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INVESTIGATION ON DISPERSION OF EFFLUENTS BY USING RADIOTRACER METHOD

The studies on evaluation of transverse mixing length of effluents discharged into natural streams have been reported. Proposed method base on measured values of transverse dispersion coefficient. Field investigations were carried out in the Vistula river at Warszawa area using radiotracer method. The procedure worked out can be used for prediction of mixing length for sewage outfall under design.

1. INTRODUCTION

Municipal and industrial wastes, when disposed to rivers and lakes, produce a danger to life in surface waters. Their continuously increasing quantities result in higher water pollution level and thus reduce water resources. The search for a method of wastes disposal, such that they could be quickly diluted in water of natural receiver, is the one of ways to solve the problem of water pollution.

Fast progress of mixing process permits dilution of wastes and reduction of toxic compounds concentration in relatively short time. This phenomenon is essential to the process of self-purification of waters. Low water pollution levels favour biological activity of the water region. Higher activity, in turn, gradually removes toxic compounds from water environment.

Radiotracer method is a useful tool for monitoring the effluent dispersion and description of mixing process taking place in natural streams. Its main advantage is that tracer detection remains unaffected by such factors as variations in chemical composition of labelled medium and the presence of deposits. Necessary condition for tracer method to be used is that the tracer does not pass to another phase different from that being labelled. The use of radiotracer methods in investigations of effluent transport processes in natural streams makes it possible to determine the factors characterizing mixing rates providing a base for studies on their dilution processes.

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2. INVESTIGATION OF EFFLUENT TRANSPORT

2.1. THEORETICAL BASIS

The wastes which have been discharged into rivers mix with ambient water in longitudinal, transverse, and vertical directions. Since the depth of natural stream is small in comparison to its width and length necessary to reach a complete mixing over its cross-section area, there occurs homogenization process along the depth in a period several times shorter than that required for a complete transverse mixing.

For this reason vertical mixing has been neglected. The factor controlling mixing intensity in the phase of process preceding a complete homogenization in cross-section areas is transverse dispersion [5].

Longitudinal dispersion should be considered in case when the distribution of instantaneously injected tracer is analysed. Therefore two-dimensional model was used to study the transport processes. General form of transport equation, assuming turbulent flow, describes mass balance of substance being transported [7]:

$$\frac{\partial C}{\partial t} = \nabla(D\nabla C) - V\nabla C \tag{1}$$

where:

C — concentration,
t — time,
D — dispersion coefficient,
V — velocity,
∇ — Laplace operator.

The solution of this equation in rectangular coordinates for infinitely wide stream is known in form [1]:

$$C = \frac{A}{4\pi Ht(D_x D_y)^{1/2}} \exp\left\{-\left[\frac{(x-ut)^2}{4D_x t} + \frac{y^2}{4D_y t}\right]\right\} \tag{2}$$

where:

A — total quantity of instantaneously injected substance,
H — depth of a stream,
D_x — longitudinal dispersion coefficient,
D_y — transverse dispersion coefficient,
O_x, O_y — coordinates of length and width,
u — linear flow velocity.

This equation describes initial phase of dispersion where the cloud of transported substance has not reached stream banks yet. This equation has been used to determine the value of dispersion coefficient.

Further analysis of a transport process requires taking into account the fact that particles of substance being transported are bounded from the banks, as well as the location of injection point in cross-section. Then the equation (1) has the following form:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D_x\frac{\partial^2 C}{\partial x^2}+D_y\frac{\partial^2 C}{\partial y^2}+A\delta(x-x^*)\delta(y-y^*)\delta(t-t^*), \tag{3}$$

$$x\in(-\infty, +\infty), \tag{4}$$

$$y\in[0, B] \tag{5}$$

with initial condition:

$$t=0; \quad C=0 \tag{6}$$

and boundary conditions

$$x\to\pm\infty; \quad C=0, \tag{7}$$

$$y=0; \quad \frac{\partial C}{\partial y}=0, \tag{8}$$

$$y=B; \quad \frac{\partial C}{\partial y}=0 \tag{9}$$

where:

B — width of the stream,
x^*, y^*, t^* — coordinates of location and time of injection,
δ — Dirac function,
A — amount of injected substance per unit depth.

