Optica Applicata VIII/3, 1978

K. Prabhakar Rao*, P. K. Mondal**

Straubel apodisation filters in coherent imagery of slit objects

Apodisation is known to reduce the fringe structures in the coherently illuminated images of straight edges and slits. This paper discusses the effects of apodisation achieved with Straubel apodisation filters on the coherent imagery of slit objects.

1. Introduction

A brief account of the importance of coherent imagery has been given in earlier papers [1, 5]. The study of imaging in coherent illumination has been carried out by BARAKAT [4] in the case of defocussed images of illuminated bars with Airy type of pupils. Studies pertaining to the effects of apodisation on the images of slit objects have been reported by GUPTA [5], and GRUBER and THOMPSON [6]. Gupta has considered the apodisation by two pupil functions

$$f(r) = \beta r^2, \tag{1}$$

and

$$f(r) = 1-\beta r^2. \tag{2}$$

He has discussed the imagery with these filters using also central obscuration. GRUBER and THOMPSON [6] have investigated the effects of Gaussian apodising pupil function on the images of slit objects. In this paper we propose to investigate the image intensity distributions of slit objects with Straubel apodisation pupils in circular apertures.

Straubel apodisation pupils have been studied earlier [1, 3] with reference to their imaging characteristics under different illumination conditions. These filters are described by a set of rotationally symmetric pupil functions

$$f(r) = (1-r^2)^P, \tag{3}$$

where P is the apodisation parameter, which for $P = 0$ gives the Airy pupil; r — as usual — is the normalised distance of a point in the aperture from its centre.

* College of Technology, Osmania University, Hyderabad, India.

** Department of Physics, Osmania University, Hyderabad, India.

2. Theory

The half width of the clear slit being represented by u_0, the two dimensional distributions of amplitude transmission of the slit can be expressed mathematically as:

$$A(u, v)\begin{cases} = 1 & \text{for} \quad |u| \leqslant u_0 \\ = 0 & \text{for} \quad |u| > u_0. \end{cases} \tag{4}$$

It is well known that the coherent imaging systems are linear with respect to amplitude distributions rather than to intensity distributions. Or, in other words, the object amplitude distributions are linearly related to the image amplitude distributions. The image amplitude distributions are obtained using the Fourier transform methods.

The object amplitude spectrum on the entrance pupil can be written as:

$$a(x, y) = \delta(y)\frac{\sin(2\pi u_0 x)}{\pi x}, \tag{5}$$

which is now modified by the use of Straubel apodising filters giving the modified spectrum at the exit pupil as

$$a'(x, y) = f(r)\cdot a(x, y) = [1-(x^2+y^2)]^P a(x, y). \tag{6}$$

The image amplitude distribution is the inverse Fourier transform of the expression (6), i.e.

$$A'(u', v') = \iint_{-\infty}^{\infty} \delta(y)[1-(x^2+y^2)]^P \frac{\sin(2\pi u_0 x)}{\pi x} \exp[2\pi i(u'x+v'y)]dxdy. \tag{7}$$

The limits of the integral are determined by the aperture radius. Simplification of the expression (7), leads us to the following result:

$$A'(u', v') = \frac{1}{\pi}\left\{\int_0^1 \frac{(1-x^2)^P}{x} \sin[2\pi(u_0+u')x]dx + \right.$$

$$\left. + \int_0^1 \frac{(1-x^2)^P}{x} \sin[2\pi(u_0-u')x]dx\right\}. \quad (8)$$

In the case of Airy pupils the expression finally is reduced to

$$A'(u', v') = \frac{1}{\pi}[\mathrm{si}\, 2\pi(u_0+u') + \mathrm{si}\, 2\pi(u_0-u')], \quad (9)$$

where $\mathrm{si}(x)$ is the sine integral of argument x.

