Environment Protection Engineering

Vol. 10 1984 No. 2

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CHARACTERISTICS OF THE MAXIMUM WATER CONSUMPTION IN AN INDUSTRIALIZED URBAN AREA

The estimation of the maximum water demand is of importance in design of some elements of water supply systems. In order to determine the degree to which the water demand for all purposes is satisfied, it is necessary to know the probability that in a given day a given water consumption will be exceeded. The paper presents a probabilistic approach to the problem of the maximum water consumption in an industralized urban area in Poland in which the number of inhabitants varied from 810,000 to 840,000. In this agglomeration water demands of all users could be fully satisfied. The probability density function of the maximum water consumption can be described by third-type Pearson distribution and, after suitable transformation normalizing the variable, by Gaussian distribution. Since the coefficient of variation for the variable studied was found to be relatively low, the characteristic values of water consumption could be estimated from small statistical samples. The effect of the duration of water consumption T on the average intensity of water consumption was insignificant for $T \in \{5,240 \text{ min}\}$. The values of $q_{\max}(T, p)$ were related to the number of inhabitants and the average annual water consumption.

NOTATIONS

A – coefficient in eqs. (6) and (7),
A_1 – coefficient in eq. (8),
A_2 – coefficient in eq. (9),
a, $\hat{a}$ – parameter of shift (limit from below) of gamma-distribution, estimator of parametr a,
B – exponent in eqs. (6) and (7),
k, $\hat{k}$ – parameter of gamma-distribution, estimator parameter k,
m – expected value of variable with Gaussian distribution,
n – number of elements of statistical sample,
n_1, n_2 – exponents in eqs. (8) and (9),

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$P_a = 1-a$ — significance level at confidence level a,
p — probability of a given value being exceeded,
Q_m — average annual water consumption [dm³/s],
q — water consumption [dm³/s], value of random variable,
$\bar{q}$ — sampling average,
$q^{60}_{\max}$ — maximum hourly water consumption [dm³/s],
$q^{T}_{\max}$ — maximum water consumption in time T,
$q^{60}_{\max_p}$ — quantile of maximum hourly water consumption,
s — estimation from standard deviation sample,
T — duration of water consumption [min],
t — standardized variable.
Y — normalized variable,
a — confidence level,
ε — size of confidence interval,
$\lambda, \hat{\lambda}$ — parameter of gamma-distribution, estimator of parameter,
$\Gamma(k)$ — gamma-function.

SUBSCRIPTS

e — empirical,
max — maximum,
Y — for normalized variable,
0.05 — at confidence level $a = 0.05$.

1. INTRODUCTION

Water consumption and water demand are subject to wide fluctuations due to a number of factors which, in some instances, are difficult to calculate. Thus, a model approach to this problem for forecasting purposes must involve probability mathematics. This refers especially to the estimation of the maximum water consumption and maximum water demand, which must be considered if a water supply system is to be designed and hydraulic calculations made. The commonly accepted opinion is that to the maximum water consumption and the maximum water demand it should be assigned the probability that these values are exceeded. Such an approach will permit us to determine the degree to which the water demand is satisfied. To determine adequate relationships, it is essential to consider the random nature of the variable under study. This paper presents a probabilistic approach to the problem of the maximum water consumption in an industrialized urban agglomeration (referred to as A) in Poland, the population of which ranged from 810,000 to

840,000 people in 1976 to 1979, and which was then capable to satisfy water requirements of all the users.

Flow rate measurements for water supplied to the piping system were performed continuously, using orifice plates with flow recorders.

2. MAXIMUM HOURLY CONSUMPTION

Water consumption and water demand can be considered as random variables limited unilaterally (from below). Thus, a statistical sample representing the random characteristic of the variable should include the data from at most one-year observations. This is because during longer periods of time the most important factors affecting the quantity of water consumption and its fluctuations may be subject to substantial changes. Therefore our analysis included the sets of data concerning the maximum hourly water consumption during successive workdays throughout the year. To eliminate the effect of a seasonal periodicity, the year was divided into two periods: May–October and November–April [3,4], which will be referred to as season I and season II, respectively. In this way, to the maximum water consumption we may assign the probability that this value is exceeded within a twenty-four hour period and not within the year which is essential in the design and analysis of piping systems (in dimensioning of the water-pipe network the flows occurring less frequently than once a year are not taken into account). Statistical samples satisfied the criteria of homogeneity (the results of the homogeneity test are reported elsewhere [2]), as well as the criterion of the independence of the sample elements. The random properties of the maximum hourly consumption can be approximated in terms of the three-parameter gamma-distribution with limit on the left (third-type Pearson distribution) [2]:

