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The joint influence of atmospheric turbulence and primary coma on the far-field diffraction of a circular aperture

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The paper is devoted to theoretical study of the Fraunhofer diffraction patterns produced by a circular aperture suffering from primary coma and illuminated by partially coherent light due to atmospheric turbulence. Gaussian form of correlation has been assumed as a representative form of the atmospheric turbulence. Numerical results of intensity distribution and encircled energy have been graphically illustrated. Results for two-point resolution based on Huber–Hopkins' criterion have been obtained, and variation of peak intensity as a function of coherence interval plotted.

Introduction

The study of diffraction in an optical system under partially space-coherent illumination is of considerable significance because the cases of complete coherence and incoherence normally do not occur in practice. That is why considerable interest has been shown [1–4] to the investigations of partially coherent diffraction patterns. A comprehensive bibliography on the subject has recently been published by SINGH and DE [5].

In the optical instruments [6–10], like astronomical telescopes, cameras used in aerial or long distance photography, the atmospheric turbulence renders the wavefront (coming from an object point) partially coherent and corresponding point spread function deviates considerably from the ideal Airy pattern that would otherwise be formed in a perfect system. The optical systems suffer also from residual aberrations which, in turn, further modifies the point spread function. Consequently, some efforts are being made [11–14] to counterbalance the effect of turbulence. SOM and BISWAS [15] have investigated the effect of partial coherence due to atmospheric turbulence on the far-field structure of a circular aperture. In their later paper [16] they have studied the joint influence of partial coherence due to turbulence and to primary and secondary spherical aberrations.

A consideration of off-axis aberration is also of special importance in reconnaissance and surveillance systems which use telescopic instrumentation.

This kind of instruments is designed to provide the maximum information in a relatively large field of view. Third order coma happens to be the most important off-axis aberration to be considered, because it varies as the first power of the object field and is the first to appear when the field extends beyond the on-axis case. The asymmetric nature of this aberration is, in general, a very undesirable feature, in particular when the position measurements such as in astronomy [17, 18] are to be made.

The influence of coma has been investigated by evaluating the optical transfer function as well as the point, line and edge spread functions [19–27]. A large number of references are available in the recent papers [24, 25] dealing with the diffraction imaging of disk, bar and edge objects in the presence of linear coma.

In view of the above, the present paper has been devoted to investigation of the joint influence of atmospheric turbulence and primary coma on the far-field diffraction patterns in terms of the intensity distribution, encircled energy, Strehl ratio and angular resolution.

Theoretical formulation

We make use of the Schell–shore integral [1–4] which is based on the Wolf–Parrent formulation of the theory of partial coherence, and facilitates the computation of irradiance distribution in the Fraunhofer diffraction pattern formed by an aperture illuminated with partially space coherent radiation. The irradiance distribution is given [1–4] by the equation

$$I(P) = \frac{A'\cos^2\Phi}{\lambda^2 R^2}\int_{\Sigma}\gamma(\vec{S})C(\vec{S})\exp[i\bar{k}\sin\Phi\hat{p}\vec{S}]d\Sigma, \tag{1}$$

where: $\bar{\lambda}$ is the mean wavelength of radiation, $\bar{k} = 2\pi/\bar{\lambda}$, A' is the area of the aperture, Σ is the range of S, $C(\vec{S})$ is the auto-correlation function of the aperture amplitude distribution, and $\gamma(\vec{S})$ is the normalized mutual intensity function.

The meaning of quantities R, Φ, S and $\hat{p}$ is clear from fig. 1. The auto-correlation function is given by

$$C(\vec{S}) = \frac{1}{A'}\int_{\sigma_S} I(\vec{S}_1)^{1/2}I(\vec{S}_1+\vec{S})^{1/2}d\sigma. \tag{2}$$

Here σ_S is the region of the aperture to which $\vec{S}_1$ is restricted so that $\vec{S}_1+\vec{S}$ lies on the aperture, $I(\vec{S}_1)$ and $I(\vec{S}+\vec{S}_1)$ are the intensities at the points S_1 and S_1+S, respectively. For small diffraction angles ($\cos\Phi \approx 1$ and

