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Letter to the Editor

Numerical recovery of the interferometrically recorded wavefront in the interscanning regions*

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Introduction

The recovery of the wavefront shape behind an examined objective by using an interferometric method has been considered in many papers [1–4]. For the one-dimensional case this problem was considered in [1] and [2] where the solutions for the wavefronts shape recovery along the diameter section were given. The two-dimensional recovery was discussed in [3, 4] which resulted in elaborating of the methods of the wavefront reconstruction at the knots of the scanning grid within the analysed region of the respective interferograms.

The purpose of this paper is to give a method for wavefront recovery at any point of the examined interferogram region, given the results of scanning along the lines parallel to a chosen direction and assuming that the wavefronts is slow-varying function of coordinates. In this paper to represent the wavefront the power polynomials have been used in contrast to the paper [4] where the Zernike polynomials were exploited.

Theory

Each interferogram contains an information concerning the optical path differences between the two interfering wavefronts – the examined, and the reference ones. As the latter any known wavefront or even the examined wavefront but transformed in a known way (shearing interferometry) may be chosen. The information about the wavefront shape $g(x, y)$ may be obtained from the analysis of the respective interferogram. For the numerical convenience it is necessary to represent the $g(x, y)$ function in the form of a finite series

$$g(x, y) = \sum_{j=0}^{M} \Big(\sum_{i=0}^{M} a_{ji} y^i \Big) x^j. \tag{1}$$

Thus to determine the wavefront $g(x, y)$ it is enough to find the coefficients a_{ij} $(i, j = 1, \ldots, M)$ of the above expansion into series.

For this purpose a well-known procedure of linear scanning may be applied. The interferogram is scanned along the lines parallel to the x-axis and defined by equations $y = y_L$ $(L = 1, \ldots, Q)$; the constants y_L being known. Denoting

$$a_j(y_L) = \sum_{i=0}^{M} a_{ji} y_L^i, \quad \text{for} \quad \begin{array}{l} L = 1, \ldots, Q, \\ j = 1, \ldots, M, \end{array} \tag{2}$$

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and assuming that the values of $a_i(y_L)$ are known from the measurement, we may consider the set of relations (2) as the one determining for the sought a_{ij} coefficients, i.e. as a system of linear equations with respect to a_{ij}. If $Q = M+1$ (2) the number of equations is the same as the number of unknows and the problem would be uniquely solved (provided that the system of linear equations (2) is consistent). However, the value of M is usually determined by an optimizing procedure used to obtain the $a_j(y_L)$ for each scanning line, thus most commonly $M \neq Q-1$. Therefore another procedure is here proposed:

Writing (2) in the form

$$
\begin{aligned}
a_j(y_1) &= \sum_{i=0}^{M} a_{ji} y_1^i \\
a_j(y_2) &= \sum_{i=0}^{M} a_{ji} y_2^i \\
&\vdots \\
a_j(y_Q) &= \sum_{i=0}^{M} a_{ji} y_Q^i
\end{aligned}
\tag{3}
$$

for each fixed j, separately, we may reformulate the problem as consisting in finding the values of a_{ji} which would fit the two experimentally determined sets of values

$$y_1, \ldots, y_Q \; y_1 \text{ and } y_1 \; a_j(y_1), a_j(y_2), \ldots, a_j(y_Q)$$

related mutually by (3). This can be done by using a method of approximation employing the orthogonal polynomials and basing on the least-square-error criterion. When performing this fitting the set of values

$$a_{j0}, a_{j1}, \ldots, a_{jM} \quad (j = 1, 2, \ldots M).$$

is determined and by repeating it M times (one for each value of a_{ij}) we may determine all the a_{ij}, which solves our problem.

Obviously, by substituting the a_{ij} found in this way into (1) we find the analytic form of the two-dimensional wavefront distribution spread continuously accros the scanned region of the interferogram, which in particular allows to recover the wavefront at any point located between the scanning lines.

Application and accuracy estimation

The method proposed above has been applied to the wavefront recovery from interferograms of the lateral shearing type. In order to check the accuracy of the results obtained by this procedure the testing data on the scanning lines $y = y_L$ ($L = 1, 2, \ldots \ldots, Q$) have been introduced to the programme of numerical calculations and the values of the wavefront at the chosen points have been calculated on the base of formula (2). The results obtained have been given in table 1. Next, the coefficients a_{ij} have been found by exploiting the method proposed in this paper and on the base of (1) the wavefront values have been calculated at the same points on the y_L lines. The results of calculations have been given in table 2. They show that the accuracy of the applied solution is sufficiently good. The errors caused by the assumed approximation are of order of 10^{-3} of the light wavelength. This solution renders a possibility of obtaining the wavefront reconstruction within all the examined region without the necessity of making denser the sampling configuration on the interferogram and solving a correspondingly greater sets of equations which would result in heavy occu-

