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APPLICATION OF THE LEAST SQUARES METHOD TO ESTIMATION OF MIXING COEFFICIENTS IN FREE-FLOWING RIVERS

1. INTRODUCTION

Rapid growth of industry in Poland affecting seriously the water body free-flowing streams calls for a sufficiently accurate mathematical description of the wastewaters dispersion in rivers. The estimation of mixing coefficients is the main problem connected with the mathematical modelling. Computer technique allows to apply the optimization procedure to evaluate the mixing coefficients. This method was first applied by THACKSTON [3] in 1967. He minimized the function connecting measurements data with the mathematical model. The method for estimation of mixing coefficients, presented in this paper and based on the tracer study is a generalization of the Thackston method on a two-dimensional case.

2. BASIC EQUATIONS

It is generally assumed that diffusion process in free-flowing rives can be described by the following equation:

$$\frac{\partial c}{\partial t}+V_x\frac{\partial c}{\partial x}=D_x\frac{\partial^2 c}{\partial x^2}+D_y\frac{\partial^2 c}{\partial y^2} \tag{1}$$

where: c — concentration of indicator; t — time; x, y — lateral and longitudinal coordinates; V_x — mean velocity in the tested cross-section; D_x, D_y — lateral and longitudinal mixing coefficients.

By assuming different initial and boundary conditions it is possible to obtain different solutions of the eq. (1). Assuming an instantaneous injection of the total mass of tracer M, at time $t=0$ in point ($x=0$, $y=0$) initial condition, and assuming also infinite width of a river and a steady-state flow as boundary conditions the solution of (1) has the form

$$c=\frac{M}{4\pi ht\sqrt{D_xD_y}}\exp-\left[\frac{(x-V_xt)^2}{4D_xt}-\frac{y^2}{4D_yt}\right] \tag{2}$$

where: M — total mass of tracer; h — average depth of the river section.

According to eq. (2) the tracer concentration at a distance x from the source and at any given time is

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described by a two-dimensional normal distribution. The time concentration plane is however, skewed because the denominator in eq. (2) varies with time.

In order to predict the distribution of a pollutant at a profile downstream from its discharge point all the parameters: h, V_x, M, D_x and D_y must be known or estimated. Generally speaking, estimation of the first two parameters is relatively easy. The estimation of the mixing coefficients D_x, D_y and the total mass of tracer flowing through the tested profile is much more complicated. The reasons of the difficulties which were given by THACKSTON [3] with respect to the mixing coefficient D refer also to the determination of the coefficient D_y.

The aim of the method presented is a simultaneous estimation of three parameters: D_x, D_y, t_{av}, based on the measurement of the tracer concentration injected once into the river. Calculations have been performed by the least squares procedure, according to the following equation

$$\Phi(D_x, D_y, t_{\text{av}}) = \sum_{i=1}^{n} (c_i - \bar{c}_i)^2 \tag{3}$$

where: c_i — the measured values of concentration in point (x_i, y_i);
$\bar{c}_i$ — values calculated from eq. (4) in point (x_i, y_i);
t_{av} — mean residence time.

An easy transformation of the eq. (2), results in the eq. (4) used to calculate the tracer concentration:

$$\frac{\bar{c}_i}{\dfrac{M}{hV_x}} = \frac{x_i}{4\pi t_i t_{\text{av}} \sqrt{D_x D_y}} \exp - \left[\frac{x_i^2 \left(1 - \dfrac{t_i}{t_{\text{av}}}\right)^2}{4D_x t_i} - \frac{y_i^2}{4D_y t_i} \right]. \tag{4}$$

The values of D_x, D_y and t_{av} have been calculated by applying the MARQUARDT's algorithm [1].

3. TRACER STUDY

It can be seen that the application of Thackston method requires some changes in the procedure of the tracer study. After Thackston's method the tracer was injected within full width of the river whereas in our investigation the tracer was injected in one point. Therefore due to the assumption of a two dimensional model the number of sampling points in each cross-section had to be increased to at least 5.

The tracer technique is based on the assumption that the diffusion of the tracer substance runs according to the diffusion conditions in the section tested. If the mass M of the substance is injected instantaneously, and the changes in its concentration are measured several km downstream, then some losses of mass ΔM will be observed from the budget. The losses in tracer mass are due to its accumulation in dead zones and to its high dilution (below the sensitivity of the instrument). In the free-flowing rivers with a small number of training structures this loss may significantly affect final results of the study.