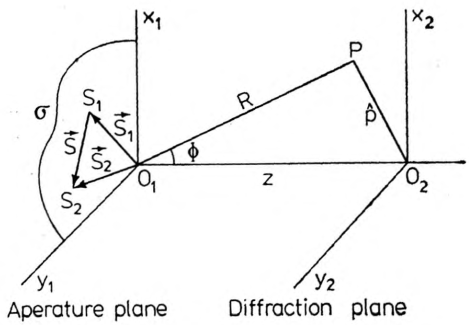

Fig. 1. Co-ordinate system and illustration of the symbols used in various formulae

$\sin \Phi \approx \Phi$) and for a spatially stationary source equation (1) can be written in the normalized form as

$$I(V, \Phi) = \frac{1}{2\pi} \int_0^2 \int_0^{2\pi} \gamma(\varrho) C(\varrho, \theta) \exp[i V \varrho \cos(\theta - \Phi) \varrho d\varrho d\theta]. \tag{3}$$

Here $\varrho = \frac{|\vec{S}_2 - \vec{S}_1|}{a'}$, is the normalized distance between two points in the aperture plane and, a' is the aperture radius. It is assumed that $a' \gg \lambda$. $V = ka'\Phi$ and Ψ is the azimuthal angle in the observation plane.

The encircled energy $E(V_0)$ is obtained by integrating equation (3) over a circle of radius V_0; i.e.

$$E(V_0) = \int_0^{V_0} \int_0^{2\pi} I(V, \Psi) V dV d\Psi. \tag{4}$$

$C(\varrho, \theta)$ – the auto-correlation of the aperture amplitude distribution in the presence of third order coma is tantamount to incoherent transfer function for a system suffering from coma, and has been calculated by GOODBODY [19], and BARAKAT and HOUSTON [20]. The transfer function in the presence of coma is a complex quantity and can be written as

$$C(\varrho, \theta) = C_r(\varrho, \theta) + iC_i(\varrho, \theta), \tag{5}$$

where C_r and C_i are the real and imaginary parts of $C(\varrho, \theta)$ and given by

$$C_r(\varrho, \theta) = C(0, 0)^{-1} \int_{-a}^{a} \int_{-b}^{b} \cos\left[\frac{2\pi}{\lambda} W\left(a + \frac{1}{2}\varrho, \beta\right) - \frac{2\pi}{\lambda} W\left(a - \frac{1}{2}\varrho, \beta\right)\right] da d\beta, \tag{6a}$$

$$C_i(\varrho, \theta) = C(0, 0)^{-1} \int_{-a}^{a} \int_{-b}^{b} \sin\left[\frac{2\pi}{\lambda} W\left(a + \frac{1}{2}\varrho, \beta\right) - \frac{2\pi}{\lambda} W\left(a - \frac{1}{2}\varrho, \beta\right)\right] da d\beta, \tag{6b}$$

where ϱ, θ are the spatial frequency variables, a and b are the limits of integration given by

$$a = \left(1 - \frac{1}{4}\varrho^2\right)^{1/2}, \quad b = (1-\beta^2)^{1/2} - \frac{\varrho}{2}, \tag{7}$$

and

$$W\left(\alpha + \frac{1}{2}\varrho, \beta\right) - W\left(\alpha - \frac{1}{2}\varrho, \beta\right) = 4\pi W_{31}\left[\frac{1}{8}\varrho^3 \sin\Phi + \right.$$
$$\left. + \frac{3}{2}\alpha^2\varrho\sin\theta + \frac{1}{2}\varrho\beta^2\sin\theta + \alpha\beta\varrho\cos\theta\right], \tag{8}$$

where W_{31} represents the aberration coefficient in units of wavelength for primary coma. Equations (6a) and (6b) can be evaluated with the help of equation (1).

