1].

4.1. THE APPROXIMATION PROGRAM

From 36 kinds of communal waters sampled along Lake Balaton, Licskó has selected 17 basic waters which he mixed in 1:1, 1:3 and 3:1 proportions, altogether resulting in 66 different mixtures. The calculations for such a large number of mixtures are rather time consuming. Therefore KONCSOS [3] has compiled an optimization programme for three different kinds of waters mixing them, in all combinations, in proportions from 1 to 10 and classifying the mixtures according to the resultant aggressive CO_2 content in 5 g/m^3 increments within the 0–35 g/m^3 range. The character of the programme imposes repeatability of samples, since the 1:1:1 mixture yields the same results as the one of 5:5:5 proportions. For optimizing the proportions of the mixtures a simplified set of data was needed.

4.2. THE ANALYTICAL PROGRAM

The program compiled according to the calculation procedure outlined in section 2 offers two possibilities:

2–5 waters in unspecified proportions,

2–3 waters in a series of predetermined mixing ratios.

4.2.1. WATER MIXTURES OF UNSPECIFIED PROPORTIONS

The input data consist of the identification code, carbonate hardness, temperature and mixing ratio of the waters to be mixed. The result is displayed as the content of total and aggressive CO_2 on the screen. A printout of the results comprises a tabulation of:

1. proportions of the components in the mixture.

For the individual components and the mixture:

2. carbonate hardness,
3. equilibrium CO_2 content,
4. temperature,
5. combined CO_2 content.

And for the mixture:

6. content of aggressive CO_2.

The parameters of a particular mixture with given proportions are thus found promptly.

4.2.2. MIXING WATER IN PREDETERMINED RATIOS

The programme outlined in section 4.2.1 becomes time-consuming in handling when the optimal mixing ratio is desired from a large number of possibilities. The present version of the program has been compiled for this purpose and it allows calculations of the results of mixing of 2 or 3 different kinds of water over a wide range of mixing ratios.

The input data needed comprise the identification code, carbonate hardness and temperature of the waters to be mixed. Waters *A* and *B* are examined in 21 preset mixing ratios, to which water *C* is added in a selected ratio. The 21 ratios are 1 : 10, 1 : 9, ..., 1 : 1.5, 1 : 1, 1.5 : 1, 2 : 1, ..., 9 : 1, 10 : 1. The results are displayed on the screen as carbonate hardness, temperature and aggressive CO_2 content for the various mixtures. The maximum value of the latter is indicated. Optionally a printout comprises the basic data of the water to be mixed (*A*, *B*, *C*) and their equilibrium and combined CO_2 content. Hereafter the carbonate hardness, temperature, combined CO_2 content, free CO_2 content, equilibrium CO_2 content and aggressive CO_2 content of the various mixtures are printed. From the tabulation, the range of the optimal mixing ratio can readily be selected for the water *C* added in a given mixing ratio.

The optimization calculation can be repeated for a different mixing ratios of the component *C* without entering the basic data. The results pertaining to five different mixing ratios of the component *C* are stored in the memory of the computer. With the help of a menu all data series can be displayed graphically on the screen and printed out on demand. The graphs can be superimposed upon each other in several versions:

carbonate hardness and/or temperature for a given ratio *C*,

combined, free, equilibrium and aggressive CO_2 in any combination for a given ratio *C*,

carbonate hardness for up to 5 different ratios *C*,

temperature for up to 5 different ratios *C*,

combined CO_2 for up to 5 different ratios *C*,

free CO_2 for up to 5 different ratios *C*,

equilibrium CO_2 for up to 5 different ratios *C*,

aggressive CO_2 for up to 5 different ratios *C*.

Aggressive CO_2 content is magnified 10 times in the graphics for the sake of clarity. The choice of the optimum mixing ratio is greatly facilitated by the combined graphs.

5. NUMERICAL EXAMPLE

The program described under 4.2 is subsequently illustrated by a specific example with tabulations and printed graphs attached. The program was run with the following basic data (table 3).

Table 3

Basic data for computations

Component water	Water temperature (°C)	Carbonate hardness (°G)	Mixing ratio
water 1	20	22.7	1
water 2	6	8.1	1
water 3	20	21.8	0.1, 2.5, 10

5.1. CALCULATION FOR WATER MIXTURES OF OPTIMAL SELECTED RATIOS

The results of the calculation performed on the basis of table 3 are shown in Annex 1 (of the tabulated data only those of waters 1 and 2 were used). The first and second columns show the data of waters 1 and 2, respectively, while the result of mixing is given in the third column. The data applying to all three waters are in one line, the aggressive CO_2 and total CO_2 values are in a separate line. No limitations are imposed on the number of calculations, but the basic data must be entered on each occasion.

5.2. CALCULATION FOR WATERS MIXED IN PRESELECTED RATIOS

The basic data are given in table 3. The calculation was repeated five times with water 3 added in the ratios 0, 1, 2, 5 and 10. The results are presented in Annexes 2–11.

$C = 0$. In the case of Annexes 2 and 3 the mixing ratio of the third (C) component was 0, so that only two waters ($A+B$) were mixed. The shapes of carbonate hardness and temperature curves are similar since water A is warmer and also considerably harder than water B. The shapes of the curves representing combined, free and equilibrium CO_2 are also similar. In contrast, the maximum of aggressive CO_2 is close to the mixture ratio $A:B = 1:1.5$.

