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## MODELLING OF THE EFFECTS OF THE COAGULATION PROCESS

A mathematical model describing the effects of surface water coagulation has been introduced. The range of the experiment has been confined to the basic coagulant, i.e. to aluminium sulphate commonly applied to coagulation process during water purification. The presented method for the interpretation of investigation is general and may be used for different kinds of water and coagulants.

### NOTATIONS

$A, B_1, B_2$  — constants,  
 $B$  — colour,  $\text{g/m}^3$  Pt,  
 $C, C_k, C_p$  — concentration of the component, final concentration, initial concentration,  
 $D, D_j$  — dosage of coagulant, unit dosage,  $\text{g/m}^3$ ,  
 $E_j$  — unit effects of the purification,  
 $M$  — turbidity,  $\text{g/m}^3$ ,  
 $m, n$  — exponents,  
 $P$  — electric charge of the particles,  $\text{val/m}^3$ ,  
 $t$  — flocculation time, s,  
 $Z$  — alkalinity,  $\text{val/m}^3$ ,  
 $\omega$  — rotational speed,  $\text{rev./min}$ .

### 1. INTRODUCTION

The so far applied methods for interpretation of the investigations on coagulation process have been based on the models in which the influence of water alkalinity and charge of particles on the dosage were taken into account (eg. 1, [4])

$$D = B_1 Z + B_2 P^m \quad (1)$$

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and on multifactorial models in which the purification effects are connected with the dose of coagulant and flocculation parameters (eg. 2, [1])

$$C/C_p = -5.94 + 2.38t - 0.012t^2 + 1.72 - 0.0085tD + 0.015t + 3.9D^{1/2} - 0.018D^2. \quad (2)$$

According to other authors [5] the charge of colloidal particles is one of the several technological criteria of the process, but it is not sufficient if the optimization conditions are to be established exactly, since it does not determine univocally the purification effects.

Model 2 indicates a high degree of difficulty in determining the independent variables, since it requires the investigations conducted in full technical scale. Investigations made on pilot or smaller technological scales cannot be approximated according to the relation (2), since the effects occurring while enlarging the scale of investigations are not known. Ambiguity of the approximation of results may be also due to the fact that the uniformity of the velocity gradients distribution in flocculation chambers has not been taken into account [3]. Optimal values of velocity gradients and flocculation time are the factors deciding upon floc size and flocculation effects [2]. The presence of floc destruction and sedimentation zones in flocculation chambers reduces the effects of the whole process and determines the efficiency of the equipment.

Thus it is fully justified to separate the modelling of the coagulation effects of suspended matter removal from the modelling of flocculation chambers efficiency. The combination of the above models will allow to establish univocally the process efficiency and direction of its optimization.

In the paper the model determining the effects of coagulation process and its verification have been presented.

## 2. MATHEMATICAL MODEL

Simplification of the procedure allowed to present the coagulation effects  $E_c$  versus unit dosage  $D_j$  and initial concentration of component  $C_p$ . The following relationship has been assumed:

$$E_j = f(C_p, D_j). \quad (3)$$

Coagulation effects have been measured by the reduction of turbidity, colour and COD of water.

It has been also assumed that the structural form of the eq. (3) is consistent with the exponential formula

$$E_j/D_j = AC_p^n \quad (4)$$

where  $n$  — is the exponent,  $A$  — constant characteristic of the coagulated water.

In the model introduced — in the contrast with these used commonly — the influence of initial concentration of the impurities removed on the unit effects of coagulation has been taken into account.