One problem in this study is to decide upon the nature of correlation in the partially coherent wave due to atmospheric turbulence that is incident on the aperture. There exists a controversy as to the form of phase structure function due to turbulence. Som and Biswas [15] have discussed various models suggested in the literature and concluded that none of the models available is well established theoretically and experimentally. The phase structure function, that is normally assumed for a locally homogeneous isotropic and stationary atmoshpere, is that due to Tatarski [28] known as Tatarski's 5/3 power law. It has, however, been shown [15] that the square law may be used for the phase structure function at the risk of very small deviations from the results predicted by Tatarski's law. The use of the square law for the phase structure function leads to the Gaussian type of correlation function

$$\gamma(\varrho) = \exp(-a^2\varrho^2), \tag{9}$$

where $a = a'/L$ is the number of correlation intervals contained in the diffracting aperture of radius a'. For a circular source, the length of the correlation interval $L = \pi f'/\lambda r$, where f' is the focal length of the collimating lens, and r is radius of the illuminating source. For telescopic systems this is a close approximation to the actual time averaging correlation fluctuations due to random phase fluctuations in the wave propagating through the turbulent media.

Results and discussions

The intensity distribution and encircled energy were numerically evaluated using a 40 point Gaussian quadrature. The results so obtained have also been utilized to study the variation of peak intensity and two point resolution, as a increases from zero, in the presence of aberration.

The irradiance distributions were calculated along three azimuths, viz. $\Psi = 0$, $\pi/4$, and $\pi/2$, for the amounts of primary coma W_{31} equal to 0.5, 1.0, 1.5, and 2.0. Various values of the correlation interval a, viz. 0.0, 0.25, 0.50, 0.75, 1.0, 2.0, 3.0, and 5.0 were taken to cover the useful range of partial coherence.

Typical results of intensity distribution have been shown in figures 2–9. For comparative reasons the Airy distribution has been shown by

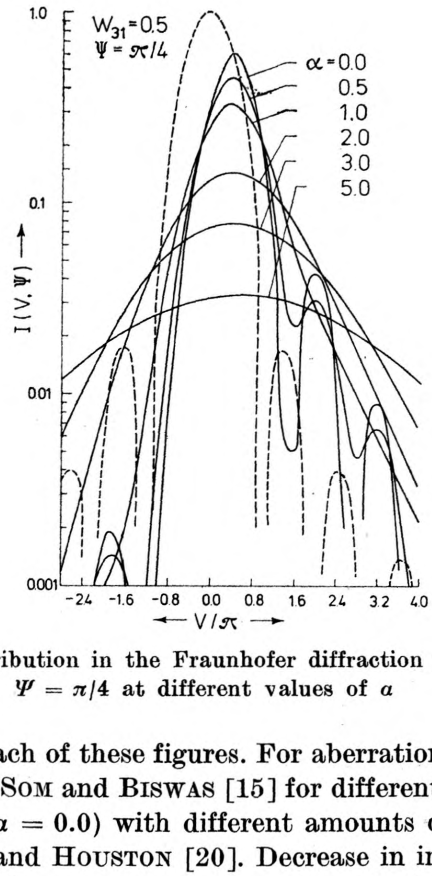

Fig. 2. Intensity distribution in the Fraunhofer diffraction patterns for $W_{31} = 0.5$, $\Psi = \pi/4$ at different values of a

the dotted line in each of these figures. For aberration-free case our results agree with those of Som and Biswas [15] for different values of a, whereas for coherent case ($a = 0.0$) with different amounts of aberration — with those of Barakat and Houston [20]. Decrease in intensity in the centre and broadening of the image in the presence of aberration are shown in figs. 2–9 for different values of a. It has been observed that in aberrated cases the maximum intensity does not occur at $V = 0$, but is displaced from the centre. However, there is no shift along zero azimuth (not shown in figures). The shift in the position of peak intensity increases when the measurement are taken along other azimuths, becoming maximum for $\Psi = \pi/2$. It is also interesting to note that for fixed values of Ψ and a, the peak intensity does not always increase with the increasing amount of aberration (say after $W_{31} = 1.0$). On the other hand, for a fixed amount of aberration but with varying a the shift initially remains almost constant but it increases when we tend towards the incoherent case (for $a > 1.0$).