To avoid this inconvenience the following procedure was applied: the measured concentrations of the tracer flowing through the given profile were plotted, assuming: t — time, and y — distance from the bank where the source is located as coordinates.

The concentrations isolines drawn (like in fig. 1 and 2) gave a spatial picture of the tracer flow through the cross-section (where concentration was the third dimension). The volume of this "solid figure" is the conventional denominater M/hV_x in the eq. (4).

The procedure was succesfully applied to studies of diffusion in a broad river, where the boundary conditions can be neglected (the stretch of the Vistula River downstream of the Kozienice Power Plant) as well as in a narrow stream (Vistula in the Oświęcim region), where the boundary conditions should be taken into account. The tests were performed within a 12.3 km long section of Vistula in the Oświęcim

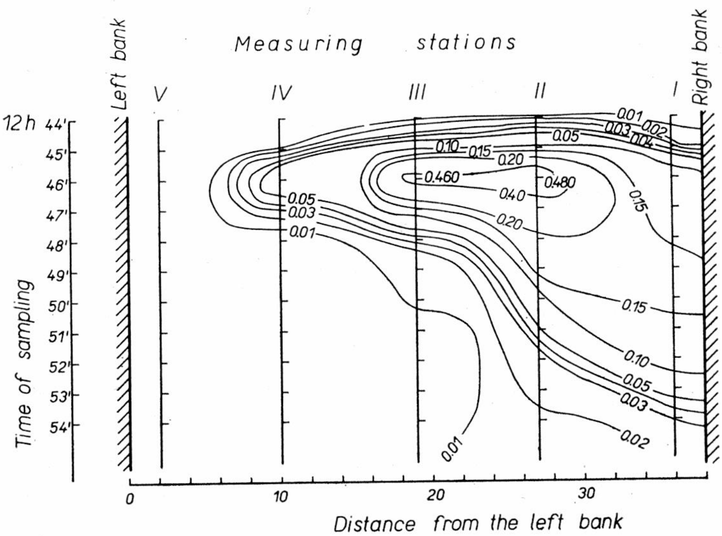

Fig. 1. Concentration of Rodamine B 1500 m downstream from the wastewater outlet

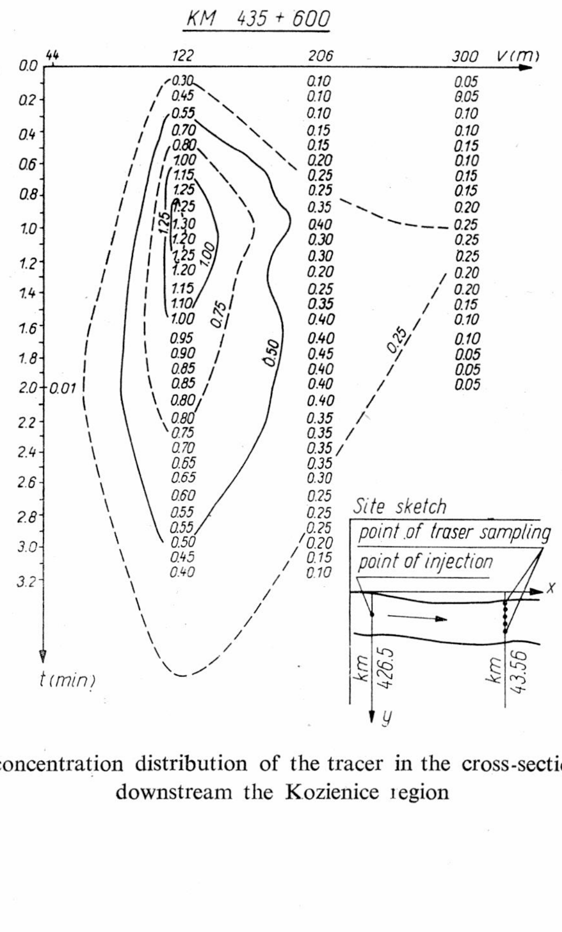

Fig. 2. An exemplary concentration distribution of the tracer in the cross-section of the Vistula River downstream the Kozienice region

region downstream of Przemsza River. Within this section the Vistula has the following tributaries: Przemsza, Soła, Monowice, Włosienica (with wastewaters) and it takes also the wastewaters discharged by Chemical Plant Oświęcim. During the measurements the water flow in the Vistula ranged within lower limit of mean values when the width of the river does not exceed 40 m. Along that section the Vistula is canalized, its bed is regular.