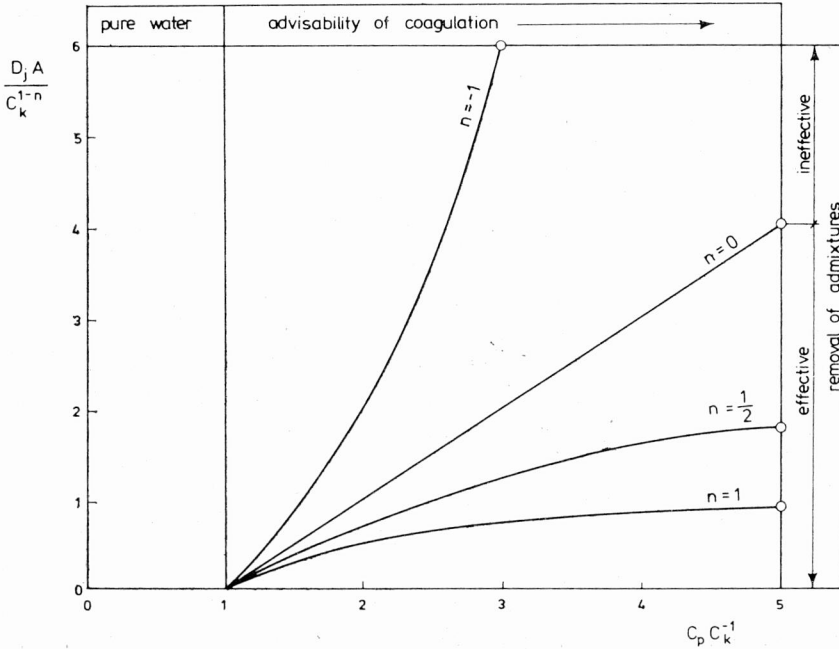


Fig. 1. Model of removal effects in coagulation process

Rys. 1. Model efektów usuwania domieszek w procesie koagulacji

The general model, if presented geometrically, is a family of curves shown in fig. 1. The curves being plotted in the Cartesian system of coordinates, described by dimensionless products, allow to use the above relationship in any system of units. The X-axis ( $C_p/C_k$ ) determines the coagulation effects, while the Y-axis ( $D_j A/C_p^{1-n}$ ) determines the coagulant dosage. Constant parameters ( $A$ ,  $n$ ) of the model have physical meaning. The values of the constants are the measures of the suspended matter removability and efficiency of coagulants.

Removability is the property of impurities and may be analyzed when several kinds of components are removed with one coagulant, while the efficiency is the property of the given coagulant and may be determined when the same components are removed by different coagulants.

From the functions presented in fig. 1 it follows that the value of the constant  $n$  affects both the impurities removability and the coagulants efficiency, whereas the constant  $A$  influences solely the efficiency of coagulants.

With the increasing value of the constant  $n$  the suspended matter removability increases, while the increase of the constant  $A$  leads to the reduction of the coagulant dosage. For  $n < 0$  the efficiency of impurities removal is not high. This means that other, more efficient coagulants should be used. For  $n = 0$  the function can be considered as a conventional borderline between the effectiveness and ineffectiveness of coagulants.

The introduced model of the purification effects may be a basis for determining the coagulant dosage. An effective dosage of a coagulant is given by the sum of unit dosages within the interval  $C_p, C_k$ . Hence, the size of dosage may be calculated from the formula for  $n \neq 1$

$$D = \frac{1}{A(n-1)C_k^{n-1}} \left[ 1 - \left( \frac{C_k}{C_p} \right)^{n-1} \right]$$

and for  $n = 1$

$$D = \frac{1}{A} \ln \frac{C_p}{C_k}$$

For the class of models, in which  $n = 1$  and the values of  $C_p/C_k$  are high, the initial concentration of the components being removed slightly affects the size of the dosage; the formula (5) is then reduced to the form

$$D = \frac{C_k^{n-1}}{A(n-1)}$$

The models introduced allow to determine the number and scope of the experiments in order to establish the model constants within the given confidence interval, which is of an essential importance in programming and optimization of research works. The determined values of the constants allow to use directly the detailed forms of the model in design calculations, e.g. when coagulant doses or treatment effects are to be determined.

The empirical models so far applied, e.g. for turbidity removal

$$D = 3.5\sqrt{M}$$

and colour removal

$$D = 4\sqrt{B}$$

result also from the general model assumed for  $n = 1/2$  and for  $A$  equal to 0.57 and 0.4 for the turbidity and colour, respectively.

### 3. VERIFICATION OF THE MODEL

#### 3.1. METHODS AND RESULTS OF INVESTIGATIONS

The purpose of the measurements performed was to obtain the data indispensable to verify the adequacy of the assumptions taken and to determine the model constants. The investigations comprised the effect of water pH and alum dosages on the water coagulation in the Kaczawa river and in infiltration pond. The experiments have been performed on a laboratory scale by the jar test method. The coagulant dosages varied from 0 to 80 g  $\text{Al}_2(\text{SO}_4)_3 \cdot 18 \text{H}_2\text{O} / \text{m}^3$ . The effects of coagulation process were controlled by commonly used methods of analytical tests, determining in raw and purified water the

H, alkalinity, turbidity, colour and COD. The pH adjustment has been made within the range of alum applicability. The values of some physicochemical components of the water are presented in table 1, and the optimal coagulant dosages, which warrant the desired treatment efficiency depending on the pH of water, are given in table 2.