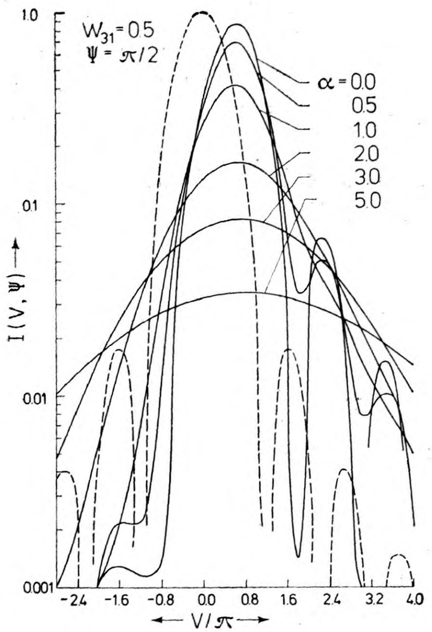

Fig. 3. Same as fig. 2 for $\Psi = \pi/2$

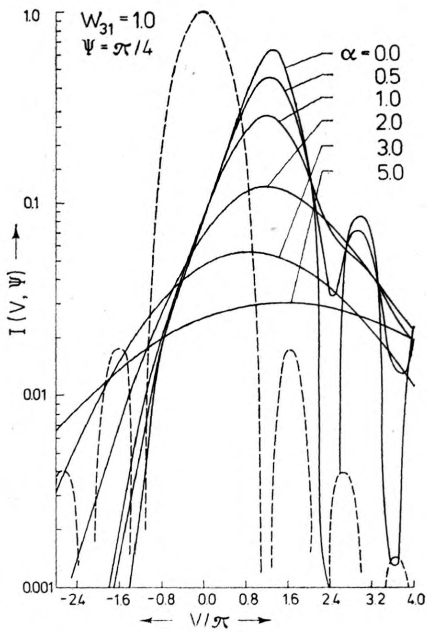

Fig. 4. Same as fig. 3 for $W_{31} = 1.0$, and $\Psi = \pi/4$

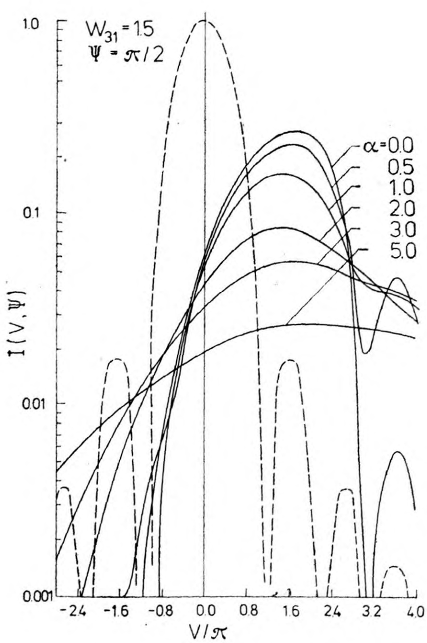

Fig. 5. Same as fig. 4 for $\Psi = \pi/2$

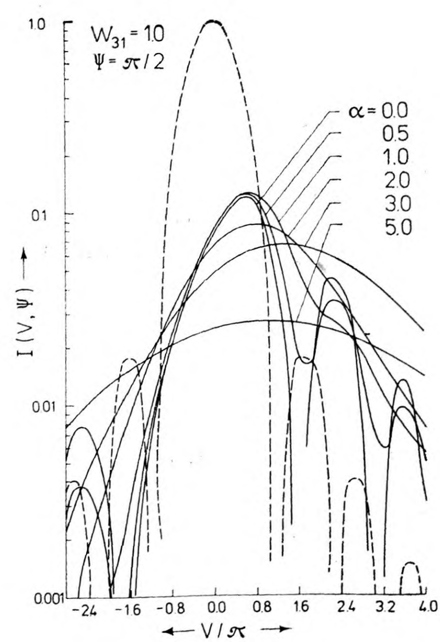

Fig. 6. Same as fig. 2 for $W_{31} = 1.5$, and $\Psi = \pi/4$

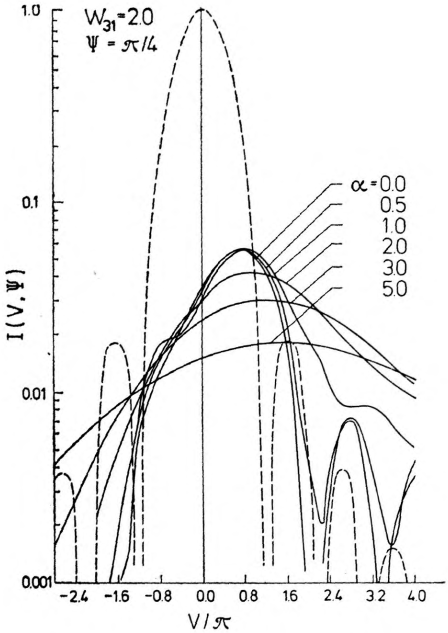

Fig. 7. Same as fig. 6 for $\Psi = \pi/2$

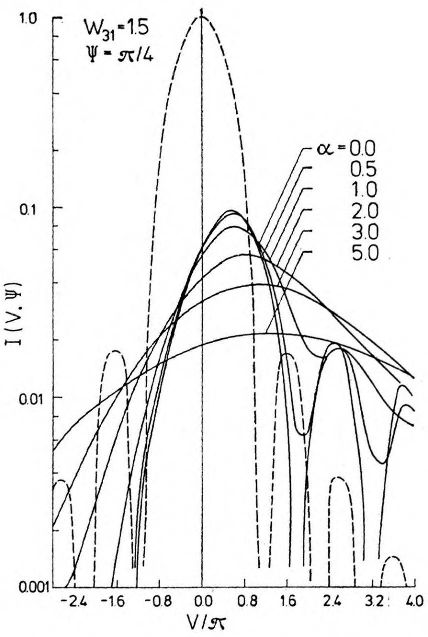

