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MATHEMATICAL MODEL DESCRIBING SEDIMENTATION OF SUSPENDED FLOC PARTICLES UNDER STATIC CONDITIONS

The effect of orthokinetic coagulation on the settling parameters of suspended floc particles is determined. To solve the problem of interest, the author makes use of some major principles dealt with in the Smoluchowski–Tuoril theory, as well as of the computer-aided simulation of the sedimentation process. The paper includes equations and curves describing variations in floc diameter, floc mass and settling rate during sedimentation as a function of fundamental decision parameters and water composition.

NOTATION

c_s – mass concentration of solid phase in the suspension, kg/m^{-3}, mg/dm^{-3},

d_{fl} – floc diameter, m,

$d_{fl}^{\min}$ – diameter of the smallest floc undergoing orthokinetic coagulation in the course of floc particle settling, m,

$d_{fl}^{(M)}$, $d_{fl}^{(D)}$ – diameter of sorbed floc, diameter of sorbing floc, m,

d_{flo}, $d_{fl}(t)$ – initial floc diameter, floc diameter after time t of sedimentation, m,

g – acceleration of gravity, m s^{-2},

m_{fl} – floc mass, kg,

m^* – coefficent of floc mass increment, kg m^{-1} s^{-1},

m_{flo}, $m_{fl}(t)$ – initial floc mass, floc mass after time of sedimentation, kg,

p_s – probability of effective collisions, dimensionless, %

t – time, s, h,

Δt – time of sampling, s,

v_{flo}, $v_{fl}(t)$ – settling velocity of floc at the beginning of the process and after time t, m s^{-1}, m h^{-1},

O_{ho}, $O_h(t)$ – hydraulic loading of tank surface described by the settling parameters of floc suspension at the beginning of the process and after time t, m^3, m^{-2} h^{-1},

T – temperature, K,

V_a – volume of attraction of settling flocs, m^3.

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1. INTRODUCTION

The settling of suspended floc particles has been analyzed so far either on an experimental or on a theoretical basis. The method of evaluating experimental data to describe the settling characteristics of suspended floc particles was developed by CAMP [1]. But the widespread acceptance of this procedure is largely the work of O'CONNOR and ECKENFELDER [2]. The theoretical approach makes use of some major principles included in Smoluchowski's, Müller's, Wiegner's and Tuoril's theories. Although the mechanism governing the sedimentation of suspended floc particles can be well enough described on this basis, the method is still inadequate as applied to the interpretation of results for the needs of water treatment. On the other hand, the empirical and experimental approach suggested by Camp or O'Connor and Eckenfelder neglects the mechanism of the process.

Having these in mind, the author of this paper thought it would be advisable to go on studying the sedimentation of suspended flocs. Such a study should aim at investigating the relations among water composition, properties of suspended flocs and the process parameters of coagulation and sedimentation [4].

2. AIM AND SCOPE OF THE STUDY

The objective of the study may be itemized as follows:

1) determination and definition of the basic variables and parameters of suspended flocs and of the sedimentation process,

2) formulation of the mathematical model with the aim to predict the course of the process as the function of water composition and floc properties,

3) investigation of the process by computer-aided simulation of discrete events.

3. MATHEMATICAL MODEL

Settling processes involving suspended floc particles should be modelled in terms of the law of motion for open systems which can increase their mass by drawing it from external sources. The flocs display different settling rates which may be attributed to their polydispersional nature. The resulting orthokinetic effect accounts for the non-stationary character of the sedimentation process. For the purpose of analysis it has been assumed that the increment in the mass of the flocs is proportional to their diameters. Hence,

$$\frac{dm}{dt} = m^* d_{fl} \tag{1}$$

which complies with Tuoril's model (via [3]). Considering relation

$$d_{fl} = \delta \cdot \sqrt[3]{m_{fl}}$$

and taking into account the initial condition of the process, $m_{fl}(t) = m_{flo}$ for $t = 0$, we obtain the floc mass after time $t = t$ of sedimentation by virtue of

$$m_{fl}(t) = m_{flo}\left(1 + \frac{2}{3}\frac{m^*}{m_{flo}^{2/3}} \cdot \delta \cdot t\right)^{3/2}. \tag{2}$$

Equation (2) may be applied in engineering practice when the value of m^* is known. Coefficient m^* describes the unit increment of the floc mass during sedimentation and depends on the following parameters: composition of the water under treatment, type and dosage of coagulants and flocculants, floc diameter, and probability of effective collision between flocs. The relation of interest takes the form

$$m^* = m^*(T, d_{fl}, \varrho_{fl}, c_s, p_s). \tag{3}$$

Water composition, as well as the type and dose of coagulants and flocculants have a strong influence on the concentration of floc suspensions and on floc density. The volume of the zone of attraction V_a between sorbed and sorbing flocs was established in terms of floc diameter by virtue of Tuoril's model (via [3]):

$$V_a = \frac{1}{4}\pi\left[(d_{fl}^{(D)} + d_{fl}^{(M)})^2 - (d_{fl}^{(D)})^2\right]. \tag{4}$$