Table 1

Physicochemical components of water Skład fizykochemiczny badanych wód			
Component	Unit	Variations of components	
		Kaczawa river	infiltration pond
Temperature	K	281-291	283-291
pH	pH	7.4-8.0	8.8-9.2
Alkalinity	g/m <sup>3</sup> CaCO <sub>3</sub>	60-115	80-105
Turbidity	g/m <sup>3</sup>	15-40	10-20
Colour	g/m <sup>3</sup> Pt	25-35	40-45
COD	g/m <sup>3</sup> O <sub>2</sub>	3.2-7.0	4.2-8.6

Table 2

Parameters of coagulation process Parametry procesu koagulacji		
pH	Optimal alum dosages g Al <sub>2</sub> (SO <sub>4</sub> ) <sub>3</sub> · 18 H <sub>2</sub> O/m <sup>3</sup>	
	Kaczawa river	infiltration pond
without adjustment of pH	25-70	—
7.5	30-60	30-50
7.0	25-50	25-50
6.5	20-40	25-40
6.0	25-50	25-40
5.5	25-70	30-50

### 3.2. DISCUSSION

Effects of coagulation process have been modelled by the relation (4) which described the exponential dependence of turbidity, colour and COD removed by unit alum dosage on the initial values of those parameters. If the equation (4) is to be applied, it is more convenient to use its logarithm form:

$$\log \frac{C_p - C_k}{D_j} = \log A + n \log C_p. \quad (10)$$

If the experimental data satisfy the exponential equation (4) then  $\log (C_p - C_k)/D$  versus  $\log C_p$  is expressed by a straight line.

Results of the coagulation process for several series are presented in figs. 2-7. The

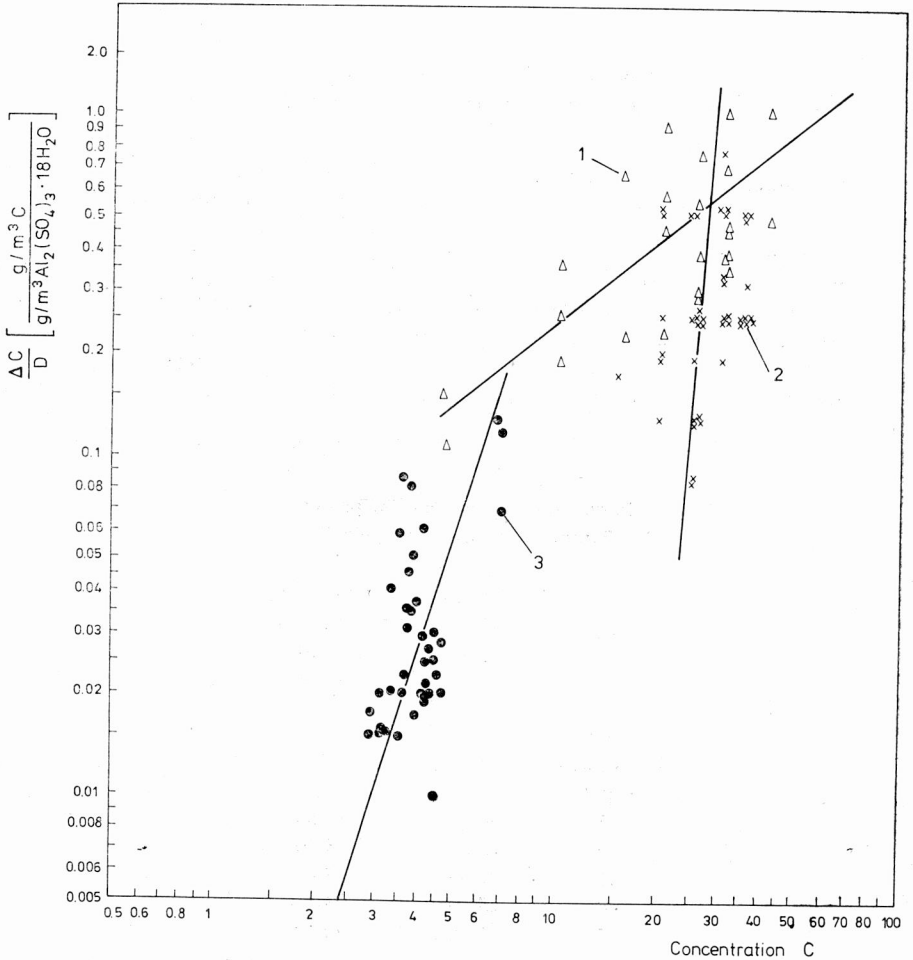


Fig. 2. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from the Kaczawa river at pH 7.7

Rys. 2. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie z rzeki Kaczawy przy pH 7,7

course of curves illustrating this process is consistent with the model assumed, and the values of constants depend on the kind of components.