The equation (3) solved by means of finite Fourier transform of x and y variables yields an ordinary differential equation related to time [6], its solution and inverse transformation give finally the solution in form $C=f(x, y, t)$. The solution runs as follows

$$C(x,y,t)=\frac{2A\exp\left[-\frac{(x-x^*-ut)^2}{4D_xt}\right]}{B(4\pi D_xt)^{1/2}}\left(1+2\sum_{N=1}^{\infty}\exp(-D_y\mu_N^2t)\cos\mu_Ny\cos\mu_Ny^*\right) \tag{10}$$

where $\mu_N=N\pi/B$.

The equation (3) is valid for initial condition (6) and boundary conditions (7)–(9) and describes concentration distribution of substance instantaneously injected into the flowing stream at the point having coordinates x^*, y^*.

The equation (10) can be used to determine the values of dispersion coefficients on the basis of measured distribution of tracer concentration.

In analysis of the wastes transport it is essential to use the equation describing transverse distribution of effluent concentration. Such a formula can be obtained by solving equation (3) for the case of continuous release or by integration of equation (10) with respect to x variable.

Eventually (10) yields

$$\frac{C(y,t)}{\overline{C}} = 1+2\sum_{n=1}^{\infty} \exp(-D_y \mu_N^2 t)\cos\mu_N y \cos\mu_N y^* \tag{11}$$

where:

$$\overline{C} = \frac{Q}{uBH}, \tag{12}$$

Q — discharge rate,
H, B — mean depth and width of stream,
$\overline{C}$ — mean concentration in cross-section area expressed as:

$$\overline{C} = \frac{1}{B}\int_0^B C(y,t)\,dy. \tag{13}$$

The equation (11) describes the concentration distribution along a stream width at time t, after release at point y^*. To analyse the effect of location of point of discharge and transport time on homogenization intensity it is convenient to use the mixing degree [9]:

$$M = 1-\frac{1}{n}\left[\sum_{i=0}^{n}\left(1-\frac{C_i}{\overline{C}}\right)^2\right]^{1/2}; \tag{14}$$

$M = 1$ means complete homogenization in cross-section area of a stream.

The length of complete mixing is given by formula:

$$L_m = \bar{u} t_m \tag{15}$$

where $\bar{u}$ is a mean flow linear velocity.

Taking into account y^* coordinate of discharge point and knowing the value of transverse dispersion coefficient and mean water flow velocity, the mixing degree at given distance from discharge point can be determined from formulae (11), (14), and (15).

Figure 1 shows mixing degree versus dimensionless transport time $D_y t/B^2$ for various dimensionless discharge locations y^*/B. $y/B = 0$ represents discharge location at the bank, while $y/B = 0.5$ in the centre of a stream. It can be seen from fig. 1, that mixing process rate strongly depends on discharge point location on y/B axis. The most favourable conditions in respect of mixing process intensity occur when discharge point is located in the centre of a stream. Moving the discharge point aside strongly decreases homogenization rate.