3. Results and discussions

We now present the results obtained with Straubel apodisation filters. The intensity distributions in the images of a slit object of various widths ($2u_0$) have been shown in the figures 1 to 7. In these figures the numbers along the curves indicate the values of

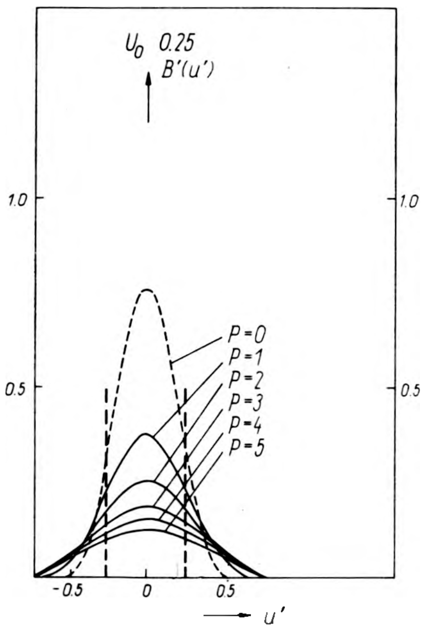

Fig. 1. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 0.25$

the apodisation parameter P. The curves corresponding to the unapodised Airy pupils are plotted as dashed lines. The vertical dashed lines show the exact geometric position of the edges of the slit object. It has been formed that for low values of the slit width the intensity within the geometrical image is maximum at the centre of the pattern. There is no central dip, the intensity gradually decreases and

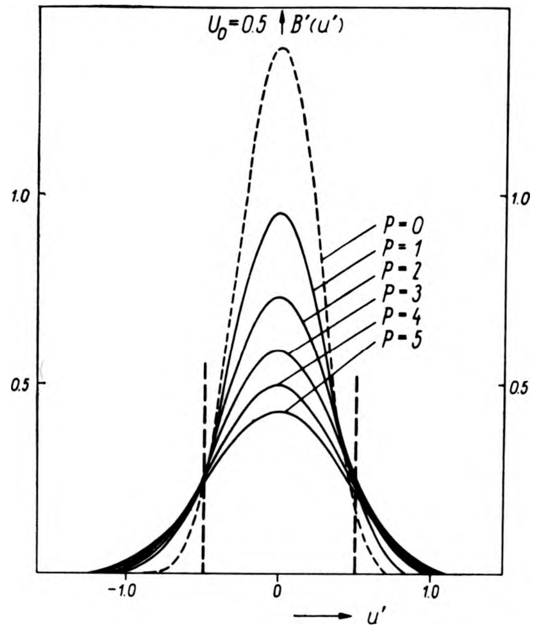

Fig. 2. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 0.5$

then spreads into the geometrical shadows on both sides of the central pattern. The amount of light flux in the geometric shadow is rather high, and for larger values of P is higher than that of the unapodised system corresponding to $P = 0$. This takes place at the cost of the central intensity whose value decreases considerably. For the increasing value P one observes a steady loss of energy passing through the centre and a considerable loss of energy throughout n the system. Furthermore, we notice that the value

Table 1

P	0	1	2	3	4	5
u_0	0.5	0.75	1.0	1.0 1.25	1.25	1.5

of P corresponding to the maximum central intensity varies with the value of u_0. This variation is shown in table 1. Thus the effects of the apodisation parameter P are very much pronounced for low values of the slit width.

The increase in slit widths is accompanied with the considerable measure of the energy contents within the geometrical images, and with a consequent reduction in the amount of light flux in the geometric shadow. Furthermore, there appear fringe structures within the geometrical image corresponding to $P = 0$. For other values of P, these structures are suppressed due to apodisation. The lowest values of P for which the effects of fringing in the images of slit are complete-

