Environment Protection Engineering

Vol. 6 1980 No. 2

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STABILITY OF MULTI-LAYER SEDIMENTATION TANKS

Hydraulic efficiency of the sedimentation tanks was discussed. It was described by the theoretical method, applying the mathematical model verified on the physical one. The amount of water flowing through the layer of the sedimentation tank was calculated. It affects the efficiency of the sedimentation in the upper and lower layers.

NOTATIONS

b — width of the settling tank,
b_0, b_z — widths of the open and closed chambers, respectively,
C_n — concentration of the labelling substance in the outflow,
g — acceleration due to gravity,
h — depth of the settling tank or chamber,
h_g, h_d — depths of the open (upper) and closed (lower) chambers, respectively,
K — constant,
L — length of the settling tank,
Δp — hydraulic losses during flow through the settling tank,
$\Delta p_0, \Delta p_z$ — hydraulic losses during flow through the open and closed chambers, respectively,
Q — rate of flow through the settling tank,
Q_g, Q_d — rates of flow through the upper and lower chambers, respectively,
Re — Reynolds number,
R_h — hydraulic radius,
t — time of flow through the model settling tank,
t_c — time for achieving the maximum concentration of the labelling substance in the outflow,
t_d — time of label dosing,
t_n — time of label appearance in the outflow,
u, V_p — velocities of flow through a horizontal-flow sedimentation tank,
V_d, V_g — velocities of flow through the lower and upper chambers, respectively,
ν — kinematic viscosity of the liquid,
ϱ — density of the liquid,
μ — kinematic viscosity.

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1. INTRODUCTION

Investigations on the design of horizontal-flow sedimentation tanks have been reported in the literature a number of times since the early 1900s. The theory of sedimentation under ideal conditions was first described in 1904 by ALLAN HAZEN who suggested that the tank be partitioned by parallel horizontal plates [3]. In this way, the water flow would be distributed into several streams, improving the effectiveness of the sedimentation process. HAZEN's theory was developed by THOMAS R. CAMP in 1947 [1]. He found that the effectiveness of sedimentation in an ideal depth tank (fig. 1a) can be doubled if a horizontal

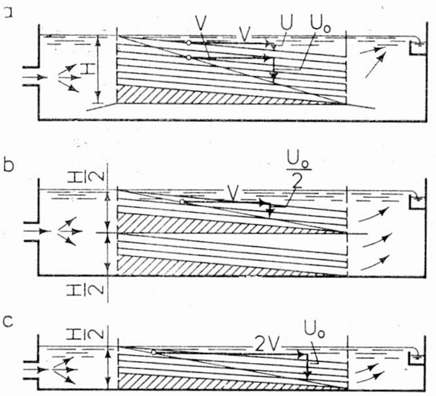

Fig. 1. Model of sedimentation conducted in an ideal settling tank (after CAMP, 1947)

Rys. 1. Model sedymentacji prowadzonej w idealnym osadniku (według CAMPA, 1947)

plate is placed in the middle of the vertical wall (fig. 1b). Such a halving of the horizontal-flow sedimentation tank depth and the twofold increase in hydraulic load yielded the same results as those achieved in a deep sedimentation tank (fig. 1a). CAMP neglected, however, both the difference in the hydraulic conditions occurring on the particular levels of the tank (fig. 1b) and the difference in sedimentation efficiency.

The purpose of present paper is to determine the hydraulic properties of multi-layer settling tanks on the basis of theoretical studies and model-scale experiments.

2. MATHEMATICAL MODEL FOR THE HYDRAULICS OF MULTI-LAYER HORIZONTAL-FLOW SEDIMENTATION TANKS

The cross-section of the lower chamber in the double-layer settling tank is rectangular. The laminar flow of water (as a Newtonian fluid) in this rectangle can be described in terms of Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{K}{\nu} \tag{1}$$

if the following boundary conditions are satisfied: $u = 0$ for $x = \pm 1/2\, b$; $y = \pm 1/2\, h_a$.

The symbols h and b, as well as the symmetry condition for water flow in the chamber, are illustrated in fig. 2 (h_d, h_g, F_d and F_g are the limits of integration).