$$f(q_{\max}^{60}) = \frac{\lambda[\lambda(q_{\max}^{60}-\hat{a})]^{k-1}\, e^{-\lambda(q_{\max}^{60}-\hat{a})}}{\Gamma(k)} \tag{1}$$

for $(q_{\max}^{60}-\hat{a}) > 0,\ k > 0,\ \hat{a} \geqslant 0,\ \lambda > 0.$

Table 1 gives the parameters of the empirical distribution for the random variable under study, as well as the estimators $\hat{\lambda}$, $\hat{k}$, and $\hat{a}$ for the parameters of distribution (1), determined by the moments method. A graphical representation of the distribution function is presented in figs. 1–4. After transformation

$$y_{\max}^{60} = (q_{\max}^{60}-\hat{a})^{0.28}, \tag{2}$$

which normalizes the variable, the compatibility between the empirical distributions of variable (2) and the Gaussian distribution was checked, using Pear-

Table 1

Parameters for empirical distribution of the maximum hourly consumption q^{60}_{max}
Parametry empirycznej dystrybuanty maksymalnego zużycia wody q^{60}_{max}

Season, year	Population thousand	Unit average annual consumption dm³/head, day	$\bar{q}^{60}_{max}$ dm³/s	s dm³/s	Estimators of gamma-distribution parameters		
					$\hat{a}$ dm³/s	$\hat{\lambda}$	$\hat{k}$
I, 1976	810.0	348.5	3946	207.1	0	0.092	363
II, 1976/77	814.2	351.8	4010	186.9	1200	0.080	226
I, 1977	818.4	350.0	4125	226.7	0	0.092	380
II, 1977/78	821.8	387.1	4467	169.2	1200	0.114	375
I, 1978	825.2	385.5	4501	241.9	0	0.077	346
I, 1979	835.0	395.3	4502	299.0	0	0.050	227
II, 1979/80	839.5	401.0	4717	131.6	1200	0.203	715

son's chi-square and Kolmogorov's lambda-tests. As shown from the data in tab. 2, the differences between the distribution of interest fall within the limits of the random error and are statistically insignificant at the confidence level $\alpha = 0.05$.

To determine the minimal number of observations (minimal number of elements of the statistical sample) sufficient to estimate, at the assumed con-

Table 2

Compatibility between distribution of normalized random variable $y^{60}_{max} = (q^{60}_{max} - \hat{a})^{0.28}$ and Gaussian distribution
Wyniki testów zgodności rozkładów zmiennej znormalizowanej $y^{60}_{max} = (q^{60}_{max} - a)^{0.28}$ z rozkładem normalnym

Season, year	$\bar{y}^{60}_{max}$	Test of compatibility between distribution of random variable and Gaussian distribution			
		χ^2 test		λ test	
		χ^2_e	$\chi^2_{0.05}$	λ_e	$\lambda_{0.05}$
I, 1976	10.158	3.94	9.49	0.28	1.36
II, 1976/77	9.235	2.98	9.49	0.32	1.36
I, 1977	10.284	8.05	9.49	0.40	1.36
II, 1977/78	9.635	1.97	9.49	0.21	1.36
I, 1978	10.539	5.09	9.49	0.30	1.36
I, 1979	10.538	4.02	9.49	0.27	1.36
II, 1979/80	9.837	7.52	9.49	0.31	1.36

fidence interval ε, the expected value m for variable (2) having a normal distribution (or of the modal value for variable Q^{60}_{max} having a gamma-distribution (1)) the following formula was applied:

$$n = \left(t_{a/2,\, n-1} \frac{S_Y}{\varepsilon_Y}\right)^2 \qquad (3)$$

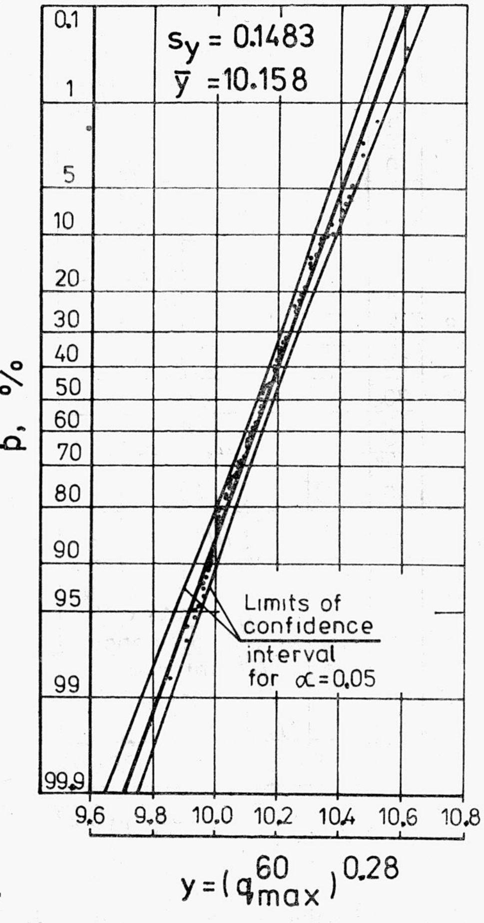

Fig. 1. Distribution function for variable Y^{60}_{max} (Gaussian distribution of probability) in the period of May 1 to October 30, 1976

Rys. 1. Wykres empirycznej dystrybuanty rozkładu prawdopodobieństwa zmiennej Y^{60}_{max} w okresie od 1.05 do 30.10.1976

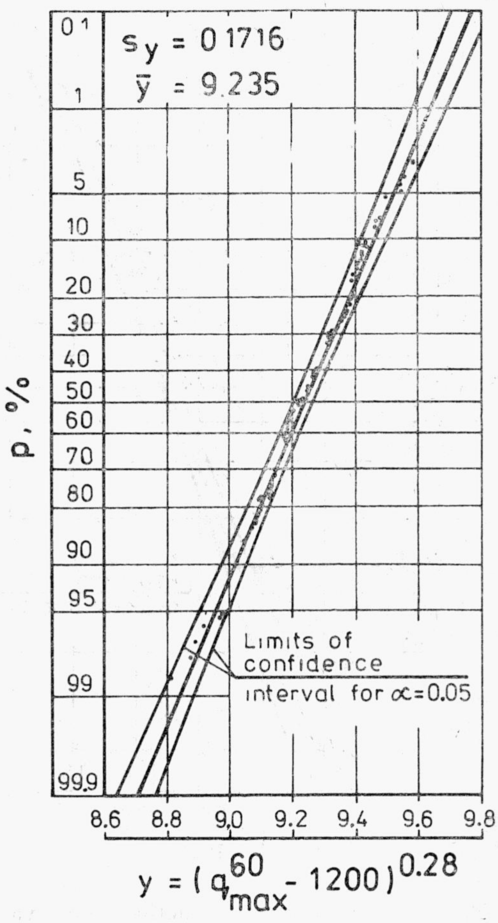

Fig. 2. Distribution function for variable Y^{60}_{max} (Gaussian distribution of probability) in the period of November 1, 1976 to April 30, 1977

Rys. 2. Wykres empirycznej dystrybuanty rozkładu prawdopodobieństwa zmiennej Y^{60}_{max} w okresie od 1.11.1976 do 30.04.1977 where $t_{\alpha/2,n-1}$ is a tabulated value which will be exceeded by the standardized variable (having a Student's distribution and a parameter $n-1$) with the probability $1-\alpha/2$. Equation (3) holds when the standard deviation $\sigma_{Y'}$ of variable Y is not known a priori (being determined from a small sample with n elements).