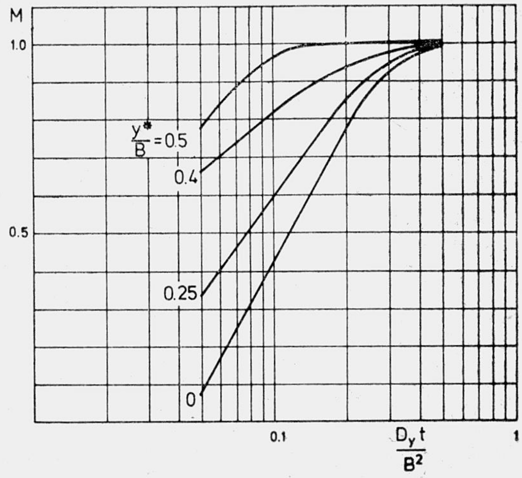

Fig. 1. Mixing degree versus nondimensional time $D_y t/B^2$ for different locations of discharge point y^*/B

Rys. 1. Stopień mieszania w zależności od czasu $D_y t/B^2$ dla różnych położeń punktu zrzutu y^*/B

2.2. EXPERIMENTAL WORK

The method described needs some confirmation by field experiments. Therefore a series of investigations on tracer distribution have been carried out in the Vistula river near Warsaw. As a tracer the ^{82}Br in a form of KBr water solution has been used [2, 4, 10].

The measurement of dispersion coefficients is based on instantaneous injection of a tracer at a given point and recording of tracer concentration distribution along transverse direction and over the time at certain distances from the injection point. Altogether a series of 5 tracer injections were performed on 24 and 30 of April, 1980, and also on 8, 10 and 11 of October, 1980, using about 11.1 GBq (300 mCi) of ^{82}Br per 1 injection.

2.2.1. MEASURING PROCEDURE

The tracer was injected at 4 points displaced along the river depth [9]. This method ensured quick vertical mixing. Downstream the injection point a number of measurement sections were established at the given distances. Each section was marked by 2 buoys located along the line perpendicular to the river stream [8]. Buoy geodesic coordinates were determined by means of triangulation.

The measuring apparatus consisted of water tight scintillation probe (NaJ/Tl) connected with the RZP-10 field radiometer. Probe measurements were recorded as a function of time by means of the Goertz RE 501 "MINIGOR" recorder. Measuring sets were installed on 2 speedboats.

Distribution of tracer activity in time at predetermined measuring section was recorded from the boat anchored at the point of maximum activity in cross-section of the stream. To achieve this the boat was guided by the other boat cruising at that time further upstream the measuring section. Activity distribution along river width was determined by means of a boat moving along measuring section and maintaining constant speed, when the maximum activity of the tracer flows by. The transverse distribution of tracer concentration was determined on the basis of recorded activity distribution curves, knowledge of the distance between bouys marking specific section, and the time which the measuring point required to cover that distance. In all two lengths of the Vistula river have been done. In April 1980, the section between 519 and 521 km of the course of the river and in October 1980, the section between 512 and 515 km.

3. RESULTS

Because of relatively short distances between injection points and measuring profiles the values of dispersion coefficient were determined from the equation (1). Variances of certain distributions of tracer concentration are the representation of the longitudinal and transverse dispersion processes. To determine the values of dispersion coefficients a Levenspiel–Smith method [3] modified to conditions of transport in open channel flow was used. The use of this method eliminates the error caused by not immediate determination of tracer concentration distribution along O_x and O_y axes. As a result formulae are obtained which connect dimensionless variance σ_l^2 and σ_d^2 of tracer concentration profiles along O_x and O_y axes, with values β and γ of dimensionless longitudinal and transverse dispersion coefficients:

$$\sigma_l^2 = \frac{4\beta K_1\left(\frac{1}{2\beta}\right)+K_0\left(\frac{1}{2\beta}\right)}{K_0\left(\frac{1}{2\beta}\right)} - \left[\frac{K_1\left(\frac{1}{2\beta}\right)}{K_0\left(\frac{1}{2\beta}\right)}\right]^2, \tag{16}$$

$$\sigma_d^2 = \frac{\frac{4\beta\gamma}{a\beta+\gamma} K_1\left(\frac{a\beta+\gamma}{2\beta\gamma}\right)+K_0\left(\frac{a\beta+\gamma}{2\beta\gamma}\right)}{K_0\left(\frac{a\beta+\gamma}{2\beta\gamma}\right)} - \left[\frac{K_1\left(\frac{a\beta+\gamma}{2\beta\gamma}\right)}{K_0\left(\frac{a\beta+\gamma}{2\beta\gamma}\right)}\right]^2 \tag{17}$$