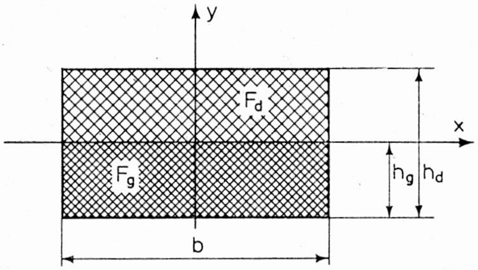

Fig. 2. Limits of integration in a cross-section of the settling tank
Rys. 2. Granice całkowania przekroju poprzecznego osadnika

The solution to eq. (1), based on the condition of symmetry, is included in eq. (2):

$$u = \frac{4Kh_d^2}{\nu\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \left[1 - \frac{\cosh \dfrac{(2n+1)\pi x}{h_d}}{\cosh \dfrac{(2n+1)\pi b}{2h_d}} \right] \cos \frac{(2n+1)\pi y}{h_d}, \tag{2}$$

and the integral defining the efficiency (2) of the chamber

$$Q = \iint_F u\,dx\,dy \tag{3}$$

is described by eq. (4):

$$Q_d = \frac{Kh_d^3 b}{12\nu} \left[1 - \frac{192}{\pi^5} \frac{h_d}{b} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^5} \tanh \frac{(2n+1)\pi b}{2h_d} \right]. \tag{4}$$

The relationship defined by eq. (4) can also be written as

$$Q_d = \frac{Kh_d^3 b}{12\nu} f\left(\frac{h_d}{b}\right) \tag{5}$$

where

$$f\left(\frac{h_d}{b}\right) = 1 - \frac{192}{\pi^5} \frac{h}{b} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^5} \tanh \frac{(2n+1)\pi b}{2h}. \tag{6}$$

In eq. (5) the value of the constant K becomes

$$K = \frac{\Delta P}{\varrho L}. \tag{7}$$

Hence, eq. (5) can be written either as

$$Q_d = \frac{\Delta p h_d^3 b}{12\nu\varrho L} f\left(\frac{h_d}{b}\right) \tag{8}$$

or as

$$\Delta p = \frac{12Q\nu\varrho L}{h_d^3 b f\left(\frac{h_d}{b}\right)}. \tag{9}$$

Eqs. (8) and (9) describe the hydraulic conditions of the laminar flow in the lower chamber of a double-layer tank of length L.

Owing to the symmetry of flow, the solution to the equation for the open chamber of the settling tank is the same as that to the equation for the half-cross-section of the closed chamber.

The flow rate in such a chamber (determined by integration of eq. (1) with the equation to the half-cross-section of the closed chamber) is described by following relations:

$$Q_g = \frac{Kh_g^3 b}{3\nu}\left[1 - \frac{192}{\pi^5}\frac{2h_g}{b}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^5}\tanh\frac{\pi b}{4h_g}\right] \tag{10}$$

and

$$Q_g = \frac{\Delta p_g h_g^3 b}{3\nu\varrho L} f\left(\frac{h_g}{b}\right). \tag{11}$$

The influence of b/h on $f(h/b)$ for the lower chamber of a double-layer settling tank is shown in fig. 3. In the figure, when $0 < b/h < 3$, the values of $f(h/b)$ increases rapidly up

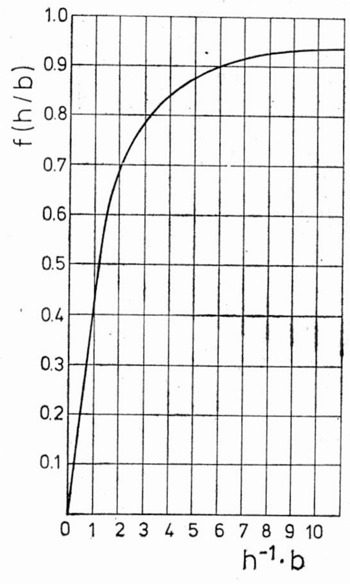

Fig. 3. Correlation between function $f(h/b)$ and width-to-length ratio
Rys. 3. Korelacja między funkcją $f(h/b)$ a stosunkiem szerokości do długości

to 0.79. Once this value is obtained, the further increase of the b/h ratio has little effect on the $f(h/b)$ value (which increases rather slowly). This leads to the condition

$$\lim_{b/h\to\infty} f(h/b) = 1,$$

tending asymptotically to the equality

$$f(h/b) = 1.$$

The relations between flow rates in the upper and lower chambers and hydraulic losses are shown in fig. 4 and table 1. It is evident from fig. 4 and table 1 that the intesity of water flow in the upper (open) chamber of the settling tank is considerably higher than that in