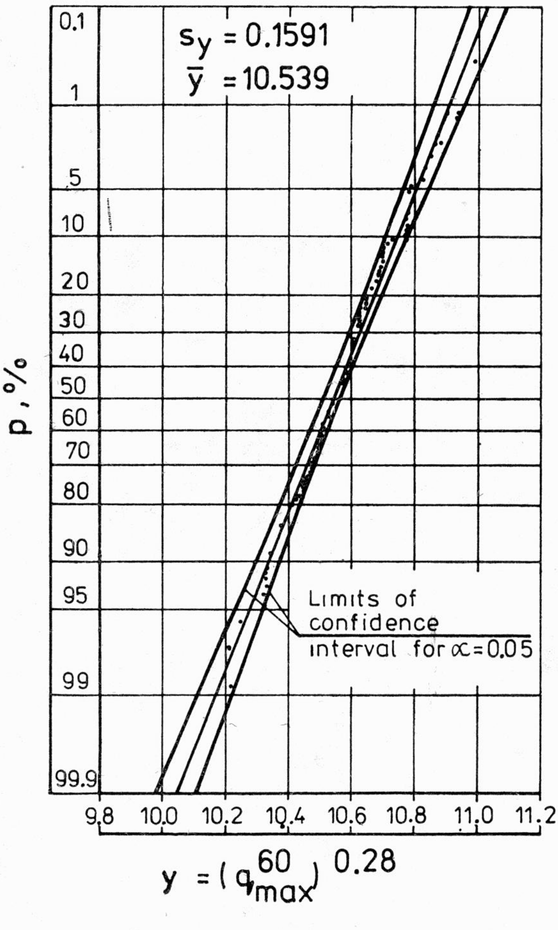

Fig. 3. Distribution function for variable Y^{60}_{max} (Gaussian distribution of probability) in the period of May 1 to October 30, 1979

Rys. 3. Wykres empirycznej dystrybuanty rozkładu prawdopodobieństwa zmiennej Y^{60}_{max} w okresie od 1.05 do 30.10.1979

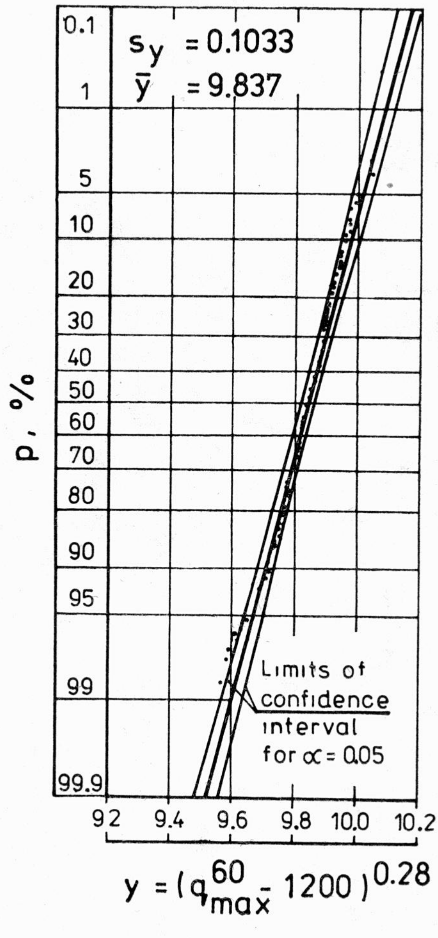

Fig. 4. Distribution function for variable Y^{60}_{max} (Gaussian distribution of probability) in the period of November 1, 1979 to April 30, 1980

Rys. 4. Wykres empirycznej dystrybuanty rozkładu prawdopodobieństwa zmiennej Y^{60}_{max} w okresie od 1.11.1979 do 30.04.1980

The confidence interval ε_Y in eq. (3) was assumed to be 2.5 and 5.0% of the average annual water consumption Q_m. The results calculated for a confidence level $\alpha = 0.05$ are shown in tab. 3. In engineering practice eq. (3) can be replaced by two simplified formulae:

$$n = 882 \frac{s}{\bar{q}^{60}_{\max}} - 18.53, \quad (4)$$

and

$$n = 192 \frac{s}{\bar{q}^{60}_{\max}} - 0.73 \quad (5)$$

for $\varepsilon = 0.025\, Q_m$ and $\varepsilon = 0.05\, Q_m$, respectively.

Table 3

Number of elementns of small statistical sample
Zestawienie liczby n elementów skróconej próby statystycznej

Season, year	Number of elements n of sample for $\alpha = 0.05$	
	$\varepsilon = 0.025\, Q_m$	$\varepsilon = 0.05\, Q_m$
I, 1976	27	9
II, 1976/77	22	8
I, 1977	32	10
II, 1977/78	16	6
I, 1978	30	10
I, 1979	40	12
II, 1979/80	10	5

The gamma-distribution of the maximum hourly consumption in agglomeration A does not significantly differ from the Guassian distribution. Therefore by introducing into eq. (3) the values s and ε of variable $Q^{60}_{\max}$ instead of $Y^{60}_{\max}$, similar values of n will be obtained, e.g. for season I of 1978:

$$n = \left(2.207 \frac{241.9}{184.1}\right)^2 = 8.71 \simeq 9,$$

$s = 241.9\ \mathrm{dm^3/s}$ was determined from 147 observations.