In equation (4), the diameter of the flocs $d_{fl}^{(M)}$ sorbed by a sorbing floc which has a diameter $d_{fl}^{(D)}$ is defined by the arithmetic mean of limit diameters belonging to the set of absorbable flocs, $\langle d_{fl}^{\min}, d_{fl}^{(D)}\rangle$. In this set, the diameters of the smallest sorbable flocs are established in terms of the Müller formula (via [3]):

$$d_{fl}^{\min} = \sqrt[4]{\frac{19.4 \cdot kT}{\pi \cdot g \cdot \varrho_{fl}}}. \tag{5}$$

The probability of effective collisions was defined as the ratio of the mass of flocs sorbed by a sorbing floc to the total mass of sorbed flocs included in the zone of attraction of the sorbing floc. When sedimentation takes place in settling tanks, the probability of collision will be primarily determined by the sorbing ability and the shape of the flocs, as well as by their rotations associated with the velocity gradient of water flow through the settling tank. Making use of the mass conservation law, at the assumption that the floc diameter values fall in the range $\langle d_{fl}^{\min}, d_{fl}^{(D)}\rangle$, and considering relation (4), we obtain the expression defining the difference between successive mass values for a floc settling at two successive moments

$$m_{fli} = m_{fl(i-1)} \left\{ 1 + \frac{\pi \left(d^{(D)}_{fl(i-1)}\right)^2 \left[\left(\frac{3}{2} + \frac{d^{\min}_{fl}}{2 d^{(D)}_{fl(i-1)}} \right)^2 - 1 \right] v_{fl(i-1)} \cdot t \cdot c_s \cdot p_s}{4 m_{fl(i-1)}} \right\}. \tag{6}$$

When Stokes Law is used, eq. (6) enables us to predict the course of floc sedimentation under conditions of agglomerability. Combining eqs. (6) and (2) makes it possible to calculate the mass increment coefficient of the floc m^*.

4. SIMULATION OF THE PROCESS

4.1. METHODS

The sedimentation process was simulated by a RIAD-computer-aided method involving a programme written in Fortran IV. The sequence of calculations was established by the superposition of function

$$v_{fl}\left(d^{(D)}_{fl}\left(m_{fl}(t)\right)\right). \tag{7}$$

To apply the computer technique of interest, it was necessary to discretize function (7) by uniform sampling and stepped approximation. For the purpose of calculations a sedimentation time of four hours and a sampling time of one second were adopted. In the first stage of calculations, internal function $m_{fl}(t)$ was substituted by a stepped curve. Thus,

$$\begin{array}{lll} m_{flo} & \text{for} & 0 < t \leqslant \Delta t, \\ m_{fl}(\Delta t) & \text{for} & \Delta t < t \leqslant 2\Delta t, \\ \multicolumn{3}{c}{\dots\dots\dots\dots\dots\dots\dots} \\ m_{fl}(i\Delta t) & \text{for} & i\Delta t < t \leqslant (i+1)\Delta t, \\ \multicolumn{3}{c}{\dots\dots\dots\dots\dots\dots\dots} \\ m_{fl}\left((n-1)\Delta t\right) & \text{for} & (n-1)\Delta t < t \leqslant n\Delta t. \end{array} \tag{8}$$

The assumed length of observation of the sedimentation process was $t = n \cdot \Delta t$, where $n = 1.4 \times 10^4$.

The curve gives a sequence of the increasing values of the settling floc mass in consecutive time intervals. Assuming that the floc is spherical in shape and has a constant density, the values included in the curve show a strong relation with the increasing floc diameter (determined in successive sampling procedures). Settling velocity was calculated in terms of internal function $v_{fl}(d^{(D)}_{fl})$ which is described by the Stokes law of sedimentation.

The simulation of the process makes it possible to calculate the hydraulic loading of the settling tank (by making use of the orthokinetic effect) and the coefficient of floc mass increment. For this purpose the following relations were used:

$$O_h(i\Delta t) = \frac{\sum\limits_{1}^{i} v_{fl}(i\Delta t)\cdot t_i}{\sum\limits_{1}^{i} \Delta t_i}, \tag{9}$$

$$m^*(i\Delta t) = \frac{3}{2\delta \sum\limits_{1}^{i} \Delta t_i}\left(m_{fl}^{2/3}(i\Delta t) - m_{flo}^{2/3}\right). \tag{10}$$

It should be notified that the assumptions made for the need of simulation, i.e. spherical shape and constant density of flocs, as well as laminar motion, are in general use. The adopted parameter values describing the suspension under study are listed in tab. 1. The data included there enable the influence of the orthokinetic effect on floc settling to be determined for a wide range of water composition variations. The floc densities and suspended floc particle concentrations adopted for the purpose of calculation are typical of coloured low turbidity waters (No. 1), of turbid waters (No. 2), and of high turbidity waters containing suspended matter at concentrations usually measured in flood water.