Fig. 8. Same as fig. 2 for $W_{31} = 2.0$, and $\Psi = \pi/4$

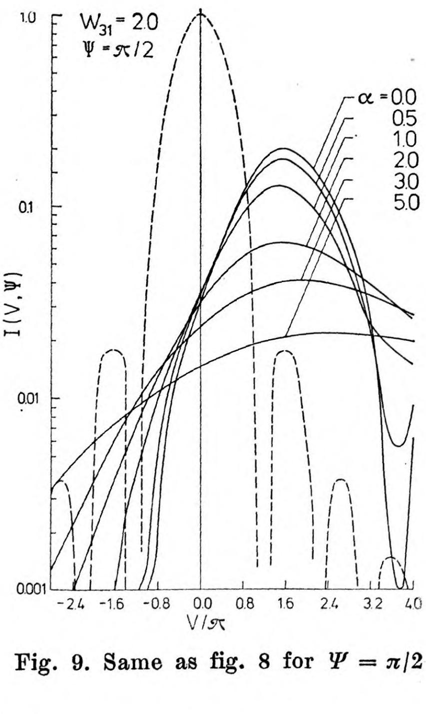

Fig. 9. Same as fig. 8 for $\Psi = \pi/2$

The results for encircled energy are shown graphically in figs. 10–13. The broken line in each of these figures represents the encircled energy distribution for the corresponding aberration free and coherently illuminated aperture. Our results for encircled energy in aberration free case agree with those of SOM and BISWAS [15] for different values of α. The