$C = 1, 2, 5$. The maximum of aggressive CO_2 content occurs at the ratios $A:B = 1:3$, $1:4$ and finally $1:8$ (Annexes 4–8 and 9).

$C = 10$. The maximum of aggressive CO_2 content is at the ratio $A:B = 1:10$ (Annexes 10 and 11).

The desired mixing ratio is thus easy to select if, for instance, it is specified that aggressive CO_2 content must be maintained below 5 mg/dm^3.

Let us assume that the $A:B$ ratio is fixed at 1:4. In this case the mixing ratio must be at least 5 for component C. If, on the other hand, an aggressive CO_2 level of 10 mg/dm^3 is also acceptable, no C component need be added.

Graphs pertaining to five different mixing ratios (0, 1, 2, 5 and 10) of the component C can be superimposed (Annexes 12–17). The numerical content of these graphs can be printed out on demand in tabulated form. The corresponding graphs and tables are represented in combination in the annexes. Annex 17 provides a perfect picture of the variation of aggressive CO_2.

6. MIXING AGGRESSIVE WATERS

The foregoing chapters have dealt consistently with the mixing of waters in which lime and carbonic acid are balanced. The calculations, programs and numerical examples included this assumption which is most of all satisfied in the case of natural waters of Hungary. No balance exists, however, in some well waters, industrial effluents in a few surface waters in Hungary and in several surface waters abroad, which already contain aggressive CO_2 before mixing. Any of the models described in the foregoing chapters can be applied without difficulty, provided that free carbonic acid content of the water samples is determined at the site, before solving the mixing problem. The weighted proportion of the original aggressive CO_2 content of the aggressive component must only be added to achieve aggressive CO_2 level calculated with due regard to the temperature, carbonate hardness and mixing ratio. The original aggressive CO_2 is found if the equilibrium CO_2 calculated is subtracted from the measured free CO_2 content. In the numerical examples (chapters 2 and 3) if the analysis of water 1 had shown 185 g/m^3 as the free CO_2 content, subtracting therefrom the calculated equilibrium CO_2 (table 2), the aggressive CO_2 would have been obtained as $185-175=10$ g/m^3.

Assuming the original mixing ratio of 1:1, the aggressive CO_2 obtained as a result, i.e. 23.3 g/m^3, would have to be increased by the weighted proportion of the original 10 g/m^3, i.e. by $1\times 10/1+1=5$ g/m^3. In this case aggressive CO_2 in the mixture would be $23.3+5=28.5$ g/m^3.

7. CONCLUSIONS

The method of calculating aggressive CO_2 in mixed waters according to the Hungarian Standard Specifications MSZ 448/23 involves the use of tabulated values. These have been compared with the model developed here, for which a computer program has also been compiled. The results obtained by the two methods agree up to 1%.

The development was prompted by the cumbersome practice and the uncertainty of the former results. Samples collected at the site are difficult to conserve in their original condition during transport and analysis. Unavoidable shaking, exposure to atmosphere and temperature changes during transport modify the dissolved CO_2 of the samples upsetting the original balance. This has lead inevitably to false results.

All this can be eliminated by calculation, since it is sufficient to determine only the temperature and carbonate hardness of the waters to be mixed, which can be performed rapidly and accurately.

The computer programs compiled yield a number of data within a brief period of time, providing a virtually complete picture of optimal possibilities of water mixing. The desired mixing ratios can be readily selected for any aggressive CO_2 level specified, or allowed.

The mathematical model offers the possibility of calculation even in situations other than those described, thus for example in automatic process control.

Consequently, the methods developed replace lengthy and uncertain mixing experiments, provide timely information and a clear picture, thus assisting decision making in selecting the mixture ratios. The model is also applicable to calculating the parameters of water mixtures containing aggressive CO_2 as described in chapter 6.

Annex 1. Calculation for water mixtures of optional selected ratios

Aggressive CO_2 content in mixtures

Parameters		1	2	Mixture
Codename		water 1	water 2	–
Carbonate hardness	(°G)	22.70	8.10	15.40
Equilibrium CO_2 content	(g/m^3)	148.55	4.52	76.54
Mixing ratio		1	1	–
Temperature	(°C)	20.00	6.00	13.00
Combined CO_2 content	(g/m^3)	178.36	63.64	121.00
Equilibrium+combined CO_2 content	(g/m^3)	326.91	68.16	197.54

Carbonate hardness in CaO/CO_2 equilibrium (°G) = 17.68.
Equilibrium CO_2 content in CaO/CO_2 equilibrium (g/m^3) = 58.60.
Aggressive CO_2 content (g/m^3) = 17.93.

