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Effect of a triangle stria in the optical system on the intensity distribution in the diffraction-limited image of a point

In the paper the influence of the triangle stria in a optical system on the intensity distribution in a diffraction limited image of a point-object has been analysed. The respective numerical calculations have been illustrated by graphs.

1. Introduction

In this paper we have examined the action of the striae in ideal (aberration-free) optical systems. Although in some papers the influence of the striae on the image quality of real (aberrated) optical systems was analysed [2, 6] it seems that this problem discussed for ideal optical systems may render cognitive results, which may be next exploited to tolerancing the striae in well-corrected optical systems (such as high class telescope objectives, laser beam collimating objectives and so on).

2. Theory

Let us assume that there exists a single stria in an optical system pupil. The position of this stria and its size may be quite arbitrary. Suppose, for the sake of convenience, that the Y-axis of the coordinate system in the stria plane is parallel to the stria. Let x_0, y_0 denote the stria centre coordinate, $2h$ — the stria length, and $2b$ — its width (see fig. 1). The coordinates in the image plane are denoted by ξ, η, whereby

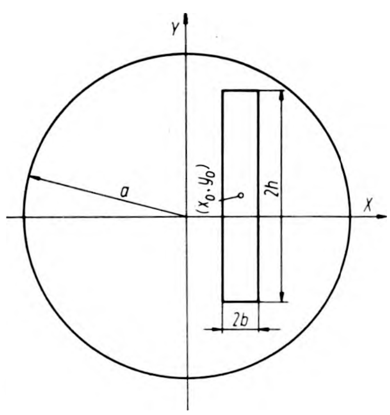

Fig. 1. The stria position in the exit pupil of the optical system: a — pupil radius, $2b$ — stria width, x_0, y_0 — the stria centre coordinates

$$\xi = q\cos\psi,$$

$$\eta = q\sin\psi$$

for

$$q = \frac{ka\varrho}{R}, \text{ and } k = \frac{2\pi}{\lambda},$$

where

λ — wavelength of the light,
R — radius of the Gaussian sphere,
ϱ — radius (dimensional) in the imaging plane,
a — radius of the exit pupil.

Amplitude distribution $U(q, \psi)$ in the diffraction image of a point-object is, apart from a multiplicative constant, a Fourier transform of the pupil function [1]. In particular, for the Gaussian plane we have

$$U(q,\psi) = C\int\int_{-\infty}^{\infty}\exp\left(iq\left(\frac{x\cos\psi+y\sin\psi}{a}\right)\right)\times$$
$$\times\exp\left(-ikV(x,y)\right)dxdy, \tag{1}$$

where

$V(x, y)$ — wave aberrations of the system.

The intensity distribution in the diffractional image of the object-point $G(q, \psi)$ is a squared modulus of $U(q, \psi)$, i.e.

$$G(q,\psi) = |U(q,\psi)|^2.$$

In this work (like in [3]) the calculations of the function $U(q, \psi)$ has been carried out in three stages, namely:

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1. The complex amplitude $U_1(q, \psi)$ from clear aperture ($V(x, y) = 0$).

2. The complex amplitude $U_2(q, \psi)$ from that part of the pupil which contains the stria assuming that there exist no wavefront deformation in this region (i. e. $V(x, y) = 0$).

3. The complex amplitude $U_3(q, \psi)$ generated by stria at presence of wavefront deformation in the stria region ($V(x, y) = 0$).