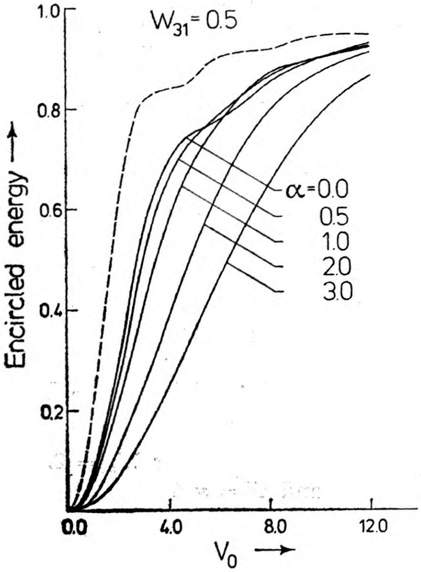

Fig. 10. Encircled energy for $W_{31} = 0.5$ at different values of α

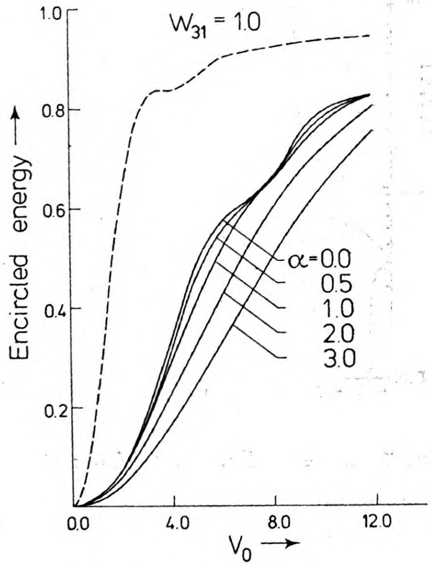

Fig. 11. Same as fig. 10 for $W_{31} = 1.0$

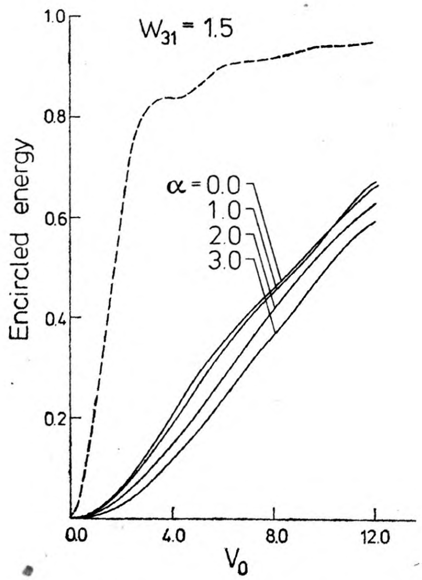

Fig. 12. Same as fig. 10 for $W_{31} = 1.5$

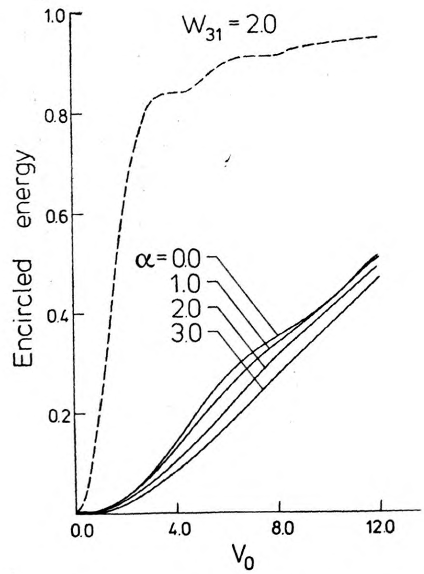

Fig. 13. Same as fig. 10 for $W_{31} = 2.0$

radius of the circle containing a given percentage of the total energy increases with the increasing amount of the aberration for the same state of coherence. It also increases with the loss of coherence for the same amount of aberration, as compared to that of aberration free case.

Figs. 14 and 15 show the decrease in peak intensity due to the lack

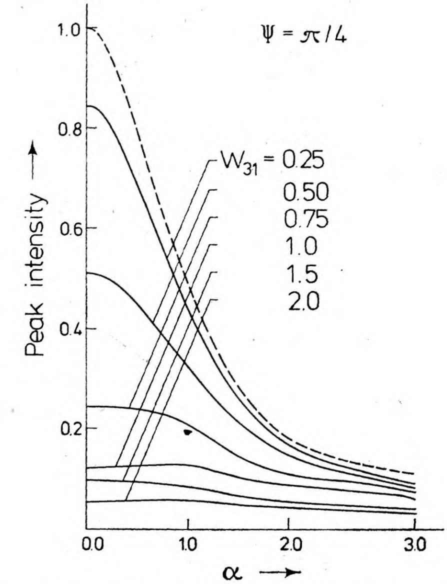

Fig. 14. Variation of peak intensity with a for different values of W_{31} and along $\Psi = \pi/4$