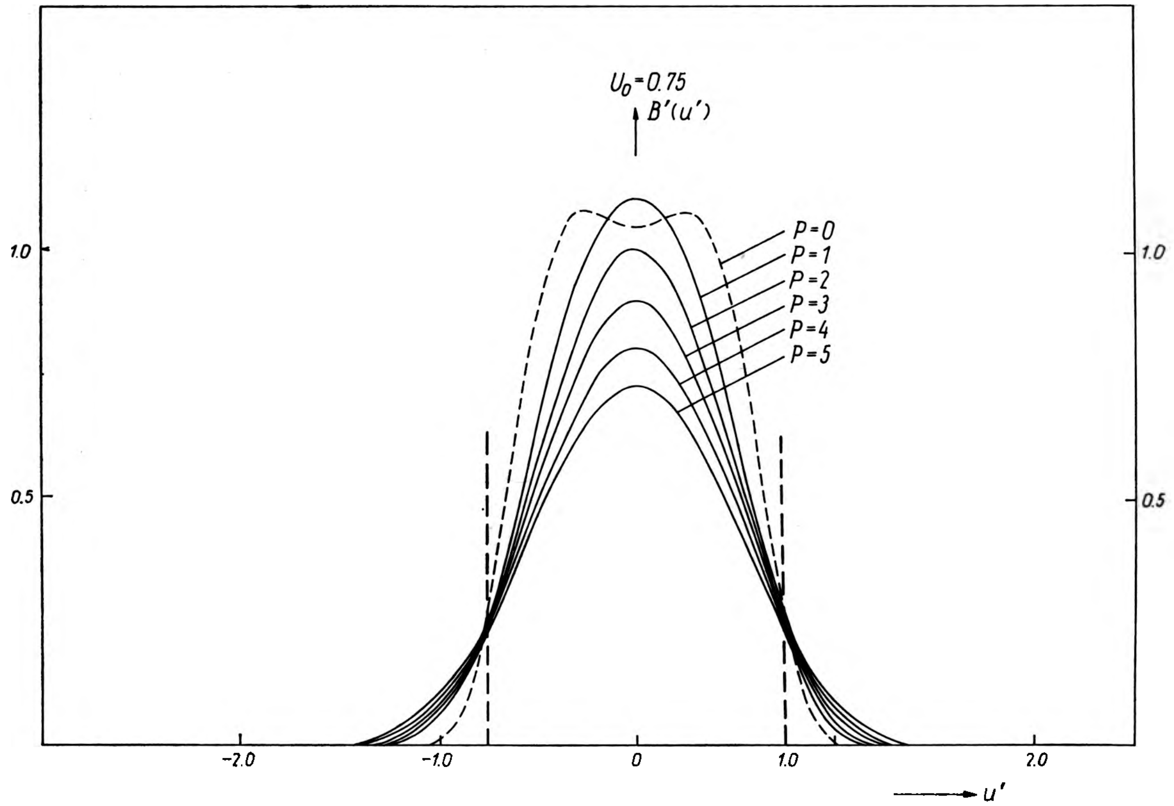

Fig. 3. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 0.75$