Table 1

Characterization of the suspension

No.	Floc density kg m^{-3}	Concentration of suspension kg m^{-3}	Probability of effective collision	Floc diameter
1	1002.9	68×10^{-3}	0.05 0.10 0.20 0.50 1.00	1.0×10^{-3}
		68×10^{-3}	1.00	0.25×10^{-3} 0.5×10^{-3} 1.0×10^{-3} 1.5×10^{-3}
2	1004.8	236×10^{-3}	1.00	1.0×10^{-3}
3	1042.0	2000×10^{-3}	1.00	1.0×10^{-3}

4.2. RESULTS AND DISCUSSION

The results are plotted in figs, 1–3 in the form of dimensionless quotients describing the following parameters: increment of floc mass and diameter, settling velocity, and hydraulic loading of a settling tank with ideal, equalized flow, as a function of time (duration of the sedimentation process). From these data it is obvious that, under certain circumstances, the orthokinetic effect may be neglected. This finding holds particularly for floc suspensions generated during alum coagulation of intensely coloured waters. The originating flocs display a density similar to that of the water, which rarely exceeds 1003.0 kg m^{-3}. In such a case (and, also, at a water temperature of 273.15 K, a settling time of 1.44×10^4 s, a floc size of 10^{-3} m and $p_c = 0.05$), neglecting the orthokinetic effect leads to errors amounting to 1.35%, 2.73%, and 1.51% for d_{fl}, v_{fl} and O_h, respectively. For $p_c = 0.10$, the error of interest takes higher values, 2.90%, 5.89% and 2.89%, respectively. When p_c reaches its maximum value ($p_c = 1.00$), the relative increment of the investigated functions becomes 25.79%, 58.23% and 30.22% for d_{fl}, v_{fl} and O_h, respectively. When water temperature increases to 293.15 K, the increments are greater, amounting to 40.37%, 97.05%, and 51.20%, respectively.

Smaller diameters of the sorbing flocs give smaller increments of the functions analyzed. Thus, at $d_{fl} = 0.25 \times 10^{-3}$ m, $p_c = 1.00$, $t = 1.44 \times 10^4$ s, and water temperature of 273.15 K, the increment amounts to 7.65%, 15.89% and 8.04% for d_{fl}, v_{fl}, and O_h, respectively. Agglomeration per unit time was found to be significantly greater during settling of flocs generated by the coagulation of particles which account for water turbidity. The flocs in question show a somewhat higher density (1004 to 1005 kg m^{-3}) which makes them settle easier. Thus, the volume of the zone from which the sorbing flocs attract the sorbed flocs in a given unit time increases. In this particular case, at $d_{fl} = 10^{-3}$ m, $p_c = 1.00$, water temperature of 273.15 K and settling time of 1.44×10^4 s, the relative increment of respective functions is 77.7%, 215.7% and 117.4% (for d_{fl}, v_{fl} and O_h, respectively). In the extreme event, which is characteristic of highly turbid waters, the increment amounts to 562.9%, 4294.9% and 2545.8%, respectively. The values obtained for very turbid water are overestimated. This is because the mathematical model used for the simulation of the sedimentation process did not include the shear resistance of flocs and the effect of settling disturbances originating from the concentration of the solid phase. Neglecting the shear resistance of flocs leads to very high values of the process parameters which are not encountered in engineering practice.

The values of the floc mass increment coefficient m^* and of the tank loading (along with the effect of orthokinetic coagulation) are listed in tabs. 2 and 3. These parameters have been related to the concentration of pollutants, the probability of effective collision, floc diameter and water temperature. As shown by the data included in the tables, the value of m^* is influenced primarily by the concentration of pollutants and floc diameter. The increase of turbidity and coloured matter