Annex 2. Calculation for waters mixed in preselected ratios

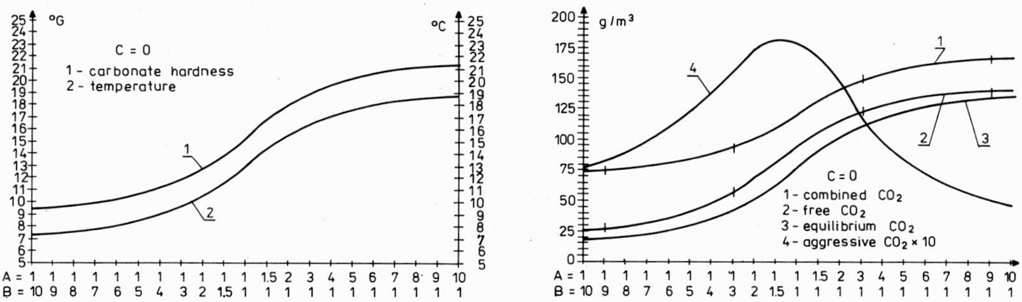
Input data

Parameters		*A*	*B*	*C*
Code		water 1	water 2	water 3
Carbonate hardness	(°G)	22.70	8.10	21.80
Temperature	(°C)	20.00	6.00	20.00
Equilibrium CO_2 content	(g/m^3)	148.55	4.52	131.57
Combined CO_2 content	(g/m^3)	178.36	63.64	171.29

Results; $C = 0$

No.	A/B	Carbonate hardness (°G)	Temperature (°C)	Combined CO_2 (g/m³)	Free CO_2 (g/m³)	Equilibrium CO_2 (g/m³)	Aggressive CO_2 (g/m³)
1	1/10	9.43	7.27	74.07	25.24	17.61	7.63
2	1/9	9.56	7.40	75.11	27.18	18.92	8.26
3	1/8	9.72	7.56	76.39	29.53	20.52	9.01
4	1/7	9.93	7.75	77.98	32.42	22.52	9.90
5	1/6	10.19	8.00	80.03	36.05	25.09	10.96
6	1/5	10.53	8.33	82.76	40.77	28.52	12.25
7	1/4	11.02	8.80	86.59	47.15	33.32	13.83
8	1/3	11.75	9.50	92.32	56.24	40.53	15.71
9	1/2	12.97	10.67	101.88	70.19	52.53	17.66
10	1/1.5	13.94	11.60	109.53	80.43	62.13	$> 18.30 <$
11	1/1	15.40	13.00	121.00	94.47	76.54	17.93
12	1.5/1	16.86	14.40	132.47	107.17	90.94	16.23
13	2/1	17.83	15.33	140.12	115.01	100.54	14.47
14	3/1	19.05	16.50	149.68	124.21	112.54	11.67
15	4/1	19.78	17.20	155.41	129.44	119.75	9.69
16	5/1	20.27	17.67	159.24	132.82	124.55	8.27
17	6/1	20.61	18.00	161.97	135.18	127.98	7.20
18	7/1	20.88	18.25	164.02	136.92	130.55	6.37
19	8/1	21.08	18.44	165.61	138.27	132.55	5.72
20	9/1	21.24	18.60	166.89	139.33	134.15	5.18
21	10/1	21.37	18.73	167.93	140.19	135.46	4.73

Annex 3. Calculation for waters mixed in preselected ratios, $C = 0$



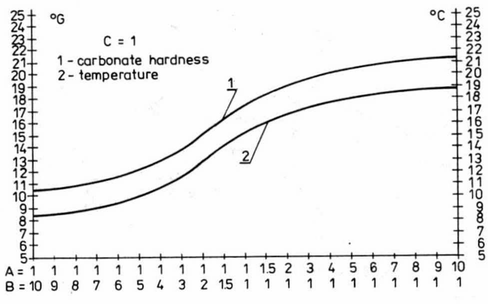
Annex 4. Calculation for waters mixed in preselected ratios

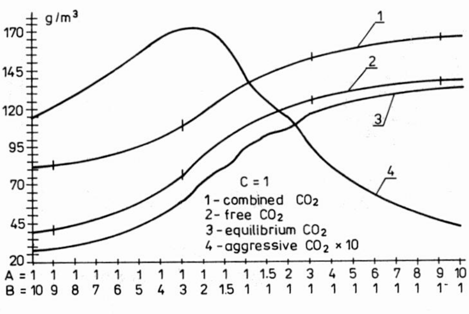
Results; $C = 1$

No.	A/B	Carbonate hardness (°G)	Temperature (°C)	Combined CO_2 (g/m³)	Free CO_2 (g/m³)	Equilibrium CO_2 (g/m³)	Aggressive CO_2 (g/m³)
1	2	3	4	5	6	7	8
1	1/10	10.46	8.33	82.17	38.62	27.11	11.51
2	1/9	10.67	8.55	83.86	41.38	29.16	12.22

1	2	3	4	5	6	7	8
3	1/8	10.93	8.80	85.88	44.64	31.63	13.01
4	1/7	11.24	9.11	88.35	48.51	34.64	13.87
5	1/6	11.64	9.50	91.44	53.20	38.40	14.80
6	1/5	12.14	10.00	95.41	59.01	43.24	15.77
7	1/4	12.82	10.67	100.70	66.38	49.70	16.68
8	1/3	13.76	11.60	108.11	76.06	58.74	> 17.32 <
9	1/2	15.18	13.00	119.23	89.31	72.29	17.02
10	1/1.5	16.19	14.00	127.17	97.98	81.97	16.01
11	1/1	17.53	15.33	137.76	108.67	94.88	13.79
12	1.5/1	18.27	16.00	143.56	115.03	102.55	12.48
13	2/1	18.83	16.50	147.91	119.63	108.30	11.33
14	3/1	19.60	17.20	154.00	125.84	116.35	9.49
15	4/1	20.12	17.67	158.06	129.86	121.72	8.14
16	5/1	20.49	18.00	160.96	132.66	125.55	7.11
17	6/1	20.76	18.25	163.13	134.74	128.43	6.31
18	7/1	20.98	18.44	164.83	136.33	130.66	5.67
19	8/1	21.15	18.60	166.18	137.59	132.45	5.14
20	9/1	21.29	18.73	167.29	138.63	133.92	4.71
21	10/1	21.41	18.83	168.21	139.48	135.14	4.34