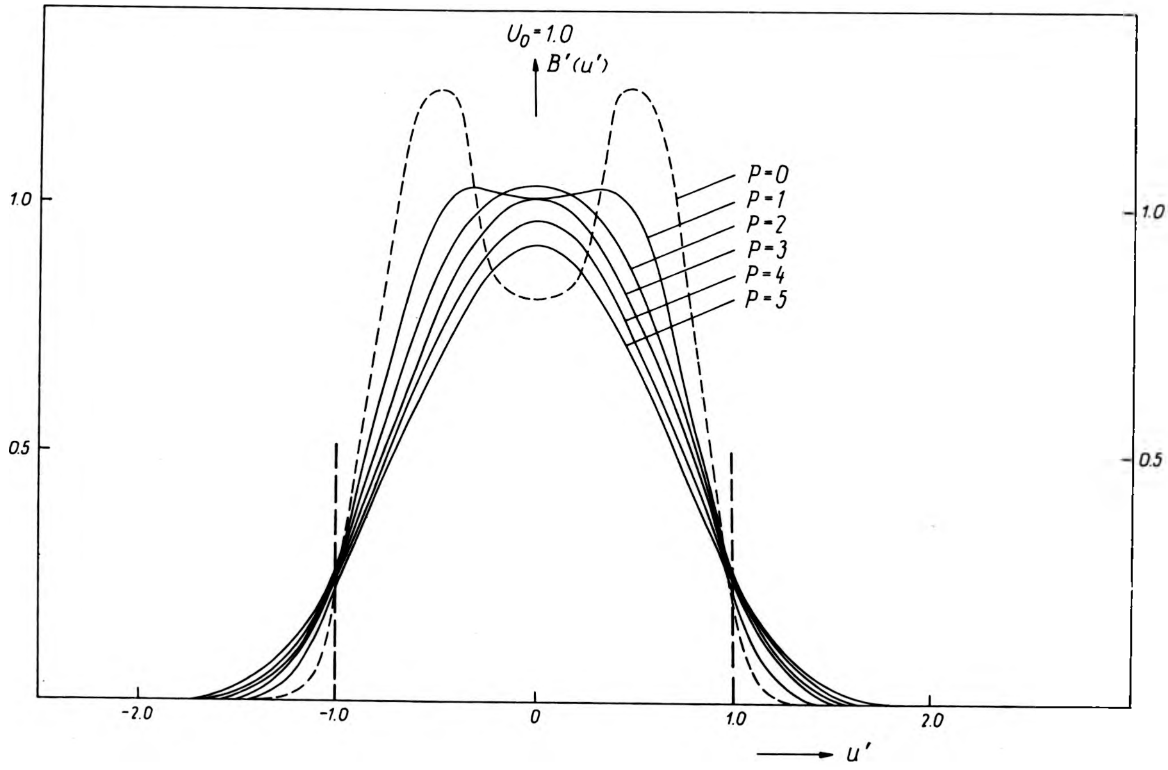

Fig. 4. Image intensity distribution for slits in coherent light $B'(u')$ versus u'; $u_0 = 1.0$