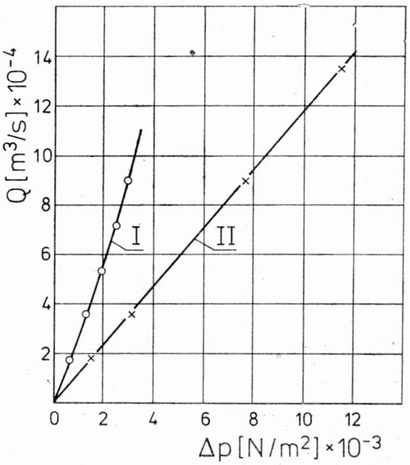

Fig. 4. Hydraulic losses in the upper and lower chambers:

I — upper chamber, II — lower chamber

Rys. 4. Straty hydrauliczne w górnej i dolnej komorze

I — komora górna, II — komora dolna

Table 1

Hydraulic parameters of a double-layer horizontal-flow settling tank

Hydrauliczne parametry dwuwarstwowego osadnika z przepływem poziomym

Flow velocity $\times 10^{-3}$ m/s	Flow rate $\times 10^{-4}$ m^3/s	Hydraulic loss $\times 10^{-3}$ N/m^2
	Lower chamber	
1.0	1.8	1.50
2.0	3.6	3.10
3.0	5.4	4.60
5.0	9.0	7.70
7.5	13.5	11.50
	Upper chamber	
1.0	1.8	0.63
2.0	3.6	1.30
3.0	5.4	1.90
4.0	7.2	2.50
5.0	9.0	3.00

the lower (closed) chamber hydraulic losses being identical. It follows that the velocity of the water flow and the hydraulic load in the upper chamber are, correspondingly, higher than in the lower chamber and the times of the flow are proportionally shorter. These conclusions can be justified qualitatively by the difference in the Reynolds numbers

$$\mathrm{Re}_d = \frac{v_p 4R_h}{\nu} = \frac{v2bh}{\nu(b+h)}$$

and

$$\mathrm{Re}_g = \frac{v_p 4R_h}{\nu} = \frac{v_p 4bh}{\nu(2h+b)}$$

for the lower and upper chambers, respectively. The calculated values of Re_d are lower than those of Re_g.

3. INVESTIGATION METHODS AND PROCEDURES

The studies on the hydraulics of the flow were carried out on a model scale in a steel doubl.-layer settling tank with parallel flow. The dimensions of both chambers being $h = 0.3$ m, $b = 0.6$ m and $L = 6.0$ m, ensured a high stability of flow in the conventional horizontal-flow sedimentation tanks.

The two chambers were fed by troughs and drained through 46 notches (of a diameter of 1.0×10^{-3} m) into an outlet trough. The schematic diagram of the model is shown in fig. 5.

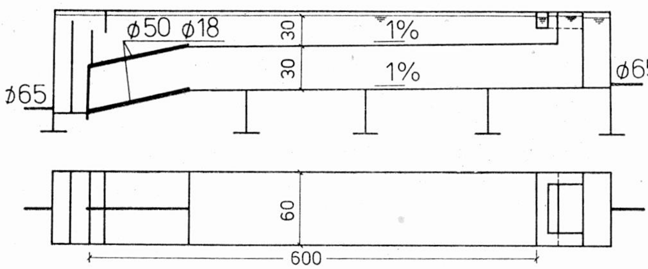

Fig. 5. Schematic diagram of a model double-layer tank
Rys. 5. Schemat modelowego dwuwarstwowego osadnika

The flow rates calculated from the theoretical curves

$$Q_g = \frac{\Delta p_g h_g^3 b f(h_g/b)}{3\mu L} \tag{12}$$

for the upper (open) chamber, and

$$Q_d = \frac{\Delta p_d h_d^3 b f(h_d/b)}{12\mu L} \tag{13}$$

for the lower (closed) chamber.

The flow being parallel, the hydraulic losses are equal. Thus we have

$$\frac{Q_g}{Q_d} = \frac{12f(h_g/b)}{3f(h_d/b)} = \frac{4\times 0.422}{0.687} \tag{14}$$

and

$$Q_g : Q_d = 2.457. \tag{15}$$

The distribution of efficiencies in the chambers of the model settling tank is shown in table 2 (for $\mu = 0.00132$ kg/m·s and a temperature of 10°C).

Table 2

Distribution of efficiencies
Rozkład efektywności

Total V_p	Hydraulic loss	Total Q	Q of the settling tank: Upper chamber	Q of the settling tank: Lower chamber	V_p of the settling tank: Upper chamber	V_p of the settling tank: Lower chamber
$\times 10^{-3}$ m/s	$\times 10^{-3}$ N/m^2	$\times 10^{-3}$ m^3/s	$\times 10^{-3}$ m^3/s		$\times 10^{-3}$ m/s	
0.69	0.7	0.25	0.18	0.07	1.0	0.39
1.39	1.3	0.50	0.36	0.14	2.0	0.78
2.78	2.5	1.00	0.71	0.29	3.0	1.56

As can be seen from eq. (15) and table 2, there is a considerable difference between the flow in the upper chamber and that in the lower one. The flow rate in the upper chamber is 2.457 times higher than that in the lower one. This indicates a difference existing in the hydraulic conditions which is of considerable importance to the sedimentation process.