As it follows from tab. 3, in order to estimate the expected value of the maximum hourly consumption on workdays in the given agglomeration, it is sufficient to perform 8 to 12 measurements, provided that the difference between the arithmetical means obtained from small sample and large sample does not exceed 5 % of Q_m with the probability of 95 %. Small statistical samples can be made representative by an adequate choice (based on the tables of random numbers) of workdays on wich measurements are to be carried out. To verify eq. (3), twenty samples (with $n = 10$ for $\varepsilon = 5.0\ \%\ Q_m$ and $n = 30$ for $\varepsilon = 2.5\ \% Q_m$) were selected for the season I of 1978. The difference between the mean values obtained from small and large statistical samples ranged from $-3.75\ \%$ to $+2.48\ \%\ Q_m$ and from $-2.44\ \%$ to $+2.08\ \%\ Q_m$ for $n = 10$ and $n = 30$, respectively, thus they did not exceed the admissible values.

3. PROBABILISTIC APPROACH TO THE PROBLEM OF THE MAXIMUM WATER CONSUMPTION

In the design of some elements of the supply system, e.g. water-pipe network, it has been assumed that the characteristic duration of the maximum water consumption is 1 h. The analysis (2) of the measured results from continuous records of water consumption in five towns (each of them being able to cover the total water demand) shows that in the successive workdays the maximum water consumption $q^T_{\max}$, lasting for 5, 10, 20, 30 120, and 240 min, is significantly correlated with the maximum hourly consumption $q^{60}_{\max} (p > 0.999)$ (see fig 5.) Having this in mind, to determine approximately the influence of the time T of the maximum water consumption on its average intensity q, we employed quantiles of the maximum hourly consumption $q^{60}_{\max_p}$ and some empirical relations:

$$q^T_{\max} = A\,(q^{60}_{\max})^B, \tag{6}$$

$$q^T_{\max} = A\,(\hat{q}^{60}_{\max})^B, \tag{7}$$

$$q^T_{\max} = A_1 p T^{-n}. \tag{8}$$

Thus, the approximation formula becomes

$$q_{\max}(T, p) = A_2 p^{-n_1} T^{-n_2} \tag{9}$$

(for $T \in \{5.240\ \text{min}\}$), the graphical representation of which are exponential curves for various probabilities which refer to particular workday in either season (examples are given in figs. 6–9). From these curves it may be stated that on a given workday the maximum water consumption in a given time T will exceed the value $q_{\max}$ with a probability p. The effect of the time T on the maximum water consumption in the urban area studied is insignificant. The $q^5_{\max}$ to $q^{60}_{\max}$ ratio ranges approximately from 1.11 to 1.14.

Analysis of the 5-year-period (1976–1979) the agglomeration under study allows us to consider the exponents n_1 and n_2, incorporated in eq. (9), as being

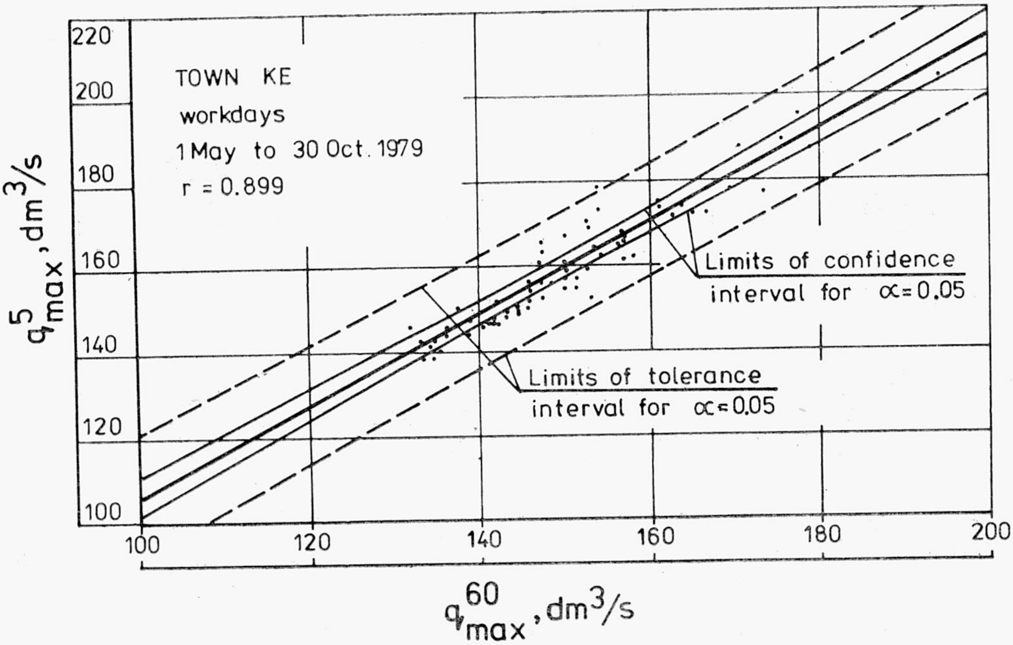

Fig. 5. Plot of $q^5_{max}(q^{60}_{max})$ for a town with the population of 41,300 inhabitants