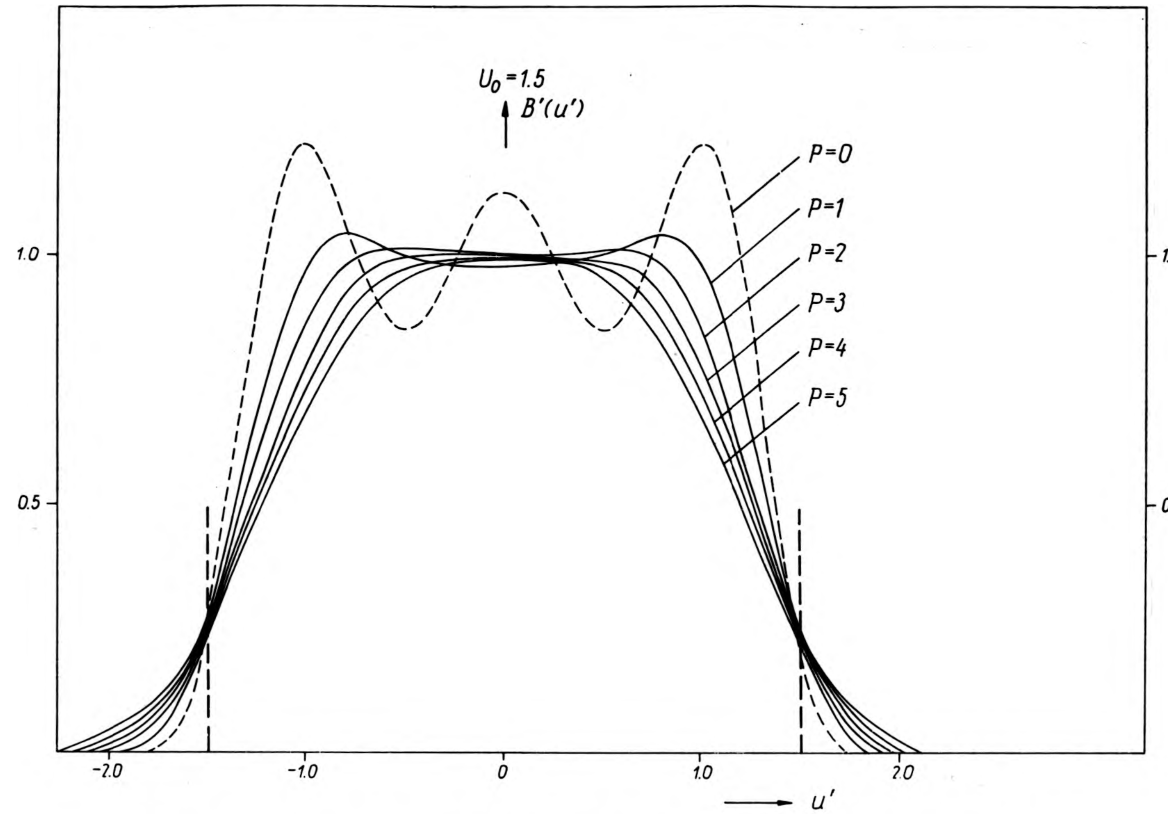

Fig. 5. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 1.5$

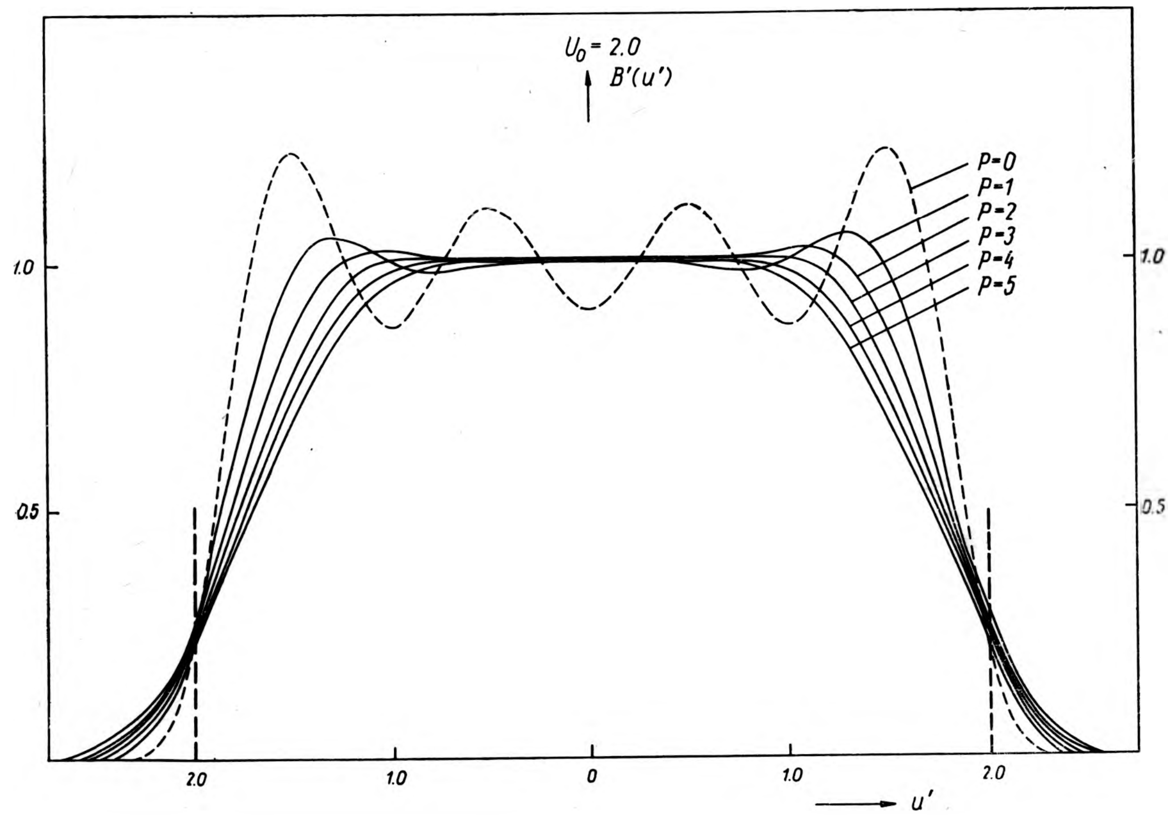

Fig. 6. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 2.0$

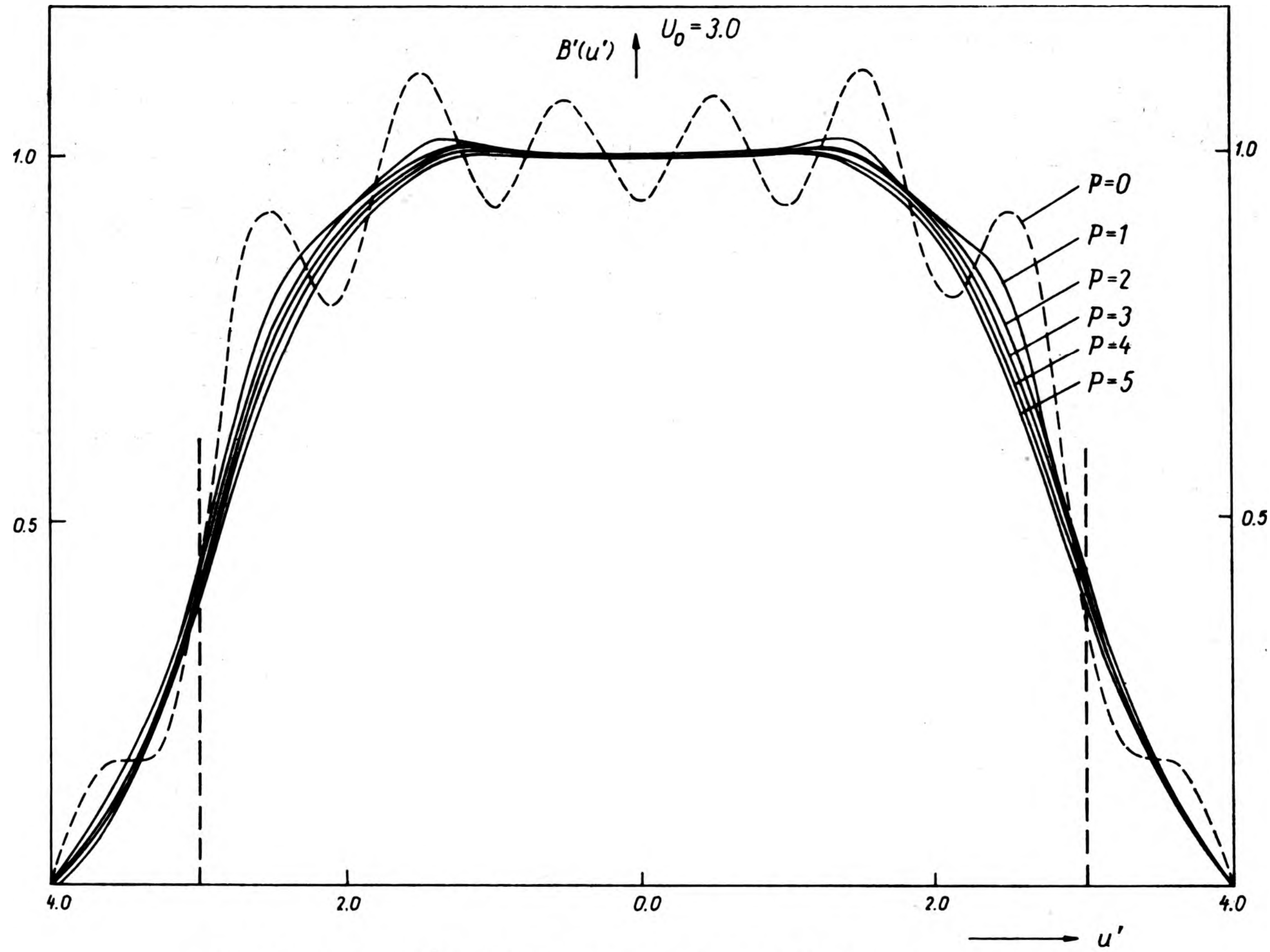

Fig. 7. Image intensity distributions for slits in coherent light $B'(u')$ versus u'; $u_0 = 3.0$

ly eliminated have been listed in table 2 for increasing slit widths. For Airy type of pupils, the number of fringes within the geometrical image is found to be more grater, the higher is the value of u_0. As a rule of thumb

Table 2

u_0	0.75	1.0	1.25	1.5
P	1	2	3	3

we find that the number of fringes which appear in the geometric image is approximately twice the value of u_0 up to $u_0 = 3.0$. The central fringe oscillates between a maximum and minimum depending on the value of the slit width $2u_0$. It is also interesting to note that the first maximum from either edge occurs at 0.5 unit for all values of the slit widths. For $u_0 > 0.5$, the fringes are equidistant. However, the contrast is not the same for all the fringes; for lower values of u_0, the contrast of