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Influence of the striae on the image quality in the ideal optical system

In this paper the effects of triangle and sinusoidal striae in ideal optical systems on the Strehl definition and optical transfer function are discussed. Some of the obtained relations are illustrated by graphs.

1. Introduction

This work is a part of a more general problem of influence of material defects on the imaging quality.

In the literature there appeared several papers concerning the effects induced by striae in ideal optical systems on the selected measures of imaging quality. Among others:

Papers [2,7] delt with the influence of rectangular stria, striae determined by forth-order polynomials and disk-shaped striae on the intensity distribution in the diffraction point. Papers [3, 4] were devoted to analysis of the effect of rectangular stria upon the optical transfer function. Papers [3,7] discussed the influence of rectangular stria and disk-like-stria on the Strehl definition.

The wavefront deformation types due to striae assumed in those papers do not reflect the reality. As it has been noted in the paper [3], and as it follows from our own observations, the typical wavefront deformation due to striae is of triangle type. Therefore, the present paper is devoted to the effect of triangle striae on Strehl definition and optical transfer function.

The papers on the influence of striae in aberrated systems on the Strehl definition have been published since 1975. The first paper which treated the influence of rectangular striae on the Strehl definition in aberrated systems was [1]. For triangle striae the same problem was solved in [6]. In the works, done at the Institute of Physics, Technical University of Wrocław, the influence of material defects upon the imaging quality of optical systems has been examined in a complex way, as they deal with striae, bubbles and birefringence [5] occurring in the real systems. As it follows from our last works (prepared for publication), the results obtained below for the ideal systems appear as particular terms in the formulae determining the quality of the real systems. Therefore they can be used (without further transformations) as ready elements for evaluation of the quality of real optical systems.

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2. Effects due to striae appearing in ideal optical systems on the Strehl definition

The Strehl definition is given by:

$$I = \frac{G(0,0)}{G_0(0,0)} = \frac{U(0,0)U^*(0,0)}{U_0(0,0)U_0^*(0,0)}, \tag{1}$$

where

$G(0,0)$ — intensity in the middle of the diffraction spot produced by the aberrated system,

$G_0(0,0)$ — intensity in the middle of the diffraction spot produced by the ideal system.

By substituting the expression for the complex amplitude $U(0,0)$ in the middle of the diffraction spot into (1), we obtain:

$$I = \frac{\iint\limits_{-\infty}^{+\infty} \exp\big(jkV(x,y)\big)dxdy \iint\limits_{-\infty}^{+\infty} \exp\big(-jkV(x,y)\big)dxdy}{S^2}, \tag{2}$$

where

k — wave number,

$V(x,y)$ — wave aberration connected with a stria located in the exit pupil plane,

x, y — normalized coordinates in the exit pupil plane,

$j = \sqrt{-1}$,

S — normalized area of the exit pupil.

From the formula (2) it can be concluded that the value of the Strehl definition does not depend upon the stria shift in the exit pupil plane.

Since the jacobian of transformation connected with the stria rotation is equal to 1, the Strehl definition does not depend upon the stria direction being related to the coordinate system associated with the exit pupil.

If there are N striae of different non-intersecting areas σ_{S_i} in the exit pupil of the optical system (fig. 1),

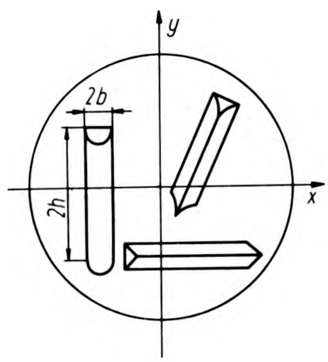

Fig. 1. An auxilliary figure showing the striae orientation in the pupil of the optical system, helpful in deriving the formula for Strehl definition

and the deformations of the wavefront $V_i(x, y)$ due to these striae differ in shape and value, then employing the method proposed by Keller [2] the formulae may be transformed into the form:

$$I = \left(1-\frac{\sigma_S}{S}\right)^2 + \frac{S-\sigma_S}{S^2}\left(\sum_{i=1}^{N}\iint_{\sigma_{S_i}} \exp\left(jkV_i(x, y)\right) dS + \sum_{i=1}^{N}\iint_{\sigma_{S_i}} \exp\left(-jkV_i(x, y)\right) dS\right) + \frac{1}{S^2}\left(\sum_{i=1}^{N}\iint_{\sigma_{S_i}} \exp\left(jkV_i(x, y)\right) dS\right) \times \left(\sum_{i=1}^{N}\iint_{\sigma_{S_i}} \exp\left(-jkV_i(x, y)\right) dS\right). \tag{3}$$

Hence, in order determine the value of the Strehl definition the wavefront deformation $V_i(x, y)$ caused by the stria must be also determined.