The investigations have shown a high removability of impurities in alum coagulation. Water impurities from the infiltration pond were more efficiently removed than those from the Kaczawa river. The removability of the components responsible for turbidity,

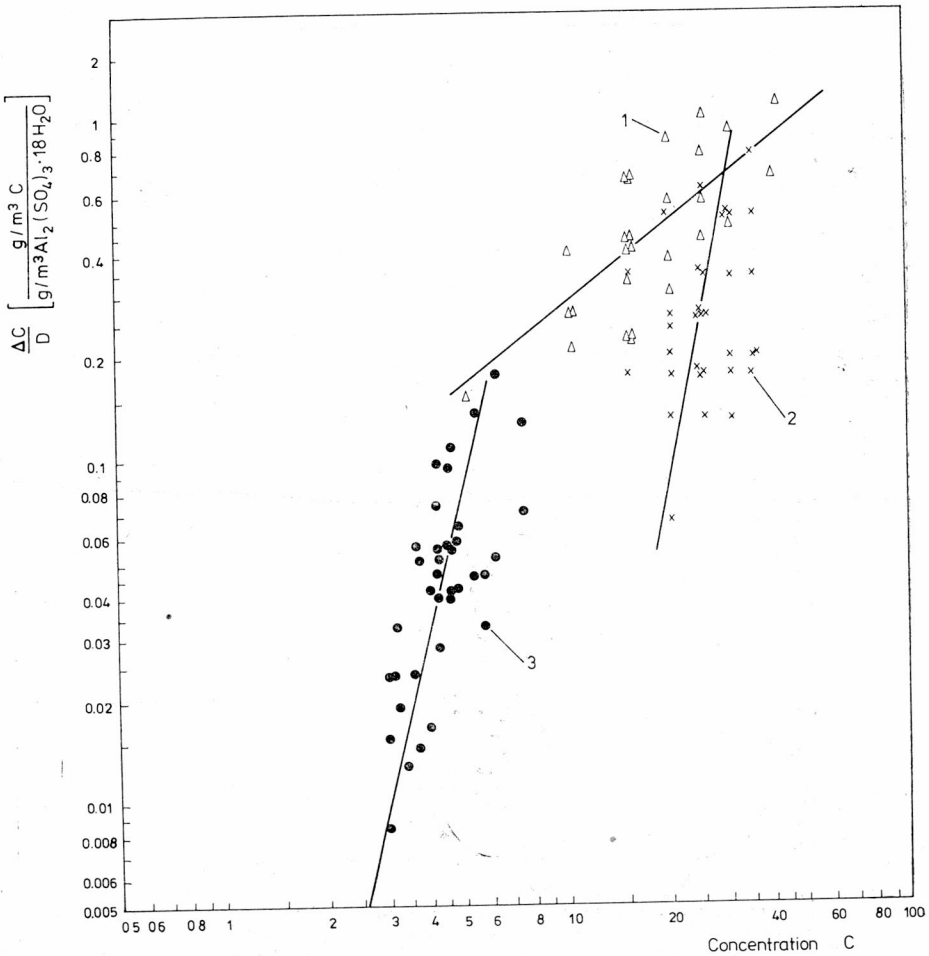


Fig. 3. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from the Kaczawa river at pH 6.5

Rys. 3. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie z rzeki Kaczawy przy pH 6,5

colour and COD increases with the value of  $n$ . The lowest value of  $n$  has been found for turbidity, and the highest one for colour. It has been also stated that the optimal pH for the impurities removal is different, and for turbidity, colour and COD its respective values are 6.5, 7.7 and 5.5 in water from the Kaczawa river and 6.5–7.0, 5.5 and 7.0 — in water from infiltration pond.

The choice of the optimal pH for the coagulation process should be based on the value of removability ( $n$ ) and the required efficiency of the process ( $C_p/C_k$ ).