where:

$$\beta = \frac{D_x}{uL}, \tag{18}$$

$$\gamma = \frac{D_y}{uL}, \tag{19}$$

$$\sigma_l^2 = \frac{\sigma_x^2}{L^2} = \frac{u^2\sigma_t^2}{L^2}, \tag{20}$$

$$\sigma_d^2 = \frac{\sigma_y^2}{L^2}, \tag{21}$$

$$K_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} \exp(-x)\left\{1 + \sum_{n=1}^{\infty} \frac{\prod_{K=1}^{n} [4x^2-(2K-1)^2]}{n!(8x)^n}\right\}, \tag{22}$$

$n = 1, 2, 3, \ldots,$

$K = 1, 2, 3, \ldots,$

L — distance between injection point and measuring section.

Due to longer distances between measuring profiles, during October 1980 measurements, it was possible to perform multiple recordings of transverse distribution of tracer concentration. This was done at two terminal measuring profiles. Taking into account the time correlation of certain tracer distribution curves and mean linear flow velocity, determined from the time between the injection and the maximum activity in the given measuring profile, it became possible to plot the tracer cloud in the form of isoconcentration curves.

Figures 2–5 present examples of tracer isoconcentration curves for 2 injections along the river width coordinate at various locations of injection point. Based on the isoconcentration curves the distribution of tracer concentration along O_x and O_y axes were plotted. The knowledge of these profiles along O_x and O_y axes made it possible to determine the values of dispersion coefficients directly from the equations:

$$D_x = \frac{\sigma_x^2}{2t}, \tag{23}$$

$$D_y = \frac{\sigma_y^2}{2t}. \tag{24}$$

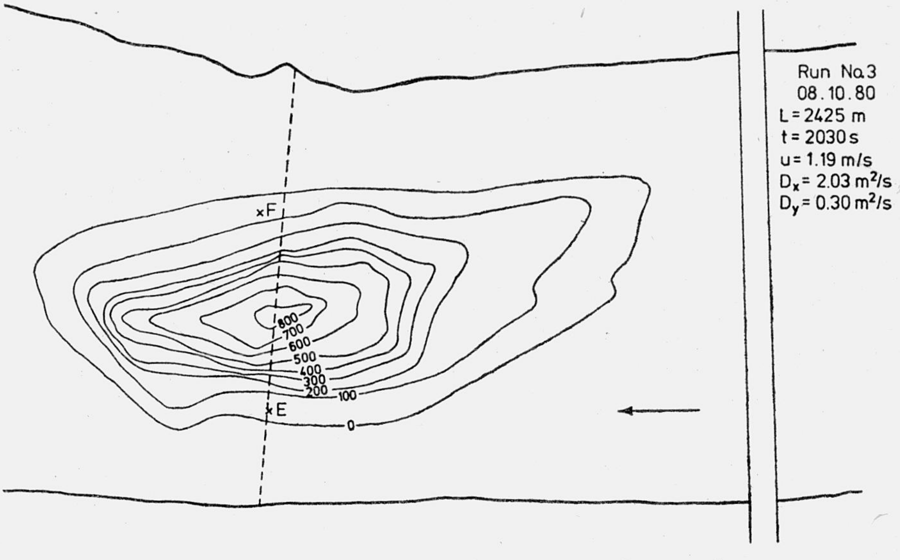

Fig. 2. An example of isoconcentration curves of tracer plume

Rys. 2. Przykład krzywych wskaźnika izotopowego o jednakowym stężeniu

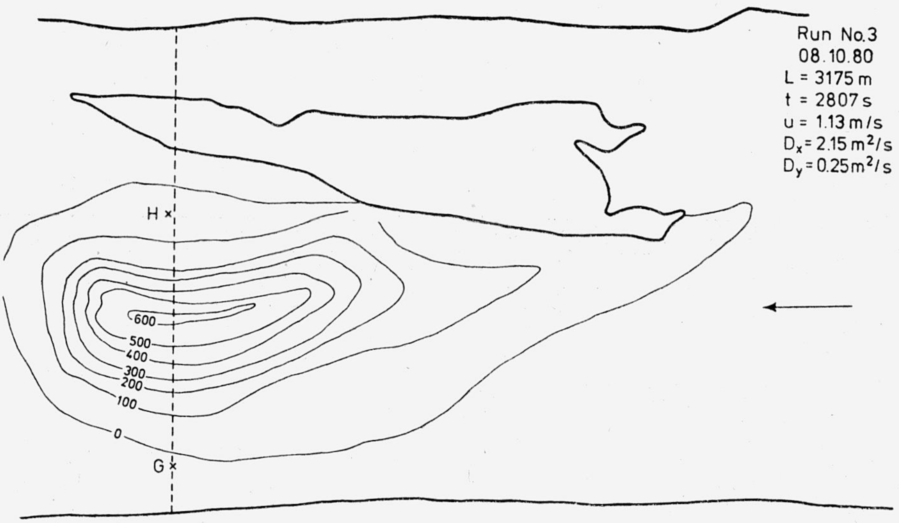

Fig. 3. An example of isoconcentration curves of tracer plume
Rys. 3. Przykład krzywych wskaźnika izotopowego o jednakowym stężeniu

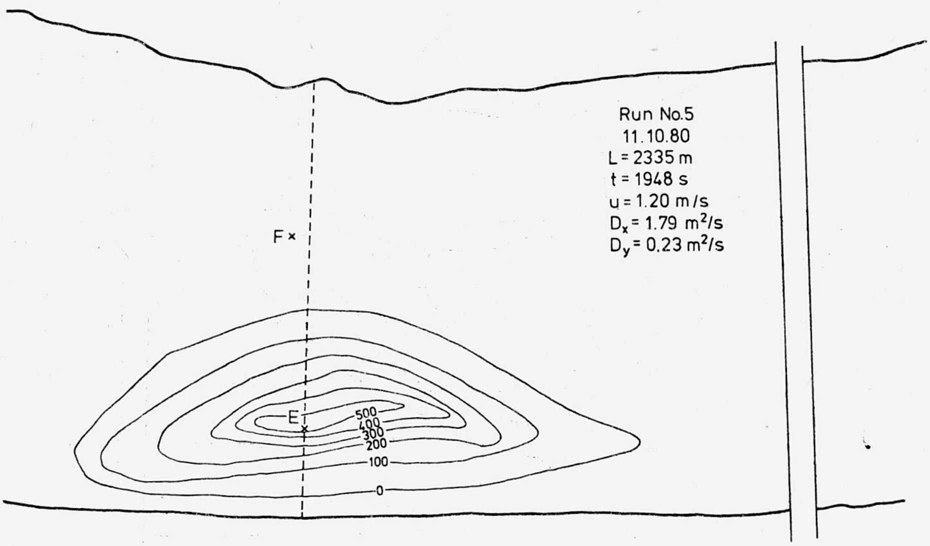

Fig. 4. An example of isoconcentration curves of tracer plume
Rys. 4. Przykład krzywych wskaźnika izotopowego o jednakowym stężeniu

Fig. 5. An example of isoconcentration curves of tracer plume
Rys. 5. Przykład krzywych wskaźnika izotopowego o jednakowym stężeniu

The results obtained by both the methods with the detailed data on injection point coordinate and distances of measuring profiles are shown in tab. 1.