Thus, the sought value of the complex amplitude in the image plane is

$$U(q, \psi) = U_1(q, \psi) - U_2(q, \psi) + U_3(q, \psi).$$

As it may be easily shown, $U_1(q, \psi)$ after normalizing (comp. [1]) is

$$U_1(q, \psi) = U_1(q) = \frac{2J_1(q)}{q}, \tag{2}$$

where $J_1(q)$ — Bessel function of the first kind and the first order. $U_2(q, \psi)$ is given by

$$U_2(q, \psi) = U_2' e^{i\varepsilon}, \tag{3}$$

where

$$U_2' = \frac{\sigma_s}{\sigma} \operatorname{sinc}\left(\frac{h}{a} q \sin\psi\right) \operatorname{sinc}\left(\frac{b}{a} q \cos\psi\right),$$

$\sigma_s = 2b\,2h$ — stria area,

$\sigma = \pi a^2$ — pupil area,

$\varepsilon = 2\pi(x_0\xi + y_0\eta)$,

x_0, y_0 — coordinates of the stria centre,

$$\operatorname{sinc} x = \begin{cases} 1 \text{ for } x = 0, \\ \dfrac{\sin x}{x} \text{ for } x \neq 0. \end{cases}$$

The value of $U_3(q, \psi)$ depends on the assumed shape wavefront deformation in the stria region. For striae discussed in [3] the respective wavefront deformation was defined as follows:

a) $$V(x, y) = V_0,$$

b) $$V(x, y) = V_0\left[\left(\frac{x^2-b^2}{b^2}\right)^2 + \left(\frac{y^2-h^2}{h^2}\right)^2 + \frac{x^2y^2}{b^2h^2} - 1\right],$$

c) $$V(x, y) = V_0\left(\frac{x^2-b^2}{b^2}\right)^2.$$

The case a) presents analytic expressions for $U_3(q, \psi)$, and $G(q, \psi)$, while the cases b) and c) have been illustrated by graphs obtained from numerical calculations.

As is follows from [4] and our own observations the stria occurring in the glass cause most frequently a triangular deformation of the wavefront. Therefore, in further considerations we have assumed that the wavefront deformation due to striae has the form

$$V(x, y) = V_0 - \frac{V_0}{b}|x - x_0| \tag{4}$$

for

$$x_0 - b \leqslant x < x_0 + b.$$

After introducing the expression (4) to the formula (1) the following formula for $U_3(q, \psi)$ has been obtained

$$U_3(q, \psi) = U_3'(q, \psi)(Ai + B)e^{i\varepsilon}, \tag{5}$$

where

$$U_3' = \frac{\sigma_s}{\sigma} \operatorname{sinc}\left(\frac{q \sin\psi}{a} h\right),$$

$$A = \frac{kV_0\left(\cos kV_0 - \cos\dfrac{q\cos\psi}{a} b\right)}{(kV_0)^2 - \left(\dfrac{q\cos\psi}{a}\right)^2},$$

$$B = \frac{kV_0 \sin kV_0 - \dfrac{bq\cos\psi}{a} \sin\dfrac{bq\cos\psi}{a}}{(kV_0)^2 - \left(\dfrac{q\cos\psi}{a}\right)^2}.$$

The resulting amplitude $U(q, \psi)$ is thus

$$U(q, \psi) = U_1(q, \psi) - U_2'(q, \psi)e^{i\varepsilon} + U_3'(q, \psi)e^{i\varepsilon}(Ai + B).$$

Hence,

$$G(q, \psi) = U_1^2 + U_2'^2 + U_3'^2(A^2 + B^2) - 2U_1 U_2' \cos\varepsilon + 2U_1 U_3'(B\cos\varepsilon - A\sin\varepsilon) - 2BU_2' U_3'. \tag{6}$$

In order to determine the Strehl definition I we must put $q = 0$ in formulas (2), (3), and (5). Then

$$U_1 = 1,$$

$$U_2' = \frac{\sigma_s}{\sigma},$$

$$U_3' = \frac{\sigma_s}{\sigma},$$

$$A = \frac{1}{kV_0}(\cos kV_0 - 1),$$

$$B = \frac{1}{kV_0} \sin kV_0.$$

By inserting the above values to the formula (6) the Strehl definition takes the form:

$$I = G(0, \psi) = \frac{1}{\sigma}\left[(\sigma-\sigma_s)^2 - 2\sigma\sigma_s\left(1-\frac{\sigma_s}{\sigma}\right)\operatorname{sinc} K V_0 + \right.$$

$$\left. + \sigma_s^2 \operatorname{sinc}^2 \frac{kV_0}{2}\right].$$

The above formula is a particular case of the formula for many stria case, derived in [5].