The optimal pH of coagulation, established for the most disadvantageous case, i.e. for the turbidity removal, was 6.5 for the Kaczawa river and 6.5–7.0 for the infiltration

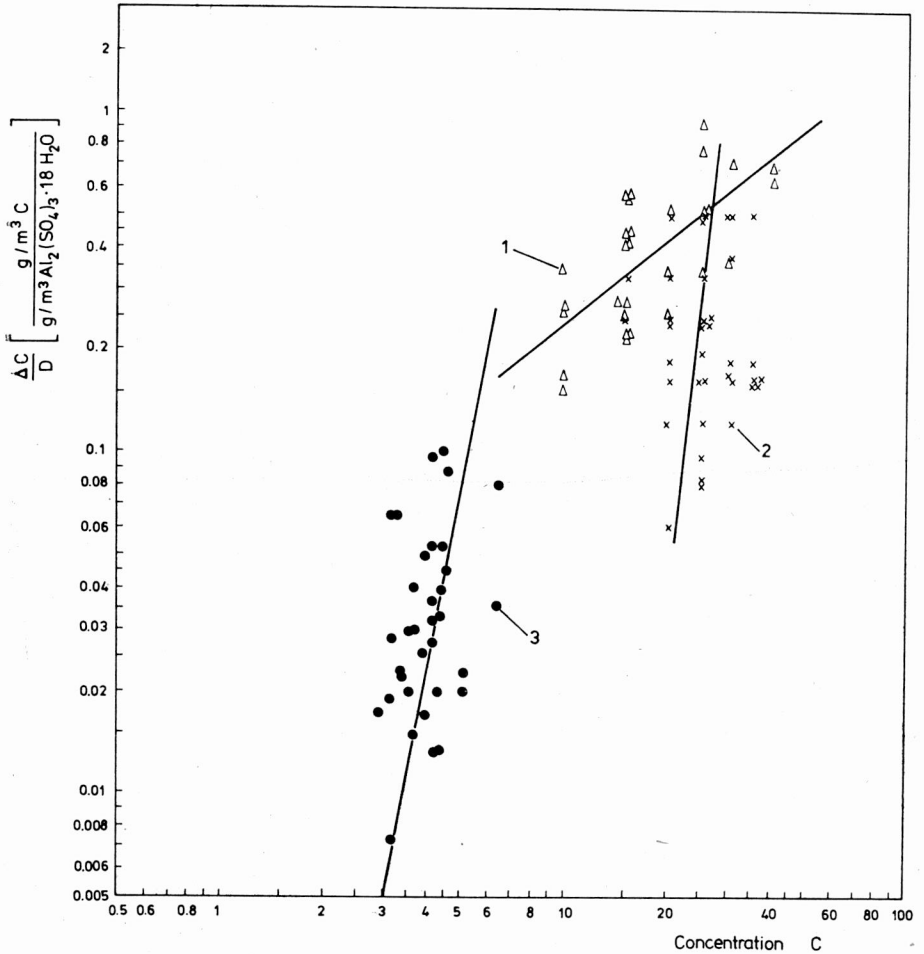


Fig. 4. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from the Kaczawa river at pH 5.5

Rys. 4. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie z rzeki Kaczawy przy pH 5,5

pond. The alum dosage, efficient in turbidity removal and established for the above pH values are:

$$D = 22.2 \log \frac{C_p}{C_k} \quad (11)$$

and

$$D = 35.7 \log \frac{C_p}{C_k} \quad (12)$$



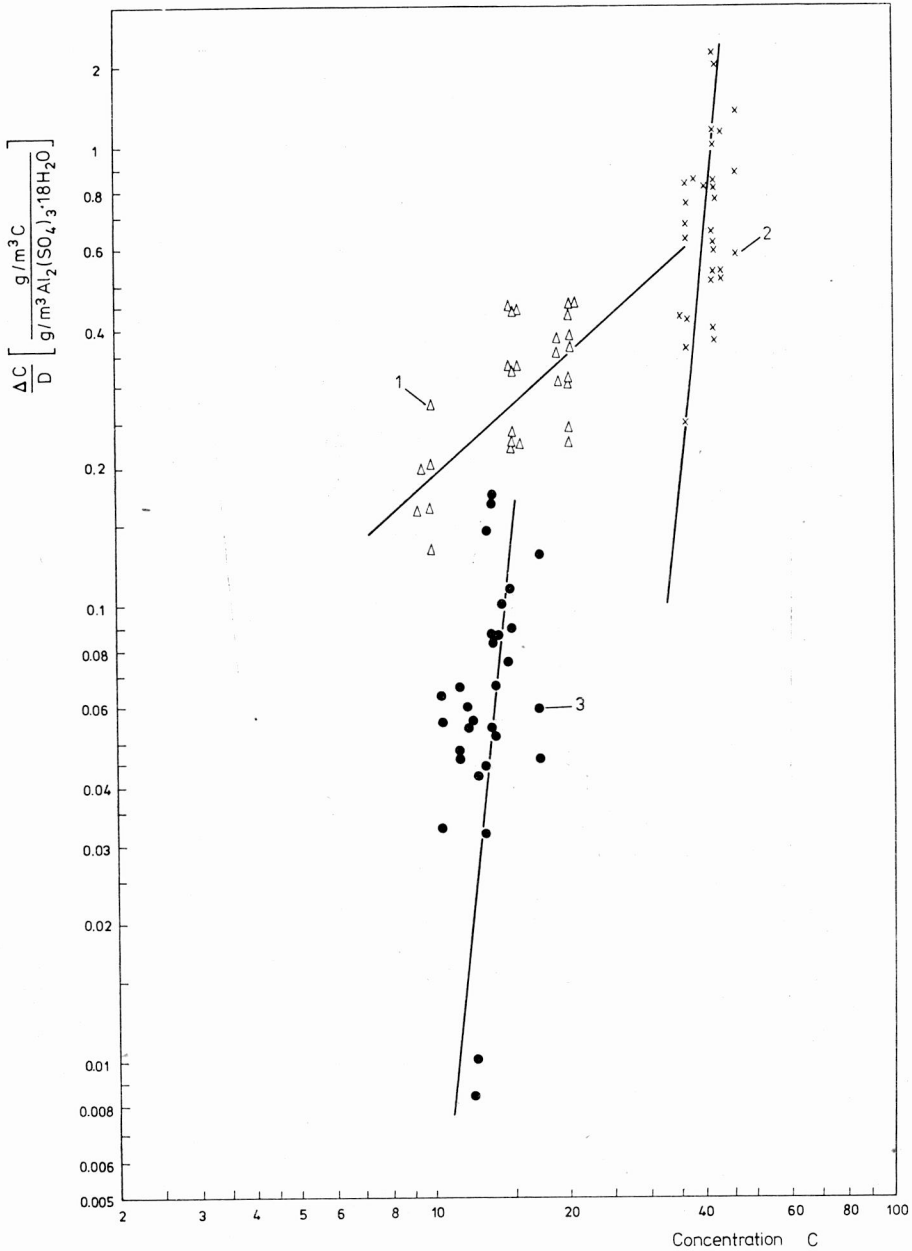


Fig. 5. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from the infiltration ponds at pH 7.0

Rys. 5. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie ze stawów infiltracyjnych przy pH 7,0