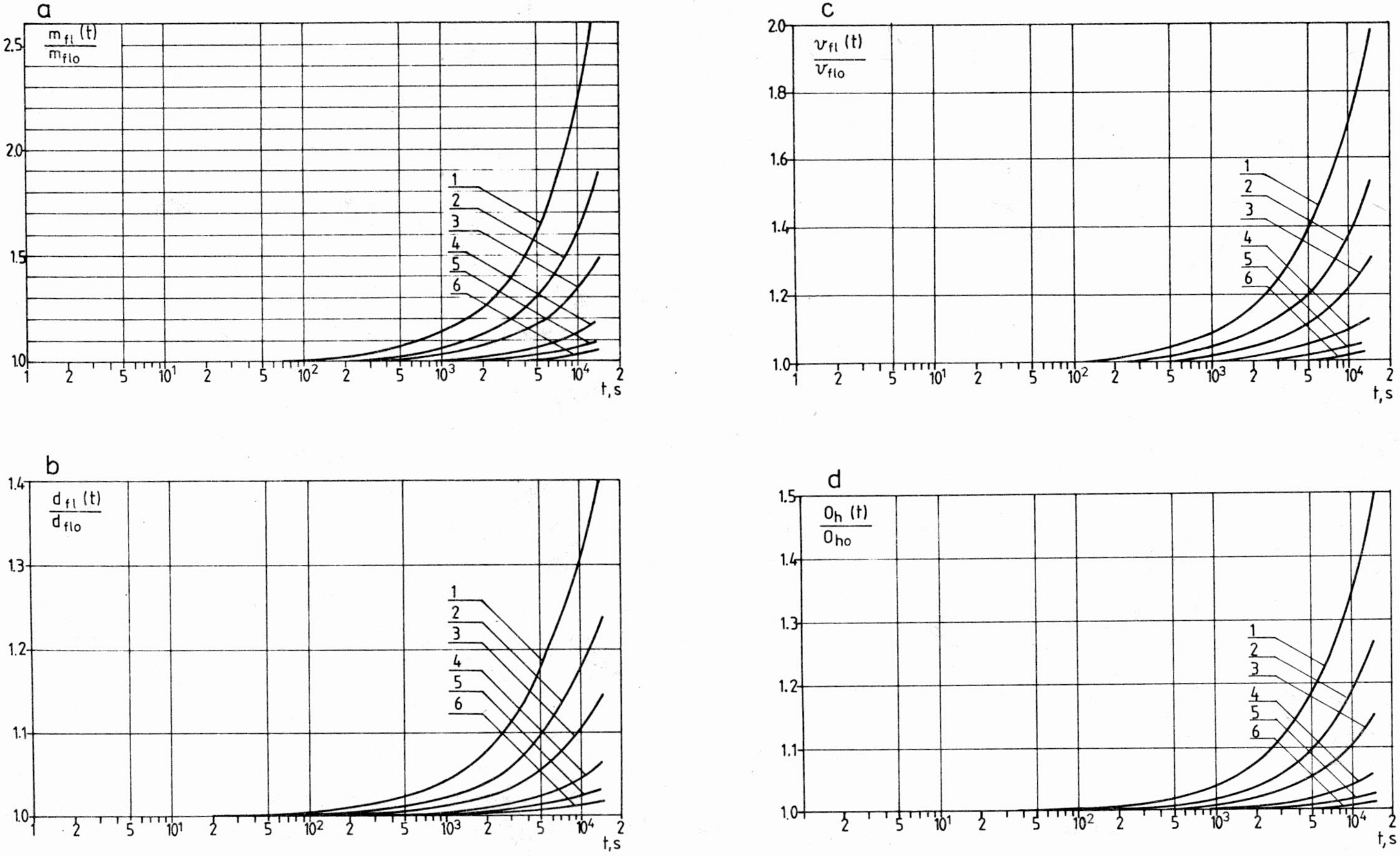

Fig. 1. Characteristics of the sedimentation of suspended floc particles

$\varrho_{fl} = 1002.9$ kg m^{-3}, $d_{flo} = 10^{-3}$ m, $c_s = 68.7$ mg dm^3, $T = 273.15$ K: a) $m_{fl}(t) \times m_{flo}^{-1} = f(p_c, t)$, b) $d_{fl}(t) \times d_{flo}^{-1} = f(p_c, t)$,

c) $v_{fl}(t) \times v_{flo}^{-1} = f(p_c, t)$, d) $O_h(t) \times O_{ho}^{-1} = f(p_c, t)$: 1) $p_c = 1.0$, 2) $p_c = 0.5$, 3) $p_c = 0.2$, 4) $p_c = 0.5$ and $T = 293.15$ K, 5) $p_c = 0.1$, 6) $p_c = 0.05$