Annex 5. Calculation for waters mixed in preselected ratios, $C = 1$





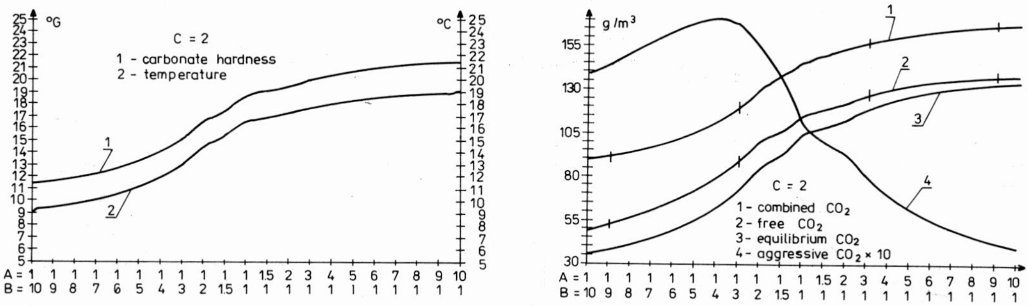
Annex 6. Calculation for waters mixed in preselected ratios

Results; $C = 2$

No.	A/B	Carbonate hardness (°G)	Temperature (°C)	Combined CO_2 (g/m³)	Free CO_2 (g/m³)	Equilibrium CO_2 (g/m³)	Aggressive CO_2 (g/m³)
1	2	3	4	5	6	7	8
1	1/10	11.33	9.23	89.03	49.01	35.14	13.87
2	1/9	11.60	9.50	91.14	52.19	37.70	14.49
3	1/8	11.92	9.82	93.64	55.84	40.71	15.13
4	1/7	12.30	10.20	96.64	60.09	44.33	15.76
5	1/6	12.77	10.67	100.31	65.01	48.76	16.35

1	2	3	4	5	6	7	8
6	1/5	13.35	11.25	104.89	71.12	54.29	16.83
7	1/4	14.10	12.00	110.79	78.44	61.40	> 17.04 <
8	1/3	15.10	13.00	118.64	87.58	70.88	16.70
9	1/2	16.50	14.40	129.64	99.31	84.15	15.16
10	1/1.5	17.43	15.33	136.98	106.55	92.00	13.55
11	1/1	18.60	16.50	146.14	115.01	104.06	10.95
12	1.5/1	19.06	16.89	149.72	119.05	108.00	10.05
13	2/1	19.42	17.20	152.59	122.22	112.95	9.27
14	3/1	19.97	17.67	156.88	126.89	118.89	8.00
15	4/1	20.36	18.00	159.95	130.15	123.13	7.02
16	5/1	20.65	18.25	162.25	132.54	126.30	6.24
17	6/1	20.88	18.44	164.04	134.40	128.78	5.62
18	7/1	21.06	18.60	165.47	135.85	130.75	5.10
19	8/1	21.21	18.73	166.64	137.05	132.37	4.68
20	9/1	21.33	18.83	167.62	138.03	133.72	4.31
21	10/1	21.44	18.92	168.45	138.86	134.86	4.00

Annex 7. Calculation for waters mixed in preselected ratios, $C = 2$

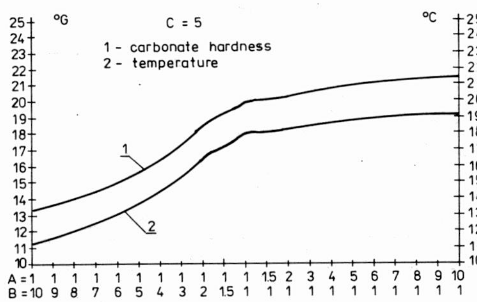


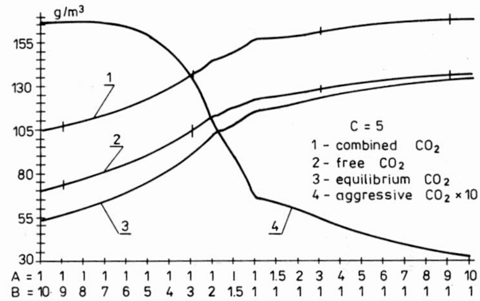
Annex 8. Calculation for waters mixed in preselected ratios, $C = 5$