The water flow velocities (in the range of laminar flow), employed in the model investigations, were:

$$V_{d\,\max} \leqslant \frac{\mathrm{Re}_{kr}\nu}{4R_h} = \frac{2\,300\times 1.306\times 10^{-6}}{4\times 0.1} = 0.0075 \text{ m/s}$$

for the lower chamber, and

$$V_{g\,\max} \leqslant \frac{\mathrm{Re}_{kr}\nu}{4R_h} = \frac{2\,300\times 1.306\times 10^{-6}}{4\times 0.15} = 0.005 \text{ m/s}$$

for the upper chamber.

Those flow velocities were determined in the same way as for the conventional horizontal-flow sedimentation tank.

The model investigations reported were to verify experimentally theoretical hypotheses derived from the model of hydraulics for settling tanks. To state the differences occurring in the hydraulic conditions of the upper and the lower chambers, the flow in a model double-layer settling tank was measured by applying the labelling method in which fluorescein was dosed at the inflow. The dosing time not exceeding 15s satisfied the condition $t_d \ll t_c - t_0$. The concentration of the labelling substance was measured at the outflow by the colorimetric method. Samples were collected at 1-5 minute intervals, depending on the intensity of the label colour. The investigations were carried out with tap water.

4. EXPERIMENTS ON THE HYDRAULICS OF WATER FLOW THROUGH THE MODEL SETTLING TANK

The flow curves $C_n = f(t_n)$ are shown in figs. 6-7. Comparison of the curves $C_n = f(t_n)$ for both chambers indicates that the flow wave in the upper (open) chamber is shifted with respect to the flow wave in the lower chamber. The flow wave delay (modal values) varying from 2 to 29 minutes can be attributed to the higher efficiency of the upper chamber. The

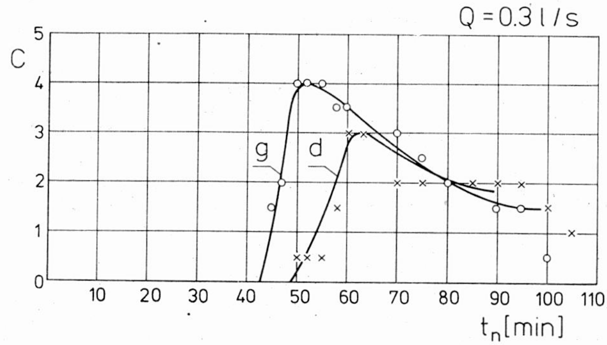

Fig. 6. Flow curves $C_n = f(t_n)$ for both chambers of a model settling tank:
g — upper chamber, d — lower chamber

Rys. 6. Krzywe przepływu $C_n = f(t_n)$ dla obu komór modelowego osadnika:
g — komora górna, d — komora dolna

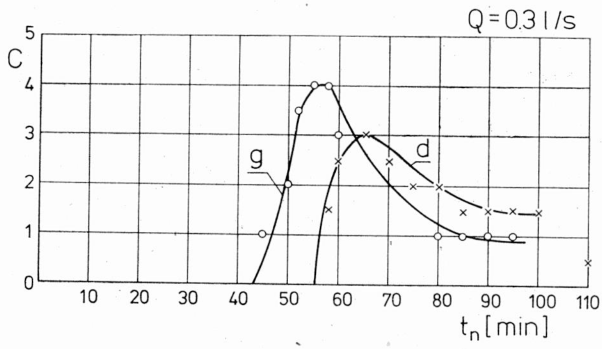

Fig. 7. Flow curves $C_n = f(t_n)$ for both chambers of a model settling tank:
g — upper chamber, d — lower chamber

Rys. 7. Krzywe przepływu $C_n = f(t_n)$ dla obu komór modelowego osadnika:
g — komora górna, d — komora dolna

time which elapsed between the introduction of the tracer and its appearance at the outlet ranged between 20 and 30 minutes in the upper chamber and from 40 to 60 minutes in the lower chamber.

The experiments carried out for the model tank have shown that the upper chamber exhibits a higher hydraulic loading and a higher flow velocity, whereas the lower chamber displays longer retention times. Thus, the theoretical relationships for the hydraulics of multi-layer settling tanks have been confirmed.