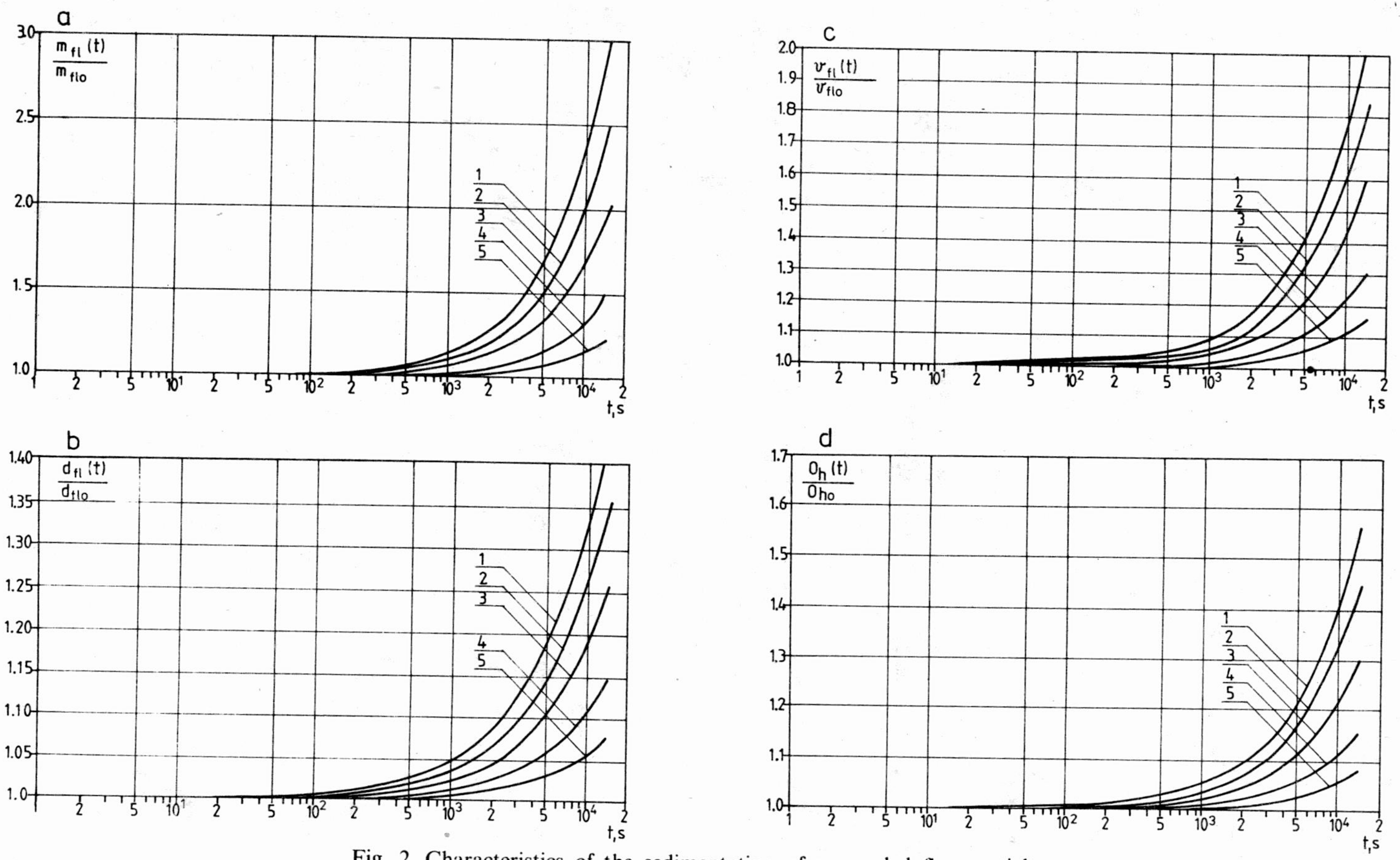

Fig. 2. Characteristics of the sedimentation of suspended floc particles

$\varrho_{fl} = 1002.9$ kg m^{-3}, $p_c = 1.0$, $c_s = 68.7$ mg dm^{-3}, $T = 273.15$ K: a) $m_{fl}(t) \times m_{flo}^{-1} = f(d_{flo}, t)$; b) $d_{fl}(t) \times d_{flo}^{-1} = f(d_{flo}, t)$, c) $v_{fl}(t) \times v_{flo}^{-1} = f(d_{flo}, t)$, d) $O_h(t) \times O_{ho}^{-1} = f(d_{flo}, t)$, 1) $d_{flo} = 2.0 \times 10^{-3}$ m, 2) $d_{flo} = 1.5 \times 10^{-3}$ m, 3) $d_{flo} = 10^{-3}$ m, 4) $d_{flo} = 0.5 \times 10^{-3}$ m, 5) $d_{flo} = 0.25 \times 10^{-3}$ m