the first fringe is from either edge greater than that of the others in the central region. For $u_0 \geqslant 3.0$, we find that the contrast of the first fringe drops and that of the next one increases.

Let us now examine more closely these curves in the neighbourhood of the geometric edge. For the values of u_0 from 0.5 to 2.0 all the curves pass through the intensity point 0.25 at the geometric edge. If we consider the edge shift and edge gradient for different values of P and take into account those slit widths for which the intensity is 0.25 at the geometric edge of the image, then $u_0 = 0.75$, and $u_0 = 1.5$ seem to have the minimum edge shifting for Airy pupil. In each case the edge gradients decrease with the increasing P of images of straight edges, and for $P = 5$ the lowest values of edge gradient occur for $u_0 = 0.75$ and 1.25. Considering the spread of the curves for P ranging from 0 to 5 at the half intensity point, it is observed that this spread is more or less the same for all values of u_0 from 0.75 to 2.0. This is also the case when these spreads for $P = 1$ to 5 are considered. A filter with the pupil function [5] $f(r) = 1-\beta r^2$ with $\beta = 1$ will be the same as a Straubel filter with $P = 1$. The results of this study fits agreable into the present work.

Acknowledgements

One of the authors, K. P. Rao, wishes to thank Prof. R. Kaparthi, of the College of Technology O. U. for providing facilities for this work.

Аподизационные фильтры Штраубеля в когерентном отображении щелевых объектов

Аподизация является известной техникой уменьшения структуры полосок в когерентно освещенных изображениях прямых ребер или щелей. В работе обсуждены аподизационные эффекты, полученные с помощью аподизационных фильтров Штраубеля в когерентном отображении щелевых объектов.

References

[1] Rao K. P., Mondal P. K., Seshagiri Rao T., Optik (to be published).

[2] Rao K. P., Mondal P. K., Optica Pura y Applicada **8**, 37 (1975).

[3] Rao K. P., Mondal P. K., Atti della Fondazione Giorgio Ronchi **30**, 355 (1975).

[4] Barakat R., Opt. Acta **16**, 205 (1969).

[5] Gupta S. S., Indian J. of Pure and Appl. Physics **12**, 133 (1974).

[6] Gruber L. S., Thompson B. J., Opt. Eng. **13**, 451 (1974).

Received, December 13, 1977