5. CONCLUSIONS

1. The hydraulic properties of the particular chambers in the multi-layer horizontal-flow sedimentation tanks differ from those in the conventional horizontal-flow sedimentation tanks.

2. In order to determine the hydraulic properties of multi-layer tanks, it is necessary to investigate the hydraulic chracteristics of the particular chambers. Different distributions of water stated in the particular chambers resulted in differences between the upper and lower chambers manifested in flow velocity, hydraulic loading and retention time. The flow velocity in the upper chamber is two times higher than in the lower one. This may be due to the higher hydraulic losses typical of water flow in the closed chamber.

3. The theoretical model of the hydraulics of multi-layer settling tanks represents adequately the laminar flow conditions in a multi-layer tank. The model allows to determine the depth ratio for the particular chambers in which the flow rates are compensated.

4. Hydraulic conditions can be estimated precisely in a full scale only, because of a transitional character of the flow.

Experiments show a considerable difference in the hydraulic conditions between the upper and the lower chambers. The change in the character of flow (from laminar to transitional) cannot decrease significantly the difference between the distribution of the flow rates in both chambers.

5. The effectiveness of sedimentation in the double-layer settling tanks can be lower than that predicted by CAMP.

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STABILNOŚĆ OSADNIKÓW PIĘTROWYCH

Celem pracy jest teoretyczna i doświadczalna analiza stabilności hydraulicznej osadników poziomych. Wyprowadzony matematyczny model ruchu wody w komorze osadnika oparto na równaniu Poissona, newtonowskich właściwościach wody i warunkach brzegowych wynikających z geometrii komory. Badania doświadczalne prowadzone w skali ułamkowo-technicznej potwierdziły słuszność wyprowadzonego modelu.

W pracy wykazano istotne różnice właściwości hydraulicznych pomiędzy górną i dolną komorą osadnika powodujące różnice prędkości przepływu, obciążeń hydraulicznych i czasów przepływu. Prędkość przepływu w komorze górnej była w warunkach i skali prowadzonych badań przeszło dwukrotnie większa niż w komorze dolnej, co uzasadnia się różnicą strat hydraulicznych. Efekty sedymentacji w osadnikach piętrowych mogą okazać się gorsze niż wynikałoby to z założeń Campa, że efekty należy odnieść do dwu krotnie większej powierzchni niż w osadnikach klasycznych.

DIE STABILITÄT VON ZWEISTÖCKIGEN ABSETZBECKEN

Die Arbeit hatte eine theoretische und empirische Analyse der hydraulischen Stabilität von horizontalen Absetzbecken zum Ziel. Das angeführte mathematische Wasserströmungsmodell basiert auf der Poisson'schen Cleichung, auf den Newton'schen Flüssigkeitseigenschaften und auf den Grenzbedingungen der Beckengeometrie. Versuche im Pilotmaßstab führten zur Bestätigung des Modells.

Auf Grund der Versuche konnte man feststellen, daß zwischen den hydraulischen Kenngrößen des oberen und des unteren Beckens wesentliche Differenzen bestehen. Diese sind: die Fließgeschwindigkeiten, die Durchflußzeiten, die hydraulische Oberflächenbelastung. Im oberen Becken war die Fließgeschwindigkeit mehr als doppelt so groß wie im unteren Becken, was mit der Differenz der hydraulischen Verluste zu erklären ist. Die Absetzwirkung in zweistöckigen Becken sollte man daher nicht so günstig beurteilen, wie das z.B. aus den Annahmen von Camp hervorgeht.

УСТОЙЧИВОСТЬ ДВУХБЯРУСНЫХ ОТСТОЙНИКОВ

Целью работы является теоретический и опытный анализ гидравлической устойчивости горизонтальных отстойников. Выведенная математическая модель движения воды в камерео тстойчика основана на уравнении Пуассона, ньютоновых свойствах воды и краевых условиях, вытекающих из геометрии камеры. Экспериментальные исследования, проводимые в частично-промышленном масштабе подтвердили правильность введённой модели.

В работе показаны существенные различия гидравлических свойств между верхней и нижней камерами отстойника, вызывающие отличия в скорости течения, гидравлических нагрузках и времени течения. Скорость течения в условиях и масштабе проводимых исследований в верхней кмере была свыше двух раз больше, чем в нижней камере, что обосновывается разницей гидравлических потерь. Эффекты седиментации в двухъярусных отстойниках могут оказаться хуже, чем это следовалобы из предположения Кэмпа, что эффекты следует отнести к два раза большей поверхности, чем в классических отстойниках.