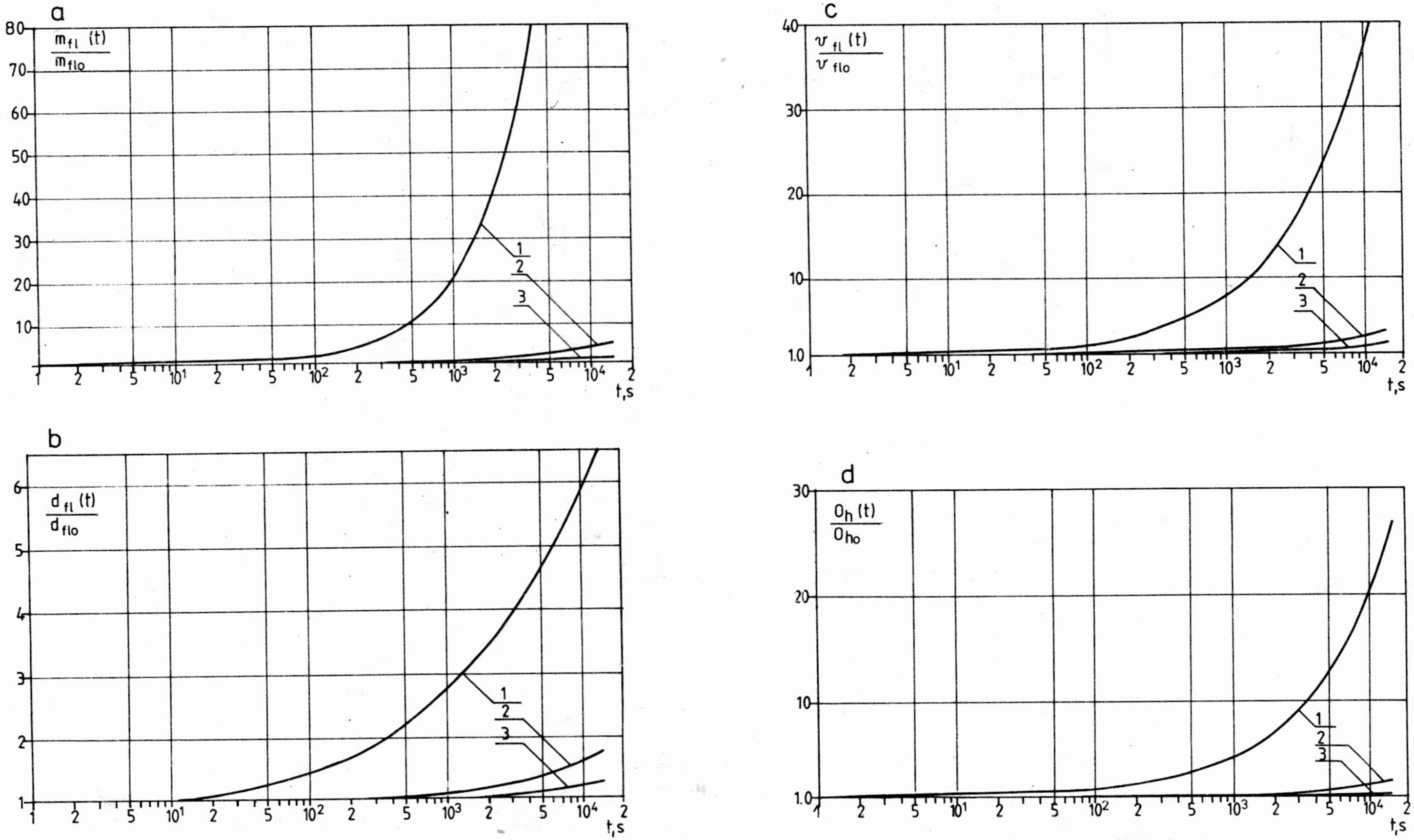

Fig. 3. Characteristics of the sedimentation of suspended floc particles

$p_c = 1.0$, $d_{flo} = 10^{-3}$ m, $T = 273.15$ K, a) $m_{fl}(t) \times m_{flo}^{-1} = f(\varrho_{fl}, c_s, t)$, b) $d_{fl}(t) \times d_{flo}^{-1} = f(\varrho_{fl}, c_s, t)$, c) $v_{fl}(t) \times v_{flo}^{-1} = f(\varrho_{fl}, c_s, t)$,

d) $O_h(t) \times O_{ho}^{-1} = f(\varrho_{fl}, c_s, t)$, 1) $c_s = 2.101$ kg m^{-3}, $\varrho_{fl} = 1042$ kg m^{-3}, 2) $c_s = 0.236$ kg m^{-3}, $\varrho_{fl} = 1004$ kg m^{-3},

3) $c_s = 0.0687$ kg m^{-3}, $\varrho_{fl} = 1002.9$ kg m^{-3}

Table 2

Coefficient m^* (kg m^{-1} s^{-1}) as a function of the investigated factors

Time	Factor													
	Effect of floc density and suspension concentration			Effect of probability of effective collisions					Effect of initial floc diameter				Effect of water temperature	
	10^8 m^* for $p_c = 1.0$ $d_{flo} = 10^{-3}$ m, $T = 273.15$ K and for ϱ_{fl} and c_s:			10^8 m^* for $\varrho_{fl} = 1002.9$ kg m^{-3} $c_s = 68.7 \times 10^{-3}$, $T = 273.15$ K, $d_{flo} = 10^{-3}$ m and for p_c					10^8 m^* for $\varrho_{fl} = 1002.9$ kg m^{-3} $c_s = 68.7 \times 10^{-3}$ kg m^{-3}, $T = 273.15$ K, $p_c = 1.0$ and for d_{flo}				10^8 m^* for $\varrho_{fl} = 1002.9$ kg m^{-3}, $c_s = 68.7 \times 10^{-3}$ kgm^{-3}, $d_{flo} = 10^{-3}$ m, $p_c = 0.50$ and for T	
	1002.9 kg m^{-3} 68.7 mg dm^{-3}	1004 kg m^{-3} 236 mg dm^{-3}	1042 kg m^{-3} 2×10^3 g dm^{-3}	0.05	0.10	0.20	0.50	1.00	0.25×10^{-3} m	0.5×10^{-3} m	1.0×10^{-3} m	1.5×10^{-3} m	273.15 K	293.15 K
1 s	6.437	16.912	1100.833	0.151	0.370	0.739	1.796	6.437	0.056	0.451	6.437	12.184	1.796	3.218
0.25 h	6.324	16.189	561.420	0.140	0.346	0.701	1.769	6.324	0.056	0.449	6.324	12.003	1.769	3.190
0.50 h	6.221	15.596	463.158	0.148	0.346	0.701	1.760	6.221	0.056	0.447	6.221	11.835	1.760	3.162
0.75 h	6.126	15.097	411.259	0.148	0.346	0.698	1.750	6.126	0.056	0.445	6.126	11.679	1.750	3.136
1.00 h	6.038	14.661	377.180	0.148	0.346	0.690	1.741	6.038	0.056	0.442	6.038	11.533	1.741	3.110
1.25 h	5.956	14.282	352.326	0.147	0.345	0.686	1.732	5.956	0.056	0.440	5.956	11.395	1.732	3.086
1.50 h	5.879	13.946	333.032	0.147	0.343	0.684	1.724	5.879	0.056	0.438	5.879	11.266	1.724	3.063
1.75 h	5.807	13.644	317.420	0.147	0.338	0.684	1.716	5.807	0.055	0.436	5.807	11.144	1.716	3.040
2.00 h	5.739	13.371	304.407	0.147	0.334	0.684	1.709	5.739	0.055	0.434	5.739	11.028	1.709	3.019
2.25 h	5.675	13.122	293.314	0.147	0.331	0.684	1.701	5.675	0.055	0.433	5.675	10.918	1.701	2.998
2.50 h	5.615	12.892	283.689	0.147	0.329	0.682	1.693	5.615	0.055	0.431	5.615	10.813	1.693	2.978
2.75 h	5.557	12.680	275.224	0.147	0.327	0.678	1.687	5.557	0.055	0.429	5.557	10.713	1.687	2.958
3.00 h	5.502	12.483	267.694	0.147	0.326	0.676	1.679	5.502	0.055	0.427	5.502	10.617	1.679	2.940
3.25 h	5.450	12.299	260.926	0.147	0.324	0.675	1.672	5.450	0.055	0.425	5.450	10.525	1.672	2.921
3.00 h	5.400	12.127	254.798	0.147	0.323	0.674	1.665	5.400	0.055	0.424	5.400	10.473	1.665	2.904
3.75 h	5.352	11.964	249.208	0.147	0.323	0.673	1.658	5.352	0.054	0.422	5.352	10.353	1.658	2.886
4.00 h	5.306	11.811	244.081	0.147	0.322	0.673	1.652	5.306	0.054	0.420	5.306	10.271	1.652	2.870