No.	*A/B*	Carbonate hardness (°G)	Temperature (°C)	Combined CO_2 (g/m^3)	Free CO_2 (g/m^3)	Equilibrium CO_2 (g/m^3)	Aggressive CO_2 (g/m^3)
1	2	3	4	5	6	7	8
1	1/10	13.29	11.25	104.45	69.72	53.23	16.49
2	1/9	13.64	11.60	107.17	73.11	56.47	16.64
3	1/8	14.04	12.00	110.28	76.88	60.18	> 16.70 <
4	1/7	14.49	12.46	113.87	81.10	64.47	16.63
5	1/6	15.03	13.00	118.05	85.83	69.46	16.37
6	1/5	15.65	13.64	123.00	91.19	75.36	15.83
7	1/4	16.41	14.40	128.94	97.33	82.45	14.88
8	1/3	17.33	15.33	136.19	104.40	91.11	13.29

1	2	3	4	5	6	7	8
9	1/2	18.49	16.50	145.26	112.67	101.93	10.74
10	1/1.5	19.18	17.20	150.70	117.37	108.43	8.94
11	1/1	19.97	18.00	156.92	122.51	115.85	6.66
12	1.5/1	20.15	18.13	158.35	124.34	118.03	6.31
13	2/1	20.31	18.25	159.60	125.92	119.94	5.98
14	3/1	20.58	18.44	161.68	128.55	123.12	5.43
15	4/1	20.79	18.60	163.35	130.62	125.66	4.96
16	5/1	20.96	18.73	164.71	132.30	127.74	4.56
17	6/1	21.11	18.83	165.85	133.70	129.48	4.22
18	7/1	21.23	18.92	166.81	134.87	130.94	3.93
19	8/1	21.34	19.00	167.64	135.87	132.20	3.67
20	9/1	21.43	19.07	168.35	136.74	133.29	3.45
21	10/1	21.51	19.13	168.98	137.50	134.25	3.25

Annex 9. Calculation for waters mixed in preselected ratios, $C = 5$





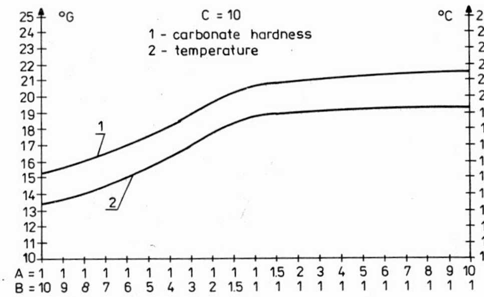
Annex 10. Calculation for waters mixed in preselected ratios

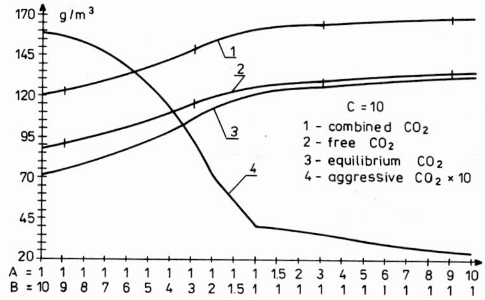
Results; $C = 10$

No.	A/B	Carbonate hardness (°G)	Temperature (°C)	Combined CO_2 (g/m^3)	Free CO_2 (g/m^3)	Equilibrium CO_2 (g/m^3)	Aggressive CO_2 (g/m^3)
1	2	3	4	5	6	7	8
1	1/10	15.32	13.33	120.36	87.85	71.88	> 15.97 <
2	1/9	15.68	13.70	123.20	90.87	75.25	15.62
3	1/8	16.08	14.11	126.33	94.12	78.97	15.15
4	1/7	16.52	14.56	129.82	97.62	83.11	14.51
5	1/6	17.02	15.06	133.71	101.41	87.73	13.68
6	1/5	17.58	15.63	138.09	105.53	92.93	12.60
7	1/4	18.21	16.27	143.05	110.02	98.82	11.20
8	1/3	18.93	17.00	148.72	114.95	105.56	9.39
9	1/2	19.76	17.85	155.27	120.38	113.33	7.05

1	2	3	4	5	6	7	8
10	1/1.5	20.23	18.32	158.93	123.32	117.69	5.63
11	1/1	20.73	18.83	162.90	126.41	122.40	4.01
12	1.5/1	20.81	18.88	163.52	127.33	123.45	3.88
13	2/1	20.88	18.92	164.09	128.16	124.41	3.75
14	3/1	21.01	19.00	165.11	129.67	126.14	3.53
15	4/1	21.13	19.07	165.00	130.96	127.63	3.33
16	5/1	21.23	19.13	166.77	132.09	128.94	3.15
17	6/1	21.31	19.18	167.45	133.08	130.09	2.99
18	7/1	21.39	19.22	168.06	133.96	131.12	2.84
19	8/1	21.46	19.26	168.60	134.75	132.04	2.71
20	9/1	21.52	19.30	169.09	135.45	132.86	2.59
21	10/1	21.58	19.33	169.53	136.08	133.61	2.47

Annex 11. Calculation for waters mixed in preselected ratios, $C = 10$

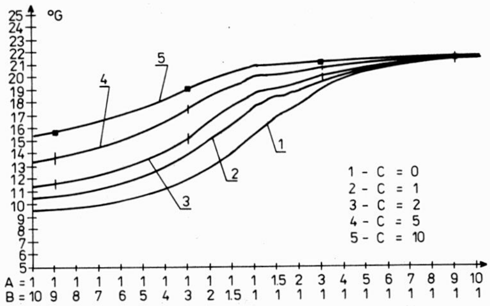




Annex 12. Calculation for water mixed in preselected ratios
Carbonate hardness (°G)