Wavefront calculated on the base of the $a_j(y_L)$ coefficients

Table 1

wavefront (μm) coordinates (mm)

y= / x=	12.000	10.000	8.000	6.000	4.000	2.000	0.000	-2.000	-4.000	-6.000	-8.000	-10.000	-12.000
15.000							22.5010						
14.000					21.2038	20.0040	19.6009	20.0040	21.2038				
13.000				20.5023	18.5034	17.3035	16.9007	17.3035	18.5034	20.5023			
12.000			20.8039	18.0021	16.0029	14.8030	14.4006	14.8030	16.0029	18.0021	20.8039		
11.000		22.1027	18.5034	15.7018	13.7025	12.5026	12.1005	12.5026	13.7025	15.7018	18.5034	22.1027	
10.000		20.0024	16.4029	13.6015	11.6022	10.4022	10.0004	10.4022	11.6022	13.6015	16.4029	20.0024	
9.000	22.5057	18.1021	14.5025	11.7013	9.7018	8.5019	8.1004	8.5019	9.7018	11.7013	14.5025	18.1021	22.5057
8.000	20.8048	16.4018	12.8021	10.0011	8.0015	6.8015	6.4003	6.8015	8.0015	10.0011	12.8021	16.4018	20.8048
7.000	19.3040	14.9015	11.3017	8.5009	6.5012	5.3012	4.9002	5.3012	6.5012	8.5009	11.3017	14.9015	19.3040
6.000	18.0033	13.6013	10.0014	7.2007	5.2010	4.0010	3.6002	4.0010	5.2010	7.2007	10.0014	13.6013	18.0033
5.000	16.9027	12.5011	8.9011	6.1006	4.1008	2.9007	2.5001	2.9007	4.1008	6.1006	8.9011	12.5011	16.9027
4.000	16.0021	11.6009	8.0009	5.2005	3.2006	2.0005	1.6001	2.0005	3.2006	5.2005	8.0009	11.6009	16.0021
3.000	15.3016	10.9007	7.3007	4.5004	2.5004	1.3004	0.9000	1.3004	2.5004	4.5004	7.3007	10.9007	15.3016
2.000	14.8012	10.4006	6.8005	4.0003	2.0003	0.8002	0.4000	0.8002	2.0003	4.0003	6.8005	10.4006	14.8012
1.000	14.5009	10.1005	6.5004	3.7002	1.7001	0.5001	0.1000	0.5001	1.7001	3.7002	6.5004	10.1005	14.5009
0.000	14.4006	10.0004	6.4003	3.6002	1.6001	0.4000	0.0000	0.4000	1.6001	3.6002	6.4003	10.0004	14.4006
-1.000	14.5005	10.1004	6.5002	3.7001	1.7000	0.5000	0.1000	0.5000	1.7000	3.7001	6.5002	10.1004	14.5005
-2.000	14.8004	10.4004	6.8002	4.0001	2.0000	0.7999	0.4000	0.7999	2.0000	4.0001	6.8002	10.4004	14.8004
-3.000	15.3004	10.9004	7.3002	4.5001	2.5000	1.3000	0.9000	1.3000	2.5000	4.5001	7.3002	10.9004	15.3004
-4.000	16.0004	11.6004	8.0002	5.2002	3.2001	2.0000	1.6001	2.0000	3.2001	5.2002	8.0002	11.6004	16.0004
-5.000	16.9006	12.5005	8.9003	6.1002	4.1001	2.9001	2.5001	2.9001	4.1001	6.1002	8.9003	12.5005	16.9006
-6.000	18.0008	13.6006	10.0004	7.2003	5.2002	4.0002	3.6002	4.0002	5.2002	7.2003	10.0004	13.6006	18.0008
-7.000	19.3011	14.9007	11.3006	8.5004	6.5004	5.3003	4.9002	5.3003	6.5004	8.5004	11.3006	14.9007	19.3011
-8.000	20.8015	16.4009	12.8008	10.0005	8.0005	6.8005	6.4003	6.8005	8.0005	10.0005	12.8008	16.4009	20.8015
-9.000	22.5019	18.1011	14.5010	11.7007	9.7007	8.5007	8.1004	8.5007	9.7007	11.7007	14.5010	18.1011	22.5019
-10.000		20.0013	16.4013	13.6008	11.6009	10.4009	10.0004	10.4009	11.6009	13.6008	16.4013	20.0013	
-11.000		22.1015	18.5016	15.7010	13.7012	12.5012	12.1005	12.5012	13.7012	15.7010	18.5016	22.1015	
-12.000			20.8019	18.0012	16.0015	14.8014	14.4006	14.8014	16.0015	18.0012	20.8019		
-13.000				20.5014	18.5018	17.3018	16.9007	17.3018	18.5018	20.5014			
-14.000					21.2021	20.0021	19.6009	20.0021	21.2021				
-15.000							22.5010						