4. VERIFICATION

To check whether the described model has been assumed correctly, mixing degree of continuously released dye tracer was measured. The concentration distribution was determined by sampling at points located along the river width, at a large distance (over 20000 m) from the release point, and by photometric measurements of dye concentration. Two experiments were performed. Mixing degree was determined from the measured tracer concentration in samples by using equations (19) and (20). The results are given in tab. 2. Figure 6 shows the diagram of mixing degree versus dimensionless time $D_y t/B^2$ and the respective measured values.

5. CONCLUSIONS

The suggested mathematical model of dispersion of effluent in natural streams does not take into account the shape of river-bed and is valid for flow in rectangular channels. The equation (2) which was used to determine dispersion coefficients disregards the effect of bottom and bank shape. Neither model takes account of the existence of dead spaces in

Table 1

Data obtained from field investigation on the Vistula river at Warsaw area
Wyniki badań polowych Wisły w rejonie Warszawy

Date	Injection point co-ordinates: km of river course	Injection point co-ordinates: Distance from the bank	Distance from injection point	Mean flow time	Mean flow velocity	Measured values of longitudinal dispersion coefficient: By Levenspiel-Smith method	Measured values of longitudinal dispersion coefficient: Directly from equation	Measured values of transverse dispersion coefficient: By Levenspiel-Smith method	Measured values of transverse dispersion coefficient: Directly from equation
		y^*	L	t	u	D_x		D_y	
	km	m	m	s	m/s	m²/s	m²/s	m²/s	m²/s
			560	479	1.17	2.12		0.24	
24 IV 80	519.62	62	980	860	1.14	1.86		0.27	
			1360	1204	1.13	2.07		0.31	
			2240	2000	1.12	2.05		0.22	
			580	487	1.19	1.74		0.32	
			960	800	1.21	1.81		0.26	
30 IV 80	519.80	112	1360	1192	1.14	1.63		0.24	
			2320	1982	1.17	1.57		0.28	
8 X 80	511.86	125	2425	2030	1.19	2.03	1.86	0.30	0.28
			3175	2807	1.13	2.15	2.07	0.25	0.29
			420	355	1.18	1.78		0.26	
10 X 80	511.88	45	2415	2127	1.13	1.99	2.07	0.13	0.18
			3165	2857	1.11	2.75	2.61	0.21	0.26
11 X 80	511.95	55	2335	1948	1.20	1.79	2.11	0.23	0.31
			3085	2530	1.22	2.48	2.26	0.28	0.32

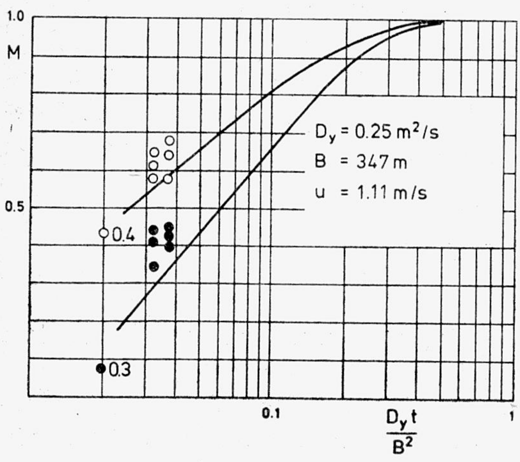

Fig. 6. Measured values of mixing degree and curves $M = f(D_y t/B^2)$ calculated for the location of discharge points $y^*/B = 0.3$ and $y^*/B = 0.4$

Rys. 6. Zmierzone wartości stopnia mieszania i krzywe $M = f(D_y/B^2)$ obliczone dla zrzutu w punktach $y^*/B = 0,3$ i $y^*/B = 0,4$

Table 2

The measured values of mixing degree of the dye and the respective predicted values
Zmierzone wartości stopnia mieszania barwnika i odpowiednie wartości przewidywane