3. Concluding remarks

The intensity distribution $G(q, \psi)$ has been calculated numerically from the formula (6). The results obtained are shown graphically in figs. 2,3, and 4.

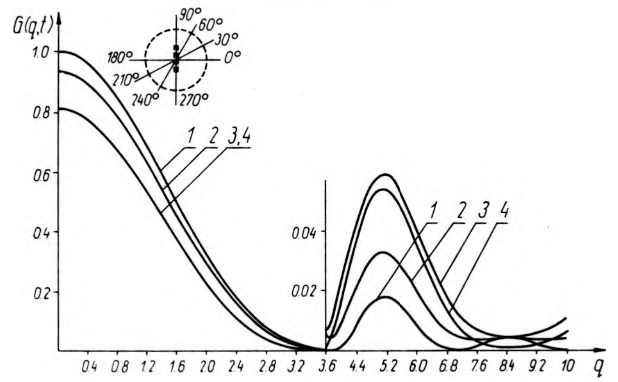

Fig. 2. The graph of the intensity distribution for the stria located at the pupil centre: $y_0 = 0$, V_0 — considered as a parameter

$1 - V_0 = 0$; $2 - V_0 = 0.25\lambda$, $x_0 = 0$; $\psi = 0°$; $3 - V_0 = 0.50\lambda$, $x_0 = 0$, $\psi = 0°$; $4 - V_0 = 1.0\lambda$, $x_0 = 0$, $\psi = 0°$

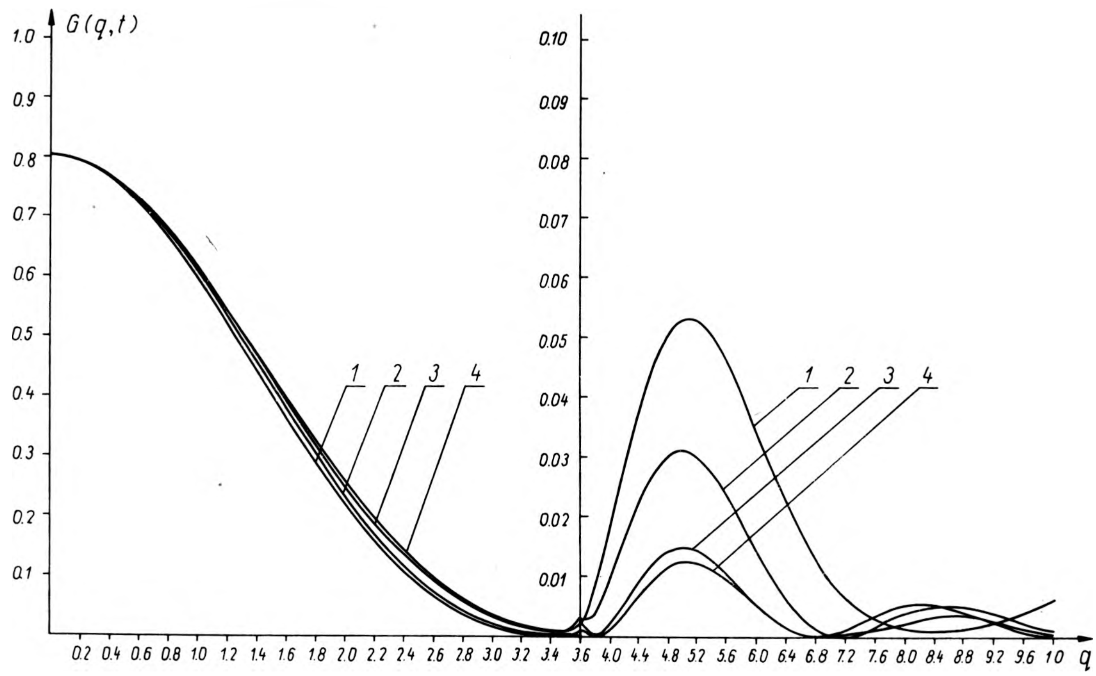

Fig. 3. The graph of the intensity distribution for stria located in the pupil centre: $V_0 = 1.0\lambda$, ψ — considered as a parameter