Rys. 5. Zależność $q^5_{max}(q^{60}_{max})$ w dniach roboczych

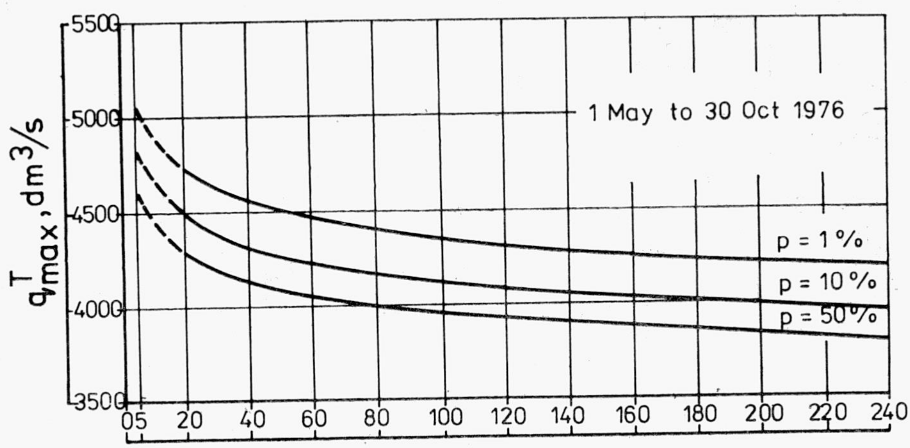

Fig. 6. Plots of $q_{max}(T, p)$ for workdays (May 1 to October 30, 1976)

Rys. 6. Wykres zależności $q_{max}(p, t)$ w dniach roboczych w okresie od 1.05 do 30.10.1976

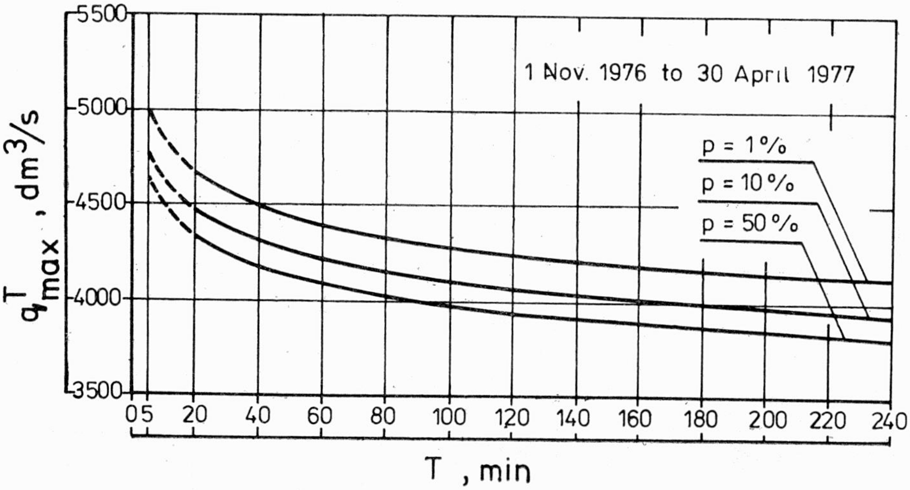

Fig. 7. Plots of $q_{max}(T, p)$ for workdays (November 1, 1976 to April 30,1977)
Rys. 7. Wykres zależności $q_{max}(T, p)$ w dniach roboczych w okresie od 1.11.1976 do 30.04. 1977