Nondimensional release point coordinate	Distance from release point	Nondimensional time	Mixing degree		
			Measured		Predicted
y/B	L	D_yt/B^2	M	M	M
0.3	20800	0.0320	0.35 0.42 0.44	0.40	0.286
	23600	0.0364	0.44 0.42 0.45	0.42	0.338
0.4	20800	0.0320	0.58 0.61 0.64	0.61	0.561
	23600	0.0364	0.58 0.64 0.67	0.63	0.592

a natural stream and mass transfer processes occurring between those areas and a bend loss. Under such conditions the measured dispersion coefficients can be overestimated. The measured values of dye mixing degree are near the values obtained from model equations (11)–(14). In conslusion, it may be assumed that the existing irregularities of river-bed have little effect on the error, when the values of dispersion coefficients are measured. These irregularities promote mixing processes, which can be confirmed by the measured values higher than the estimated ones. The average positive deviation from the estimated values is 0.067 on the scale of M from 0 to 1. The results obtained lead to the conclusion that method presented can be useful for prediction of mixing degree basing on the measured dispersion coefficients. Due to inconsiderable discrepancies between the results, the method of measurement of dispersion coefficients based on modified Levenspiel–Smith method of variance analysis is equivalent to their direct determination from equation (2), based on variance of tracer concentration distribution along O_x and O_y axes.

Tracer methods are a useful tool in investigations of the transport phenomena in natural streams. They provide quick and relatively accurate estimation of mixing parameters. The methods presented make it possible to predict mixing degree for natural stream, taking into account location of discharge point in cross-section of the stream.

They can be used when the outfall location of industrial and municipal wastes is to be selected. The authors are of the opinion that further work should include the investigations of dispersion of effluent in close vicinity of discharge point and the effect of design features of sewage outfall initial dilution.

It seems advisable to investigate the effect of multi-port diffusers which considerably improve the rate of dillution process directly after their introduction in natural water receiver and provides further reduction of water pollution.

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BADANIE DYSPERSJI ŚCIEKÓW METODĄ ZNACZNIKA PROMIENIOTWÓRCZEGO

Przedstawiono prace nad wyznaczeniem odcinków całkowitego wymieszania poprzecznego zanieczyszczeń odprowadzonych do naturalnych cieków wodnych. Podstawą proponowanej metody są pomierzone wartości współczynników dyspersji poprzecznej. Prace doświadczalne wykonano na Wiśle w rejonie Warszawy metodą znaczników promieniotwórczych. Opracowana procedura może być zastosowana w prognozowaniu odcinków całkowitego wymieszania dla projektowanych zrzutów ścieków.

DIE UNTERSUCHUNG DER ABWASSERVERTEILUNG MITTELS MARKIERUNGSISOTOPEN

Es werden Versuche dargestellt, die zur Bestimmung einer Abschnittslänge die zur vollen Vermischung der in ein fliessendes Gewässer eingeführten Verunreinigungen führt. Die Grundlage der vorgeschlagenen Methode bilden Meßwerte der Vermischungskoeffiziente. Die Versuche wurden an der Weichsel in der Nähe von Warschau mit Hilfe von radioaktiven Markierungsisotopen durchgeführt. Die Methode kann man bei der Projektierung von Abwasserableitungen und zur Berechnung von Abschnittslängen der vollen Vermischung gut gebrauchen.

ИССЛЕДОВАНИЕ ДИСПЕРСИИ СТОЧНЫХ ВОД МЕТОДОМ РАДИОАКТИВНОГО СЛЕДОУКАЗАТЕЛЯ

Представлены результаты работ, касающихся определения участков полного поперечного перемешивания примесей, отводимых в естественные водные потоки. Основой предлагаемого метода являются измеренные значения коэффициентов поперечной дисперсии. Эксперименты проводились на реке Висле около Варшавы методом радиоактивных следоуказателей. Разработанная процедура может применяться в прогнозировании участков полного перемешивания для проектированных сбросов сточных вод.