Table 3

$O_h(t) \times O_{ho}^{-1}$ as a function of the investigated factors

Time, h	Factor													
	Effect of composition of floc suspension		Effect of probability of effective collisions					Effect of initial floc diameter					Effect of water temperature	
	$O_h(t) \times O_{ho}^{-1}$ for $T = 273.15$ K, $d_{flo} = 10^{-3}$ m, $p_c = 1.0$ and ϱ_{fl} and c_s		$O_h(t) \times O_{ho}^{-1}$ for $\varrho_{fl} = 1002.9$ kg m^{-3}, $c_s = 68.7$ mg dm^{-3}, $T = 273.15$ K, $d_{flo} = 10^{-3}$ m and for p_c					$O_h(t) \times O_{ho}^{-1}$ for $\varrho_{fl} = 1002.9$ kg m^{-3} $c_s = 68.7$ mg dm^{-3}, $T = 273.15$ K, $p_c = 1.0$ and for d_{flo}					$O_h(t) \times O_{ho}^{-1}$ for $p_c = 1.0$, $\varrho_{fl} = 1002.9$ kg m^{-3}, $c_s = 10^{-3}$ m and for T	
	1002.9 kg m^{-3} 68.7 mg dm^{-3}	1004 kg m^{-3} 236 mg dm^{-3}	0.05	0.10	0.20	0.50	1.00	0.25×10^{-3} m	0.50×10^{-3} m	10^{-3} m	1.5×10^{-3} m	2.0×10^{-3} m	273.15	293.15
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.020	1.093	1.001	1.002	1.004	1.010	1.020	1.005	1.010	1.020	1.030	1.040	1.020	1.036
0.50	1.039	1.182	1.002	1.003	1.007	1.019	1.039	1.010	1.020	1.039	1.060	1.080	1.039	1.071
1.0	1.080	1.348	1.003	1.007	1.015	1.039	1.080	1.020	1.040	1.080	1.119	1.159	1.080	1.140
1.25	1.098	1.427	1.004	1.009	1.019	1.049	1.098	1.025	1.050	1.098	1.147	1.192	1.098	1.174
1.5	1.119	1.504	1.005	1.011	1.023	1.059	1.119	1.030	1.060	1.119	1.176	1.229	1.119	1.207
2.0	1.157	1.649	1.007	1.015	1.031	1.078	1.157	1.040	1.080	1.157	1.231	1.299	1.157	1.271
2.25	1.176	1.719	1.008	1.017	1.035	1.088	1.176	1.045	1.090	1.176	1.258	1.334	1.176	1.303
2.5	1.194	1.788	1.009	1.019	1.038	1.097	1.194	1.050	1.099	1.194	1.284	1.368	1.194	1.334
3.0	1.231	1.921	1.011	1.022	1.046	1.116	1.231	1.060	1.119	1.231	1.336	1.432	1.231	1.394
3.25	1.249	1.986	1.012	1.024	1.050	1.125	1.249	1.065	1.128	1.249	1.362	1.467	1.249	1.424
3.5	1.267	2.049	1.013	1.026	1.054	1.134	1.267	1.070	1.138	1.267	1.388	1.499	1.267	1.454
4.0	1.302	2.174	1.015	1.029	1.061	1.153	1.302	1.080	1.157	1.302	1.438	1.562	1.302	1.512

concentration in the water under treatment brings about both a rise in the density of originating flocs and an increase in the concentration of the post-coagulation suspensions. Flocs produced by the coagulation of high turbidity water display m^* values twice as high as do flocs generated by the coagulation of intensely coloured water. The m^* value of the flocs produced during coagulation of highly turbid water is also influenced by their shear resistance and by the disturbances coming from the hydrodynamic interference of the water streams. These factors have not been included in the model, and that is why the m^* values listed in column 3 of tabl. 2 are overestimated; they only show the tendency of floc mass increment influenced by the parameters of interest.