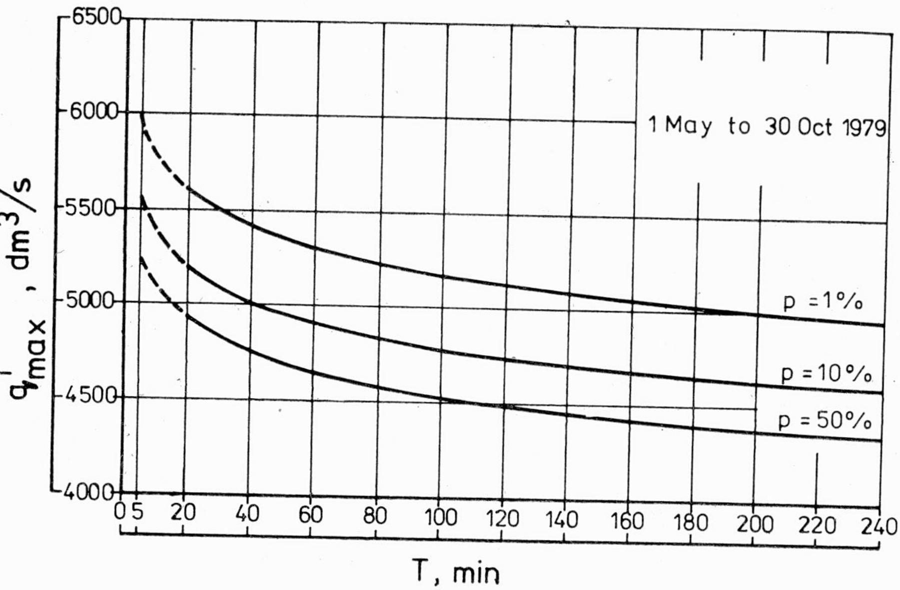

Fig. 8. Plots of $q_{max}(T, p)$ for workdays (May 1 to October 30, 1979)
Rys. 8. Wykres zależności $q_{max}(T, p)$ w dniach roboczych w okresie od 1.05 do 30.10.1979

constant. The coefficient A_2 varies with time being chiefly influenced by the number of inhabitants M and the average annual water consumption Q_m (water losses due to leakage in the household fittings in the period examined did not undergo substantial changes). Thus, eq. (9) will become

$$q_{max}(T, p) = (2.84 \times 10^{-3} M + 1.175 Q_m - 1.11) p^{-0.0223} T^{-0.048} \quad (10)$$

and, after having neglected the influence of T,

$$q_{max}^{60} = 0.82(2.04 \times 10^{-3} M + 1.175 Q_m - 1.11) p^{-0.0223}. \quad (11)$$

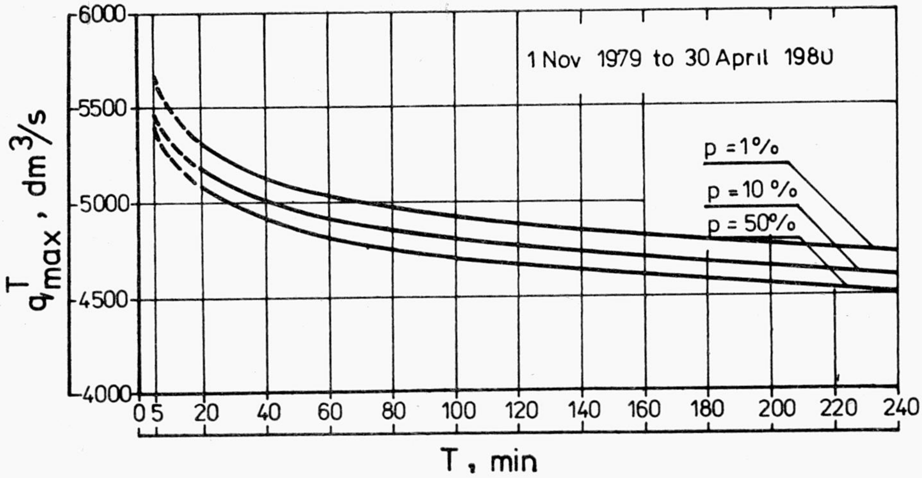

Fig. 9. Plots of $q_{max}(T, p)$ for workdays (November 1, 1979 to April 30, 1980)
Rys. 9. Wykres zależności $q_{max}(T, p)$ w dniach roboczych w okresie od 1.11. 1979 do 30.04. 1980

The errors of approximation of the quantiles $q_{max_{50}}^{60}$ by means of eq. (11), which result from mathematical model of water consumption (in which the errors involved in the evaluation of M and Q_m are neglected), are listed in tab. 4. These errors are relatively small, their mean values amount to 2.29% and the maximum value does not exceed 5% of the value determined from the empirical distribution function. Similar values of error were obtained for other quantiles of q_{max}^{60}.