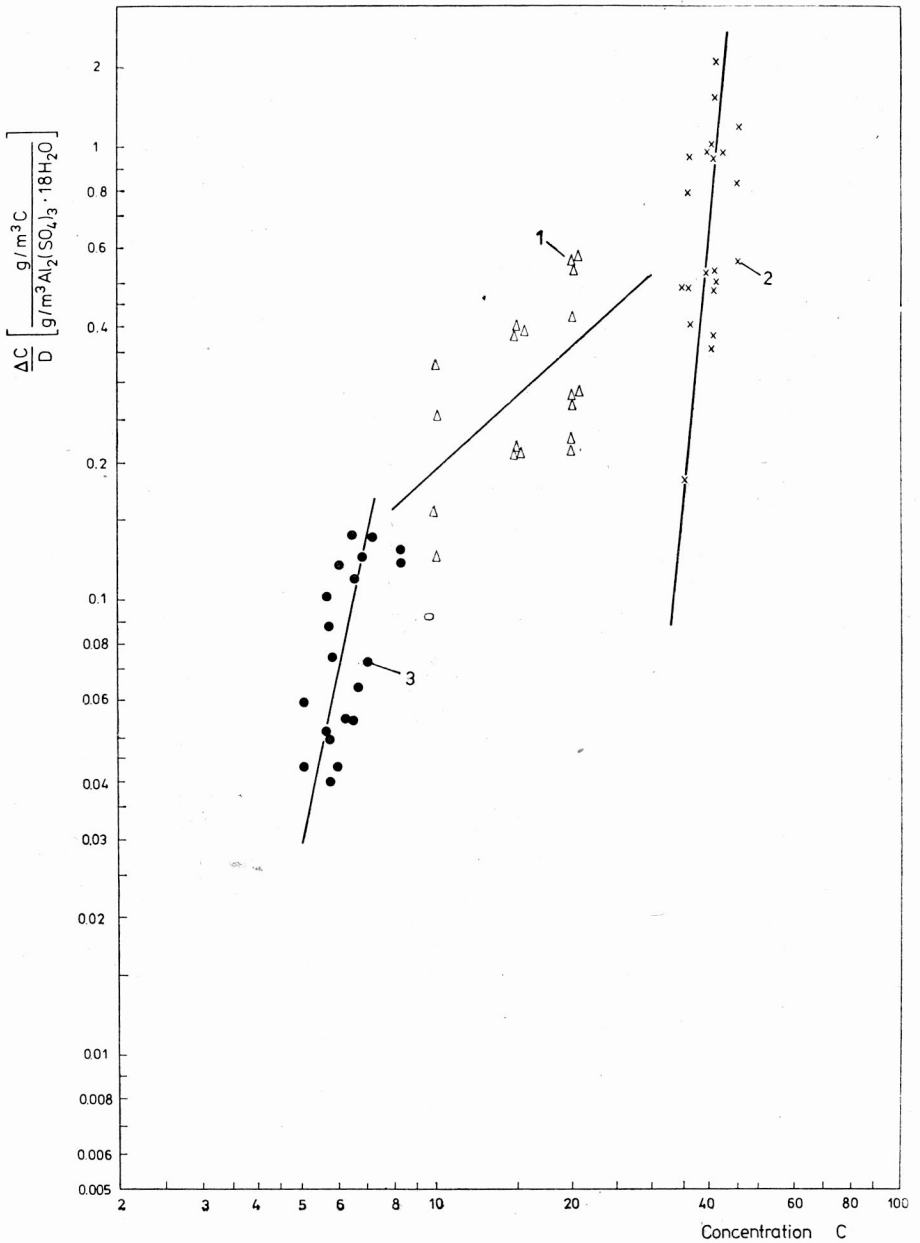


Fig. 6. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from the infiltration ponds at pH 6.5

Rys. 6. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie ze stawów infiltracyjnych przy pH 6,5