Wavefront calculated on the base of the a_{ji} coefficients

Table 2

wavefront (μm) coordinates (mm)

x= \ y=	12.000	10.000	8.000	6.000	4.000	2.000	0.000	-2.000	-4.000	-6.000	-8.000	-10.000	-12.000
15.000							22.5028						
14.000					21.2024	20.0024	19.6023	20.0024	21.2024				
13.000				20.5021	18.5020	17.3020	16.9019	17.3020	18.5020	20.5021			
12.000			20.8019	18.0017	16.0017	14.8016	14.4016	14.8016	16.0017	18.0017	20.8019		
11.000		22.1017	18.5015	15.7014	13.7013	12.5013	12.1013	12.5013	13.7013	15.7014	18.5015	22.1017	
10.000		20.0014	16.4013	13.6011	11.6010	10.4010	10.0010	10.4010	11.6010	13.6011	16.4013	20.0014	
9.000	22.5013	18.1012	14.5010	11.7009	9.7008	8.5007	8.1007	8.5007	9.7008	11.7009	14.5010	18.1012	22.5013
8.000	20.8011	16.4009	12.8008	10.0007	8.0006	6.8005	6.4005	6.8005	8.0006	10.0007	12.8008	16.4009	20.8011
7.000	19.3009	14.9008	11.3006	8.5005	6.5004	5.3003	4.9003	5.3003	6.5004	8.5005	11.3006	14.9008	19.3009
6.000	18.0008	13.6006	10.0004	7.2003	5.2002	4.0002	3.6002	4.0002	5.2002	7.2003	10.0004	13.6006	18.0008
5.000	16.9007	12.5005	8.9003	6.1002	4.1001	2.9001	2.5000	2.9001	4.1001	6.1002	8.9003	12.5005	16.9007
4.000	16.0006	11.6004	8.0002	5.2001	3.2000	2.0000	1.6000	2.0000	3.2000	5.2001	8.0002	11.6004	16.0006
3.000	15.3006	10.9004	7.3002	4.5001	2.5000	1.2999	0.8999	1.2999	2.5000	4.5001	7.3002	10.9004	15.3006
2.000	14.8006	10.4004	6.8002	4.0001	2.0000	0.7999	0.3999	0.7999	2.0000	4.0001	6.8002	10.4004	14.8006
1.000	14.5006	10.1004	6.5002	3.7001	1.7000	0.5000	0.0999	0.5000	1.7000	3.7001	6.5002	10.1004	14.5006
0.000	14.4006	10.0004	6.4003	3.6002	1.6001	0.4000	0.0000	0.4000	1.6001	3.6002	6.4003	10.0004	14.4006
-1.000	14.5007	10.1005	6.5004	3.7003	1.7002	0.5001	0.1001	0.5001	1.7002	3.7003	6.5004	10.1005	14.5007
-2.000	14.8009	10.4007	6.8005	4.0004	2.0003	0.8002	0.4002	0.8002	2.0003	4.0004	6.8005	10.4007	14.8009
-3.000	15.3010	10.9008	7.3007	4.5006	2.5005	1.3004	0.9004	1.3004	2.5005	4.5006	7.3007	10.9008	15.3010
-4.000	16.0012	11.6010	8.0009	5.2008	3.2007	2.0006	1.6006	2.0006	3.2007	5.2008	8.0009	11.6010	16.0012
-5.000	16.9015	12.5013	8.9011	6.1010	4.1009	2.9008	2.5008	2.9008	4.1009	6.1010	8.9011	12.5013	16.9015
-6.000	18.0017	13.6015	10.0014	7.2013	5.2012	4.0011	3.6011	4.0011	5.2012	7.2013	10.0014	13.6015	18.0017
-7.000	19.3020	14.9019	11.3017	8.5016	6.5015	5.3014	4.9014	5.3014	6.5015	8.5016	11.3017	14.9019	19.3020
-8.000	20.8024	16.4022	12.8020	10.0019	8.0018	6.8018	6.4018	6.8018	8.0018	10.0019	12.8020	16.4022	20.8024
-9.000	22.5028	18.1026	14.5024	11.7023	9.7022	8.5021	8.1021	8.5021	9.7022	11.7023	14.5024	18.1026	22.5028
-10.000		20.0030	16.4028	13.6027	11.6026	10.4026	10.0025	10.4026	11.6026	13.6027	16.4028	20.0030	
-11.000		22.1034	18.5033	15.7031	13.7031	12.5030	12.1030	12.5030	13.7031	15.7031	18.5033	22.1034	
-12.000			20.8038	18.0036	16.0035	14.8035	14.4035	14.8035	16.0035	18.0036	20.8038		
-14.000				20.5041	18.5041	17.3040	16.9040	17.3040	18.5041	20.5041			
-15.000					21.2046	20.0046	19.6045	20.0046	21.2046				
							22.5051						

pation of the computer memories. The analytic description of the wavefront obtained in this way anables an optimal choice of the reference sphere, which is very important for computation of wave aberration of the examined objectives.

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