Floc diameter exerts a substantial influence on the m^* value. This is an indication that the operation of the flocculation tanks has an important contribution to the parameters of the orthokinetic coagulation which occurs during settling of suspended floc particles. A 2-fold increase of floc diameter may account for an as much as 8-fold increase of the m^* value.

Also water temperature was one of the major factors affecting the value of m^*. As it is shown by the data in tab. 2, a rise in water temperature from 273.15 K to 293.15 K increases the m^* value by some $70^0/_0$. This offers further support for the higher effects of orthokinetic coagulation obtained in secondary settling tanks in the summer season.

In this paper, attempts are also made to determine the relation between the probability of effective collisions and the m^* coefficient for $p_c \in \langle 0.05, 1.0 \rangle$. The relationship in question is of an approximately linear nature. One of the typical features of coefficient m^* is its poor time-dependence. This holds for a wide range of t-values (which is $t \in \langle 1, 1.4 \times 10^4 \text{ s} \rangle$), except the m^* values determined for waters of high turbidity ($\leqslant 1000$ g m^{-3}, tab. 2, column 3). These curtail the range of application for the model derived here.

The data in tab. 3 give the relative increment of the hydraulic loading value due to the orthokinetic effect. The variation of this parameter as a function of the investigated factors has already been discussed (figs. 1–3). The values of $O_h(t) \times O_{ho}^{-1} = f(\varrho_{fl}, c_s, p_c, d_{flo}, T)$ calculated in terms of the model offer sufficient support for neglecting the orthokinetic effect when the water to be treated carries intensely coloured matter ($\leqslant 50$ g Pt m^{-3}), has very low turbidity and does not contain suspended solids. The calculated values of this function also justify the need of including the orthokinetic effect when the water under treatment displays an increasing turbidity. When turbidity is increased, the values listed in column 2 of tab. 3 describe the relative increment of hydraulic loading – by $65^0/_0$ for 2 hour sedimentation and by $117^0/_0$ for 4 hour sedimentation. But these values have been determined on the assumption that $p_c = 1.0$, and such a value fails to occur under flow conditions found in a settling tank, where p_c falls in the range $\langle 0, 0.2 \rangle$. It is impossible to determine theoretically the accurate values of p_c for the flow of suspended floc particles through the settling tanks. Floc shape other than spherical and perturbation of flow in the tanks (which can primarily be attributed to the variable and high velocity gradients) are amongst the major factors contributing to the increase of the p_c value, but they may also account for the destruction of the flocs. Estimations of the p_c values for settling tanks have not been reported in the available literature. Estimates of that kind require either full-scale investigations or pilot-plant procedures enabling visualization.

5. CONCLUSIONS

1. To evaluate the settling parameters of suspended floc particles it is convenient to make use of an iterative approach which enables construction of a mathematical model even if not all of the data required are at hand.

2. The orthokinetic coagulation that occurs during settling of suspended floc particles is influenced by the following factors: water composition, floc diameter, probability of effective collisions and water temperature.

3. When alum coagulation is applied to the treatment of intensely coloured ($\leqslant 50$ g Pt m^{-3}) and low turbidity waters, the effect of orthokinetic coagulation on the settling parameters of suspended floc particles is insignificant and may, therefore, be neglected. The opposite holds for the floc suspension generated by the coagulation of water with increased turbidity level and solids concentration; thus the orthokinetic effect must be taken into account, as otherwise calculation errors are obtained. The assessment of those errors requires knowledge of the probability of effective collisions or the value of the floc mass increment coefficient.

4. The orthokinetic coagulation effect on the settling of suspended floc particles is especially pronounced in the summer months. This holds primarily for water treatment plants where a well-designed flocculation is involved to produce large and settleable flocs.

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MATEMATYCZNY MODEL PROCESU SEDYMENTACJI CZĄSTEK ZAWIESINY KŁACZKOWATEJ W WARUNKACH STATYCZNYCH

Określono wpływ koagulacji ortokinetycznej na parametry sedymentacji cząstek zawiesiny pokoagulacyjnej. Zagadnienie rozwiązano w oparciu o wybrane elementy teorii Smoluchowskiego–Tuorila oraz metodę cyfrowej symulacji procesu sedymentacji na maszynach matematycznych. Podano równania i wykresy określające zmianę średnicy, masy i prędkości opadania kłaczków w czasie sedymentacji w funkcji podstawowych parametrów decyzyjnych procesu i składu wody.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ПРОЦЕССА СЕДИМЕНТАЦИИ ЧАСТИЦ ХЛОПЬЕВИДНОЙ СУСПЕНЗИИ В СТАТИЧЕСКИХ УСЛОВИЯХ

Определено влияние ортокинетической коагуляции на параметры седиментации частиц послекоагуляционной суспензии. Решение проблемы основано на избранных элементах теории Смолюховского–Ториля, а также на методе цифрового моделирования процесса седиментации на математических машинах. Даны уравнения и диаграммы, определяющие изменение диаметра, массы и скорости оседания хлопьев во время седиментации в зависимости от основных параметров решения процесса и состава воды.