No.	*A/B*	$C=0$	$C=1$	$C=2$	$C=5$	$C=10$
1	2	3	4	5	6	7
1	1/10	9.43	10.46	11.33	13.29	15.32
2	1/9	9.56	10.67	11.60	13.64	15.68
3	1/8	9.72	10.93	11.92	14.04	16.08
4	1/7	9.93	11.24	12.30	14.49	16.52
5	1/6	10.19	11.64	12.77	15.03	17.02
6	1/5	10.53	12.14	13.35	15.65	17.58
7	1/4	11.02	12.82	14.10	16.41	18.21
8	1/3	11.75	13.76	15.10	17.33	18.93
9	1/2	12.97	15.18	16.50	18.49	19.76
10	1/1.5	13.94	16.19	17.43	19.18	20.23
11	1/1	15.40	17.53	18.60	19.97	20.73
12	1.5/1	16.86	18.27	19.06	20.15	20.81

1	2	3	4	5	6	7
13	2/1	17.83	18.83	19.42	20.31	20.88
14	3/1	19.05	19.60	19.97	20.58	21.01
15	4/1	19.78	20.12	20.36	20.79	21.13
16	5/1	20.27	20.49	20.65	20.96	21.23
17	6/1	20.61	20.76	20.88	21.11	21.31
18	7/1	20.88	20.98	21.06	21.23	21.39
19	8/1	21.08	21.15	21.21	21.34	21.46
20	9/1	21.24	22.29	21.33	21.43	21.52
21	10/1	21.37	21.41	21.44	21.51	21.58



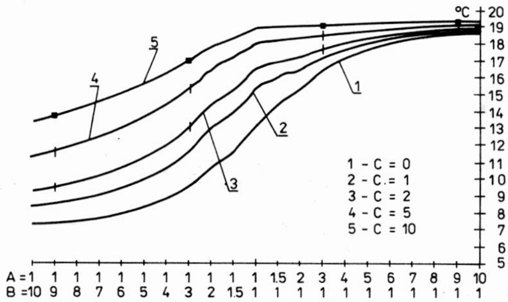
Carbonate hardness

Annex 13. Calculation for water mixed in preselected ratios

Temperature

No.	A/B	$C=0$	$C=1$	$C=2$	$C=5$	$C=10$
1	2	3	4	5	6	7
1	1/10	7.27	8.33	9.23	11.25	13.33
2	1/9	7.40	8.55	9.50	11.60	13.70
3	1/8	7.56	8.80	9.82	12.00	14.11
4	1/7	7.75	9.11	10.20	12.46	14.56
5	1/6	8.00	9.50	10.67	13.00	15.06
6	1/5	8.33	10.00	11.25	13.64	15.63
7	1/4	8.80	10.67	12.00	14.40	16.27
8	1/3	9.50	11.60	13.00	15.33	17.00
9	1/2	10.67	13.00	14.40	16.50	17.85
10	1/1.5	11.60	14.00	15.33	17.20	18.32
11	1/1	13.00	15.33	16.50	18.00	18.83
12	1.5/1	14.40	16.00	16.89	18.13	18.88
13	2/1	15.33	16.50	17.20	18.25	18.92
14	3/1	16.50	17.20	17.67	18.44	19.00
15	4/1	17.20	17.67	18.00	18.60	19.07

1	2	3	4	5	6	7
16	5/1	17.67	18.00	18.25	18.73	19.13
17	6/1	18.00	18.25	18.44	18.83	19.18
18	7/1	18.25	18.44	18.60	18.92	19.22
19	8/1	18.44	18.60	18.73	19.00	19.26
20	9/1	18.60	18.73	18.83	19.07	19.30
21	10/1	18.73	18.83	18.92	19.13	19.33

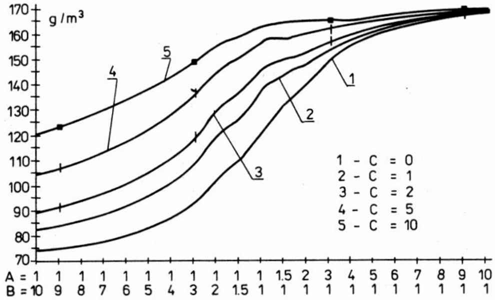


Temperature

Annex 14. Calculation for water mixed in preselected ratios

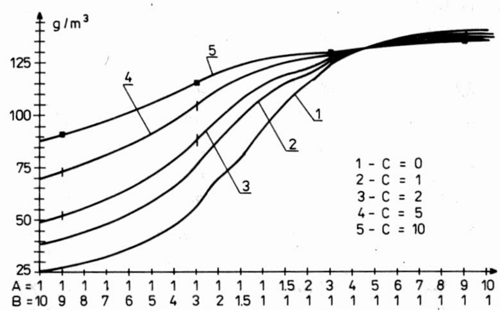
Combined CO_2 (g/m^3)