Equations (10) or (11) can be used to forecast the maximum water demand in the agglomeration of interest, one year to five years ahead. It is essential, however, that the model of forecasting combine the estimate of maximum water demand for a given year with the annual consumption for one or more previous years. The term Q_m (incorporated in eqs. (10) and (11) to represent the average water demand throughout the year and treated as a predictor) can be evaluated, e.g. from the total unit coefficient of water demand. The errors of forecast may be greater than those of the approximation of the quantiles $q^{60}_{\max_p}$ due to the errors of the demographic forecast and to the errors of the forecast of the Q_m value. The numerical values of the parameters occurring in eqs. (10) and (11) may, moreover, be somewhat changed when water losses due to leakage in household fittings radically decrease.

Table 4

Approximation errors (eq. (11)) for quantiles $q^{60}_{\max_{50}}$

Zestawienie błędów aproksymacji kwantyli $q^{60}_{\max_{50}}$ za pomocą wzoru (11)

Season, year	Actual value of $\bar{q}^{60}_{\max}$ dm³/s	Value calculated using eq. (11) dm³/s	Approximation error %
I, 1976	3946	4135	4.74
II, 1976/77	4010	4182	4.27
I, 1977	4125	4188	1.53
II, 1977/78	4467	4518	1.14
I, 1978	4501	4523	0.49
I, 1979	4502	4660	3.51
II, 1979/80	4177	4735	0.37

4. CONCLUSIONS

The estimation of the maximum water demand is of a great importance in the design of some elements of the water supply system. To determine the degree to which the water demand for all purposes is satisfied, it is necessary to know the probability of the fact that a given value of water consumption is exceeded on a given day. This paper presents a probabilistic approach to the problem of the maximum water consumption $q_{\max}$ measured in the years 1976–1979 in an industrialized urban area which had, then, a population of 810,000 to 840,000 and was able to cover the water needs of all users. The investigations show that the probability density of the maximum water consumption can be described by the gamma-distribution (third-type Pearson distribution) and, after suitable transformation normalizing the random variable, by the Gaussian distribution. Considering the relatively low values of the coefficient of variation for the variable in question, it seemed advisable to employ estimations of the characteristic values of water consumption, based on small statistical samples. The effect of water consumption T on the average intensity of the maximum water consumption was found to be insignificant for $T \in \{5.240$ min$\}$. The values of the parameters $q_{\max}(T, p)$ or $q^{60}_{\max}(p)$ were for the five-year period of interest related to the number of inhabitants N and the annual average water consumption Q_m. The relative errors involved in the approximation of the quantiles $q^t_{\max_p}$ (which should be attributed to the structure of the mathematical model of water consumption) in general did not exceed 5%.

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PROBABILISTYCZNA CHARAKTERYSTYKA MAKSYMALNEGO ZUŻYCIA WODY W AGLOMERACJI MIEJSKO–PRZEMYSŁOWEJ

Sformułowano probabilistyczny model zużycia wody. Losowe właściwości zmiennej opisano rozkładem gamma, po zastosowaniu zaś przekształcenia normalizującego – rozkładem Gaussa. Zaprezentowano równanie wiążące natężenie maksymalnego zużycia wody z czasem jego trwania i prawdopodobieństwem przewyższenia.

PROBABILISTISCHE CHARAKTERISTIK DES MAXIMALEN WASSERVERBRAUCHS IM STÄDTISCH–INDUSTRIELLEN BALLUNGSGEBIET

Es wurde ein probabilistisches Wasserbrauchsmodell formuliert. Die stochastischen Eigenschaften der Variable wurden mit der Gammaverteilung beschrieben, und nach Einsatz der Normalisierungstransformation mit der Gaußverteilung. Es wurde die Gleichung vorgelegt, die die Intensität des maximalen Wasserverbrauchs mit seiner Zeitdauer sowie mit der Überhöhungswahrscheindlichkeit verbindet.

ВЕРОЯТНОСТНАЯ ХАРАКТЕРИСТИКА МАКСИМАЛЬНОГО РАСХОДА ВОДЫ В ПРОМЫШЛЕННО–ГОРОДСКОЙ АГЛОМЕРАЦИИ

Сформулирована вероятностная модель расхода воды. Случайные свойства переменной описаны гамма-распределением, после применения же нормализующего преобразования — гауссовым распределением. Представлено уравнение, связывающее интенсивность максимального расхода воды с его продолжительностью и вероятностью превышения.