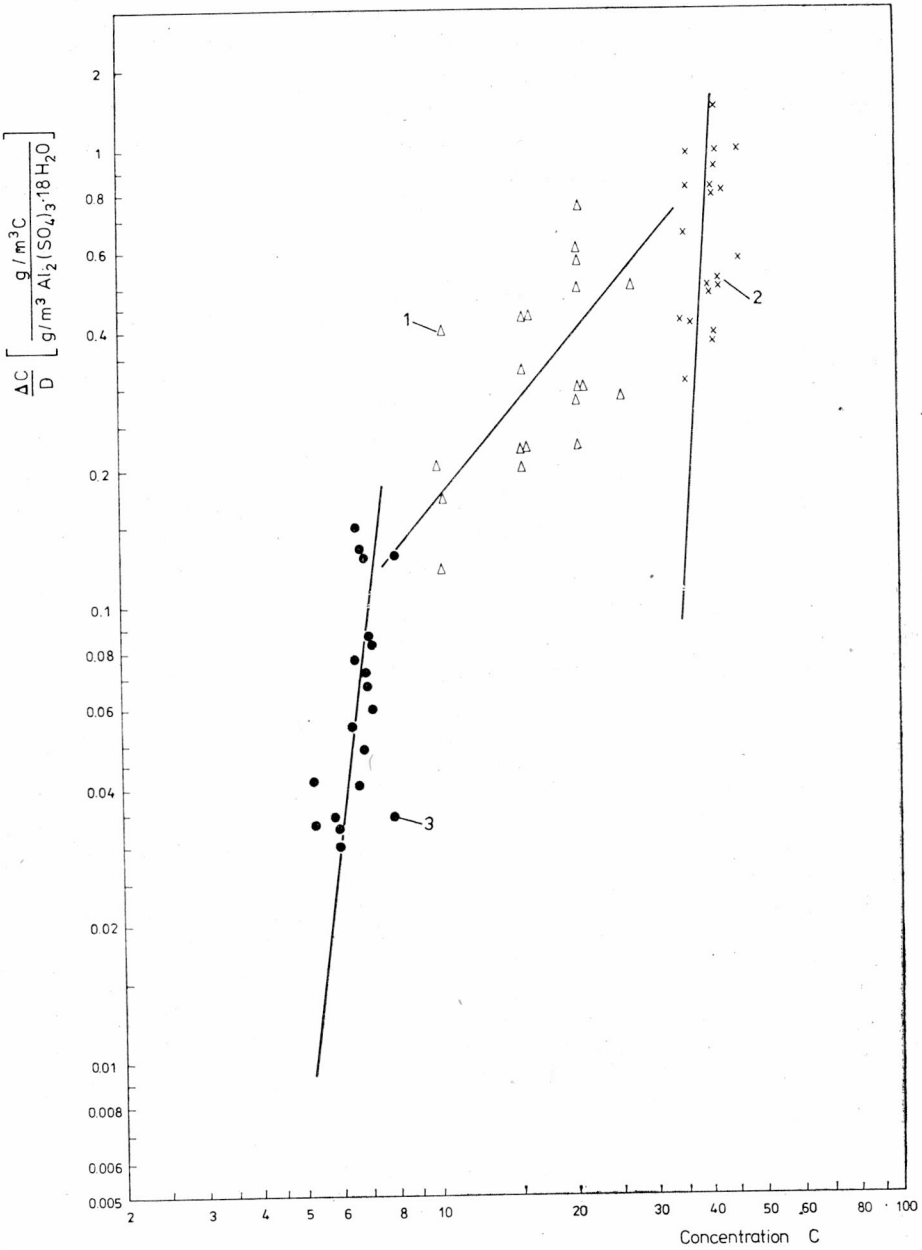


Fig. 7. Unit effects of the removal of turbidity (1), colour (2) and COD (3) in water from infiltration ponds at pH 5.5

Rys. 7. Jednostkowe efekty usuwania mętności (1), barwy (2) i utlenialności (3) w wodzie ze stawów infiltracyjnych przy pH 5,5

for the water for the Kaczawa river and infiltration pond, respectively.

It follows from the above formulae that the effect of initial turbidity on the coagulant dosage is higher for water from the Kaczawa river.

Coagulant doses determined for turbidity removal for the Kaczawa river and infiltration pond guarantee also the colour and COD removal. Doses determined from the two latter indices are charged with high computational errors, cf. the curves in figs. 2-7.

#### 4. CONCLUSIONS

1. The method presented for interpretation of results is general and may be used for different kinds of water and different coagulants.

2. The effects of water coagulation are approximated adequately by an exponential function.

3. The model introduced allows to determine univocally the optimal pH of the coagulation process and efficient dosages of coagulants, taking account of the kind of components.

4. Removability of pollutants on removal and the efficiency of coagulants are determined by the model constants.

5. The model of unit effects of the components removal may be helpful in optimization of criteria of the coagulation process.

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#### MODELOWANIE EFEKTÓW PROCESU KOAGULACJI

Przedstawiono ogólną metodę interpretacji wyników badań procesu koagulacji. W tym celu wprowadzono model matematyczny określony funkcją potęgową. Weryfikację modelu przeprowadzono dla podstawowego koagulantu — siarczanu glinowego, stosowanego powszechnie w uzdatnianiu wód powierzchniowych, oraz dla domieszek wód z rzeki Kaczawy i stawu infiltracyjnego. Na podstawie stałych modelu ustalono podatność usuwania domieszek i skuteczność siarczanu glinowego w oczyszczaniu badanych wód. Analiza stałych modelu pozwoliła także określić optymalny odczyn procesu z uwzględnieniem rodzaju domieszek.

## EINE MODELLIERUNG VON ERGEBNISSEN IM KOAGULATIONSVERFAHREN

Dargestellt wird eine allgemeine Methode der Interpretation von Modellergebnissen im Koagulationsverfahren. Zu diesem Zweck wurde ein mathematisches Modell mit einer Potenzfunktion formuliert. Die Nachprüfung des Modells wurde für Aluminiumsulphat, für die Wasserinhaltsstoffe der Kaczawa sowie für ein en Infiltrationsteich ausgeführt. Anhand der Modellkonstanten wurde die Beseitigungsmöglichkeit der Wasserinhaltsstoffe sowie die Wirkung des Aluminiumsulphats bestimmt. Eine weitgehende Analyse der Modellkonstanten ermöglicht auch die Bestimmung der optimalen pH-Reaktion bei Berücksichtigung der Art der Inhaltstoffe.

## МОДЕЛИРОВАНИЕ ЭФФЕКТОВ ПРОЦЕССА КОАГУЛЯЦИИ

Предложен общий метод интерпретации результатов исследований процесса коагуляции. Для этой цели была введена математическая модель, определённая степенной функцией. Проверка моделей была проведена для основного коагулянта — сульфата алюминия, обычно применяемого в подготовке поверхностных вод, а также для примесей вод из реки Качавы и инфильтрационного пруда. На основе постоянных модели была установлена податливость удаления примесей и эффективность сульфата алюминия в очистке исследуемых вод. Анализ постоянных моделей позволил также определить оптимальную реакцию процесса с учётом вида примесей.