No.	A/B	$C=0$	$C=1$	$C=2$	$C=5$	$C=10$
1	1/10	74.07	82.17	89.03	104.45	120.36
2	1/9	75.11	83.86	91.14	107.17	123.20
3	1/8	76.39	85.88	93.64	110.28	126.33
4	1/7	77.98	88.35	96.64	113.87	129.82
5	1/6	80.03	91.44	100.31	118.05	133.71
6	1/5	82.76	95.41	104.89	123.00	138.09
7	1/4	86.59	100.70	110.79	128.94	143.05
8	1/3	92.32	108.11	118.64	136.19	148.72
9	1/2	101.88	119.23	129.64	145.26	155.27
10	1/1.5	109.53	127.17	136.98	150.70	158.93
11	1/1	121.00	137.76	146.14	156.92	162.90
12	1.5/1	132.47	143.56	149.72	158.35	163.52
13	2/1	140.12	147.91	152.59	159.60	164.09
14	3/1	149.68	154.00	156.88	161.68	165.11
15	4/1	155.41	158.06	159.95	163.35	165.00
16	5/1	159.24	160.96	162.25	164.71	166.77
17	6/1	161.97	163.13	164.04	165.85	167.45
18	7/1	164.02	164.83	165.47	166.81	168.06
19	8/1	165.61	166.18	166.64	167.64	168.60
20	9/1	166.89	167.29	167.62	168.35	169.09
21	10/1	167.93	168.21	168.45	168.98	169.53



Combined CO_2

Annex 15. Calculation for water mixed in preselected ratios



Free CO_2

Free CO_2 (g/m^3)

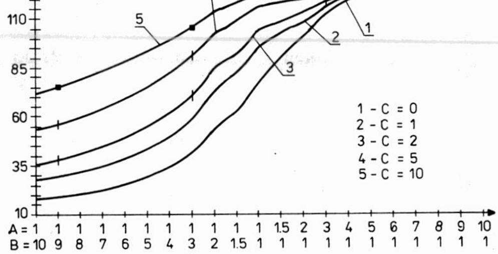
No.	*A/B*	*C* = 0	*C* = 1	*C* = 2	*C* = 5	*C* = 10
1	2	3	4	5	6	7
1	1/10	25.24	38.62	49.01	69.72	87.85
2	1/9	27.18	41.38	52.19	73.11	90.87
3	1/8	29.53	44.64	55.84	76.88	94.12
4	1/7	32.42	48.51	60.09	81.10	97.62
5	1/6	36.05	53.20	65.01	85.83	101.41
6	1/5	40.77	59.01	71.12	91.19	105.53
7	1/4	47.15	66.38	78.44	97.33	110.02
8	1/3	56.24	76.06	87.58	104.40	114.95
9	1/2	70.19	89.31	99.31	112.67	120.38
10	1/1.5	80.43	97.98	106.55	117.37	123.32
11	1/1	94.47	108.67	115.01	122.51	126.41
12	1.5/1	107.17	115.03	119.05	124.34	127.33
13	2/1	115.01	119.63	122.22	125.92	128.16
14	3/1	124.21	125.84	126.89	128.55	129.67

1	2	3	4	5	6	7
15	4/1	129.44	129.86	130.15	130.62	130.96
16	5/1	132.82	132.66	132.54	132.30	132.09
17	6/1	135.18	134.74	134.40	133.70	133.08
18	7/1	136.92	136.33	135.85	134.87	133.96
19	8/1	138.27	137.59	137.05	135.87	134.75
20	9/1	139.33	138.63	138.03	136.74	135.45
21	10/1	140.19	139.48	138.86	137.50	136.08

Annex 16. Calculation for water mixed in preselected ratios

Equilibrium CO_2 (g/m³)

No.	A/B	$C=0$	$C=1$	$C=2$	$C=5$	$C=10$
1	1/10	17.61	27.11	35.14	53.23	71.88
2	1/9	18.92	29.16	37.70	56.47	75.25
3	1/8	20.52	31.63	40.71	60.18	78.97
4	1/7	22.52	34.64	44.33	64.47	83.11
5	1/6	25.09	38.40	48.76	69.46	87.73
6	1/5	28.52	43.24	54.29	75.36	92.93
7	1/4	33.32	49.70	61.40	82.45	98.82
8	1/3	40.53	58.74	70.88	91.11	105.56
9	1/2	52.53	72.29	84.15	101.93	113.33
10	1/1.5	62.13	81.97	92.00	108.43	117.69
11	1/1	76.54	94.88	104.06	115.85	122.40
12	1.5/1	90.94	102.55	108.00	118.03	123.45
13	2/1	100.54	108.30	112.95	119.94	124.41
14	3/1	112.54	116.35	118.89	123.12	126.14
15	4/1	119.75	121.72	123.13	125.66	127.63
16	5/1	124.55	125.55	126.30	127.74	128.94
17	6/1	127.98	128.43	128.78	129.48	130.09
18	7/1	130.55	130.66	130.75	130.94	131.12
19	8/1	132.55	132.45	132.37	132.20	132.04
20	9/1	134.15	133.92	133.72	133.29	132.86
21	10/1	135.46	135.14	134.86	134.25	133.61

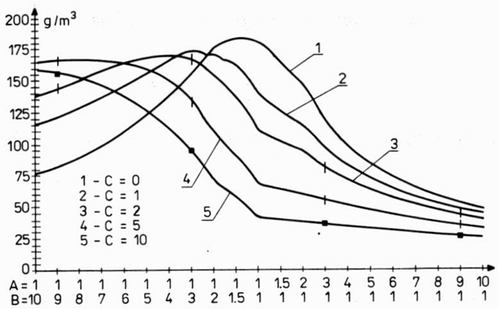


Equilibrium CO_2

Annex 17. Calculation for water mixed in preselected ratios

Aggressive CO_2 (g/m^3)