The aim of the study was to determine the length of mixing zones for water from tributaries and Vistula. The tracer Rodamine B was instantaneously injected in the outlet centerlines of the tributaries. The samples were taken from the cross-section 1500 m downstream of the source. The concentrations of the samples were measured by "Aminco" flourometer. Because of the high level of the background caused by the wastewaters discharges, the concentration of the samples was measured with the accuracy of 0.01 mg/dm³. The results of measurements shown in fig. 1 were used to estimate the value of M/hV_x. The remaining parameters being found from direct measurements were used to compute the value of D_x and D_y. Final results are shown in table 1.

Table 1

Mixing coefficients D_x and D_y calculated from tracer study in Vistula near Oświęcim

Tributaries of Vistula	Flowrate Vistula Q_r m³/s	Mean flow velocity V_x m/s	Mean residence time t_{av} s	Mixing coeff. longit. D_x m²/s	Mixing coeff. lateral D_y m²/s
Przemsza River	57.0	1.016	1476	4.877	0.208
Soła River	57.7	1.100	1364	2.075	0.118
Wastewaters from factory	60.1	1.036	1448	1.174	0.127
Manowice creek	60.2	0.695	2158	1.535	0.113
Włosienica creek	61.7	0.894	1677	1.260	0.264

In Kozienice region the investigations comprised a 10 km long section. The purpose of the investigations was to determine the conditions for mixing of heated water discharged from the plant with that from river.

Along the stretch examined the Vistula is characterized by an irregular bed with movable sand bank and irregular banklines. During the measurements the river regulation was made in the region adjacent to the power plant. The width of the river ranged from 150 m in the outlet profile up to 450 m in the river section with irregular banklines. The radioactive isotope ^{82}Br and dye Rodamine B were used as tracers. These substances were injected instantaneously in the centerline of the outlet. The main cross-section was located at a distance of 9.9 km from the injection point downstream the Radomka confluence. An example of the tracer distribution obtained from the radiometric measurements is shown in fig. 2. The values of mixing coefficients calculated for 3 runs are presented in table 2. It should be noted that the 1st run was performed by using isotope ^{82}Br and the remaining two — by Rodamine. B

From the results obtained it follows that in the Kozienice and Oświęcim regions $D_x \approx 2D_y$ and $D_x \approx (10\text{–}20)D_y$, respectively, whereas according to Morris [2] $D_x \approx 70D_y$.

Hence, it be stated that the ratio D_x/D_y increases with the increasing of the bed compartness. The computed values of mixing coefficients confirm in the field the usefulness of the procedure presented above. Thus it seems that it can be recommended for application whenever the value of lateral mixing coefficient should be known.

Table 2

Mixing coefficients D_x and D_y calculated from tracer study in the Vistula, downstream Kozienice Power Plant

Run	Flow rate Q_r m³/s	Mean residence time t_{av} s	Mean flow velocity V_x m/s	Mixing coeff. longit. D_x m^2/s	lateral D_y m^2/s	Tracer
1	205	4325*	0.577	1.062	0.557	^{82}Br
2	434	11450	0.795	1.464	0.760	Rodamine B
3	365	16420 s	0.554	1.366	0.356	Rodamine B

* Samples were taken in the profile 3.4 km downstream the injection point.

4. CONCLUSIONS

The following conclusions may be drawn from the investigations performed:

1. The procedure of the least squares estimation of nonlinear parameters allows to obtain an acceptable accuracy of the mixing coefficients values.
2. The computer programme based on the Marquardt algorithm is a quick, easy and economical tool for that purpose.

REFERENCES

[1] MARQUARDT D. W., *An algorithm for least squares estimation of nonlinear parameters*, J. Soc. Ind. Appl. Math. Vol. 11, No. 2, 1963.

[2] MORRIS J. S., KRENKEL P. A., THACKSTON E. L., *Investigation on turbulent diffusion in inland waterways*, Techn. Rep. No. 14, Vanderbilt Univ. 1967.

[3] THACKSTON E. L., HAYS J. R., KRENKEL P. A., *Least squares estimation of mixing coefficient*, Journ. San. Div. SA3, 1977.