$1 - x_0 = 0$, $V_0 = 1\lambda$, $\psi = 0°$; $2 - x_0 = 0$, $V_0 = 1\lambda$, $\psi = 30°$; $3 - x_0 = 0$, $V_0 = 1\lambda$, $\psi = 60°$; $4 - x_0 = 0$, $V_0 = 1\lambda$, $\psi = 90°$

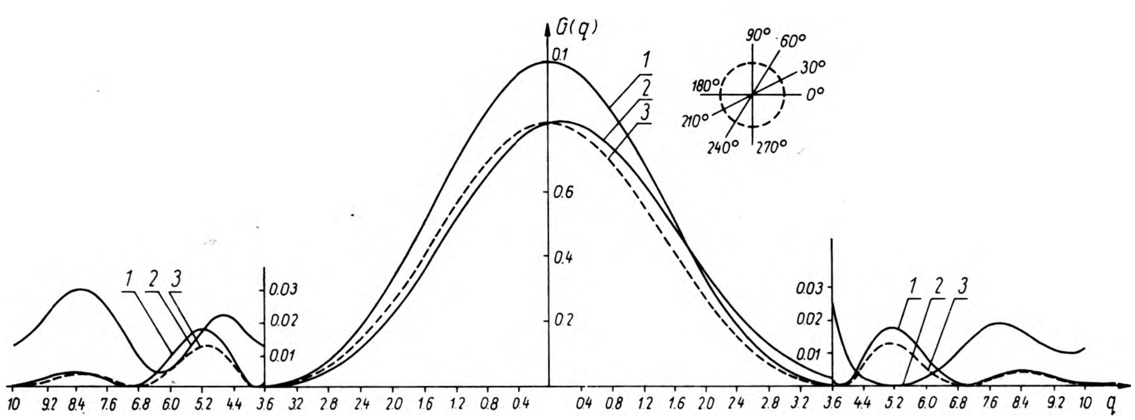

Fig. 4. The graph of the intensity distribution for stria located outside the pupil centre: $x_0 = 0.5$, $V_0 = 0.5\lambda$, ψ — considered as a parameter

$1 - V_0 = 0$; $2 - V_0 = 0.5\lambda$, $x_0 = 0.5$, $\psi = 180°$; $1 - V_0 = 0$; $2 - V_0 = 0.5\lambda$, $x_0 = 0.5$, $\psi = 0°$; $3 - V_0 = 0.5\lambda$, $x_0 = 0.5$, $\psi = 30°$ $3 - V_0 = 0.5\lambda$, $x_0 = 0.5$, $\psi = 270°$

From fig. 2 it follows that as V_0 increases, the intensity in the middle of the diffraction spot (Strehl definition) decreases, while the intensity in the secondary maxima increases. It is characteristic that the action of stria causing maximal deformation $V_0 = 0.5\lambda$ is similar to that evoking stria $V_0 = 1.0\lambda$.

The intensity distribution in the diffraction image of a point-object is shown in fig. 3 for a stria located in the middle of the pupil for different ψ. From the graph it follows that the intensity distribution is disturbed most strongly in the direction perpendicular to the stria.

For a stria located outside the pupil centre an asymmetry of the diffraction spot is observed (fig. 4).

Влияние треугольной полосы в оптической системе на распределение интенсивности в дифракционном изображении точки

В работе определено влияние треугольной полосы в оптической системе на распределение интенсивности в дифракционном изображении. Численные расчеты проиллюстрированы графиками.

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