No.	A/B	$C=0$	$C=1$	$C=2$	$C=5$	$C=10$
1	1/10	7.63	11.51	13.87	16.49	> 15.97 <
2	1/9	8.26	12.22	14.49	16.64	15.62
3	1/8	9.01	13.01	15.13	> 16.70 <	15.15
4	1/7	9.90	13.87	15.76	16.63	14.51
5	1/6	10.96	14.80	16.35	16.37	13.68
6	1/5	12.25	15.77	16.83	15.83	12.60
7	1/4	13.83	16.68	> 17.04 <	14.88	11.20
8	1/3	15.71	> 17.32 <	16.70	13.29	9.39
9	1/2	17.66	17.02	15.16	10.74	7.05
10	1/1.5	> 18.30 <	16.01	13.55	8.94	5.63
11	1/1	17.93	13.79	10.95	6.66	4.01
12	1.5/1	16.23	12.48	10.05	6.31	3.88
13	2/1	14.47	11.33	9.27	5.98	3.75
14	3/1	11.67	9.49	8.00	5.43	3.53
15	4/1	9.69	8.14	7.02	4.96	3.33
16	5/1	8.27	7.11	6.24	4.56	3.15
17	6/1	7.20	6.31	5.62	4.22	2.99
18	7/1	6.37	5.67	5.10	3.93	2.84
19	8/1	5.72	5.14	4.68	3.67	2.71
20	9/1	5.18	4.71	4.31	3.45	2.59
21	10/1	4.73	4.34	4.00	3.25	2.47



Aggressive $CO_2 \times 10$

REFERENCES

[1] LICSKÓ I., *Water mixing experiments in the Lake Balaton area* (in Hungarian), Hungarian Hydrological Soc. V Junior Section, Győr, 1978.

[2] Hungarian Standard Specifications MSZ 448/23: *Drinking water qualification by physical and chemical testing* (in Hungarian), 1978.

[3] TÓTH A., *Studies on mixing waters of different origin and quality* (in Hungarian) VITUKI Project report, 1980.

KOMPUTEROWA OPTYMALIZACJA MIESZANIA WODY

Mieszanie wód naturalnych, które są bliskie stanu równowagi, często powoduje ich agresywność lub wytrącanie, czasem połączone z nieprzyjemnym zapachem i smakiem. Gdy nie mamy wystarczającej do zmieszania objętości wody, wskazany jest odpowiedni dobór składników i ich proporcji tak, aby zmniejszyć agresywność mieszaniny. Aby dokonać optymalizacji komputerowej, wyprowadza się równanie trzeciego rzędu, odnoszące się do zawartości równoważnego CO_2 (a) i do twardości węglanowej (k)

$$a = (0{,}67 + 0{,}03t)10^{-2}k^3,$$

gdzie t oznacza temperaturę. Zawartość związanego CO_2 (b) może być obliczona z dobrze znanego wzoru

$$b = \frac{k}{2{,}8}22.$$

Obliczenia prowadzą do równania trzeciego rzędu. Kiedy równanie to jest rozwiązane zgodnie ze wzorem Cardano, otrzymuje się wartości zawartości CO_2 agresywnej i równoważnej w mieszaninie. Programy komputerowe pracujące na podstawie tej metody umożliwiają otrzymanie w krótkim czasie ogromnej liczby danych w formie rysunków i tabel. Można przeprowadzać wyczerpujące badanie proporcji mieszania, aby ułatwić optymalizację. Łatwo można też wybrać właściwe proporcje mieszania, spełniające zalecane lub dozwolone poziomy agresywnego CO_2 w wodzie. Metoda ta może być także stosowana do mieszania wód, które są agresywne ze swej natury.

ОПТИМИЗАЦИЯ С ПОМОЩЬЮ ЭВМ СМЕШИВАНИЯ ВОДЫ

Смешивание натуральных вод, находящихся в состоянии равновесия, часто вызывает их агрессивность или оседание, связанное с неприятным запахом и вкусом. Когда нет достаточного для смешения количества воды, необходим соответствующий подбор компонентов и их пропорций так, чтобы уменьшить агрессивность смеси. Для оптимизации с помощью ЭВМ введено уравнение третьего порядка, относящееся к содержанию эквивалентного CO_2 (a) и к углекислой жесткости (k)

$$a = (0{,}67 + 0{,}03t)10^{-2}k^3$$

где t обозначает температуру. Содержание связанного CO_2 (b) может быть вычислено из хорошо известной формулы

$$b = \frac{k}{2{,}8}22.$$

Расчеты ведут к уравнению третьего порядка. Когда это уравнение решено согласно формуле Кардано, получаются значения агрессивного и эквивалентного содержаний CO_2 в смеси. Компьютерные программы, работающие на основе этого метода дают возможность получения в течение короткого времени большого числа данных в виде рисунков и таблиц. Можно провести полное исследование пропорций смешения, чтобы улучшить оптимизацию. Легко можно также выбрать соответствующие пропорции смешения, отвечающие рекомендуемым или допускаемым уровням агрессивного CO_2 в воде. Этот метод может быть также применен для смешивания вод агрессивных по своей натуре.