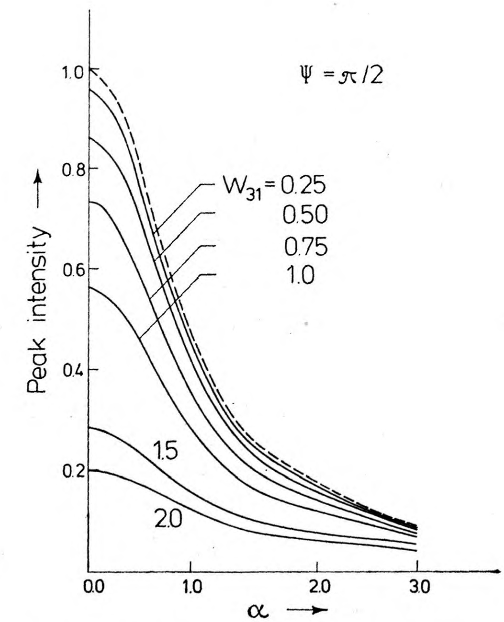

Fig. 15. Same as fig. 14 for $\Psi = \pi/2$

of perfect coherence in azimuths $\pi/4$ and $\pi/2$, respectively. The value of a which would produce pictures of questionable quality and those which would produce decidedly inferior quality can be determined.

Finally, we have studied the resolution capability of the optical system with circular aperture in the presence of third order coma with the loss of coherence. We have the Huber–Hopkins criterion [2] for this purpose because the other criteria, such as Rayleigh or the Sparrow one are not convenient to use since the intensity distribution is not circularly symmetric [29]. Huber–Hopkins criterion states that the resolving power of a lens is related to the radius of a circle which contains 25 percent of the total energy. We can take the angular resolution as twice the angular radius of the circle encircling 25 percent of the energy. The results are illustrated in fig. 16. Broken line represents the aberration free case and is consistent

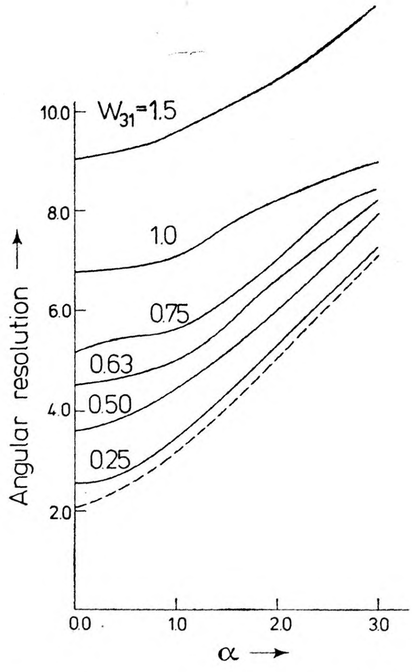

Fig. 16. Variation of angular resolution with a for different values of W_{31}

with that of SOM and BISWAS [15]. The decrease of the angular resolution with the increasing amount of aberration can be seen in figure 16, for the same state of coherence.

It is interesting to note that the expression (3) is analogous to the one which will be obtained by incoherent imaging of Gaussian source in an optical system with circular aperture suffering from primary coma. The finite width of Gaussian source gives rise to partially coherent illumination on the aperture.

Additional remarks

We would like to make a mention of a number of recent investigations that are being made on topics related to the Schell theorem in the analysis of radiometry and correlation properties of bounded planar sources [30], far-field coherence and radiant intensity of light scattered from liquid-crystals [31] and scattering from rough surfaces etc. [30, 32].

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Совместное влияние атмосферной турбулентности и первичной комы на дифракционный спектр поля, далекого от круговой диафрагмы

Проведены теоретические исследования диффузионного спектра Фраунгофера, образованного круговым отверстием, возмущенным первичной комой и освещенным частично когерентным светом при наличии атмосферной турбулентности. В качестве представительного для атмосферной турбулентности принят гауссов вид корреляции. Численные результаты, полученные для распределения интенсивности и энергии, изображены графически. Приведены результаты для двухточечной разрешающей способности, основанной на критерии Губера-Гопкинса, и вычерчены изменения пиковой интенсивности как функции интервала когерентности.