2.1. The influence of the striae, producing triangle wavefront deformation, on the Strehl definition

The real stria produces a triangle wavefront deformation, which may be described by formula:

$$V_i(x, y) = V_{0_i} - \frac{V_{0_i}}{b}|x - x_{0_i}|, \tag{4}$$

$$x_{0_i} - b \leqslant x \leqslant x_{0_i} + b,$$

where V_{0_i} — maximal value of wavefront deformation in the stria,

b — half-width of the stria,

x_{0_i} — x-coordinate of the stria centre.

After inserting (4) into (3) the Strehl definition for an optical system containing many triangle striae is given by the formula

$$I = \left(1 - \sum_{i=1}^{N} A_i\right)^2 + \left(\sum_{i=1}^{N} B_i\right)^2, \tag{5}$$

where

$$A_i = \frac{\sigma_{S_i}}{S}(1 - \mathrm{sinc}\, kV_{0_i}), \tag{6}$$

$$B_i = \frac{\sigma_{S_i}}{S}\frac{kV_{0_i}}{2}\mathrm{sinc}^2\frac{kV_{0_i}}{2}. \tag{7}$$

Practically two cases may appear:

1. There are few striae in the exit pupil of an optical system ($\sigma_S \ll S$), and the formula (5) can be simplified to the form:

$$I = 1 - 2\sum_{i=1}^{N} A_i. \tag{8}$$

2. There are many striae in the exit pupil of the optical system, that may occupy an arbitrary area.

This case concerns the striae being undiscovered by the applied glass controll technique. Therefore it is assumed that all the undiscovered striae cause the same maximal wavefront deformation V_0 equal to the limiting value discoverable by the given method. This assumption allows to reduce the formula (5) to the form:

$$I = (1-A)^2 + B^2, \tag{9}$$

where

$$A = \frac{\sigma_S}{S}(1 - \mathrm{sinc}\, kV_0), \tag{10}$$

$$B = \frac{\sigma_S}{S}\frac{kV_0}{2}\mathrm{sinc}^2\frac{kV_0}{2}. \tag{11}$$

The influence of maximal wavefront deformation due to striae on the Strehl definition for fixed relative area σ_S/S (fig. 2) has been examined together with relation $I=f(\sigma_S/S)$ for fixed maximal wavefront deformation on the striae (fig. 3). From fig. 2 it follows that for an arbitrary relative area the striae causing the maximal wavefront deformation of about 0.7 λ are most disadvantageous.

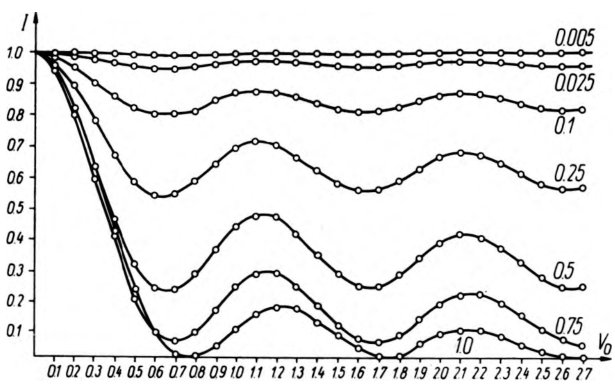

Fig. 2. Strehl definition vs. the relative surface of the striae at fixed wavefront deformation value, for triangle striae

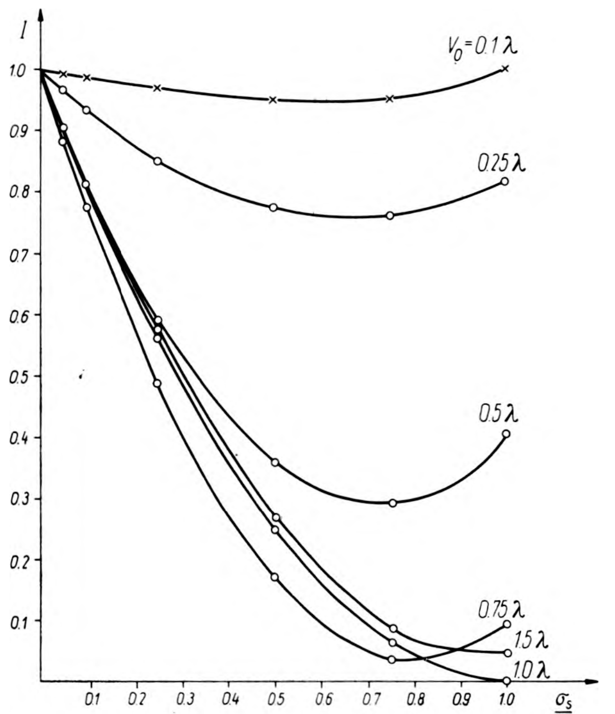

Fig 3 Strehl definition vs. the maximal wavefront deformation due to striae at fixed relative striae area, for triangle striae

From the examination of the relation $I=f(\sigma_S/S)$ (fig. 3) it is clear that the greatest reduction of the Strehl definition is due to the striae occupying about 75% of the pupil area.

2.2. The influence of the striae causing sinusoidal wavefront deformation upon the Strehl definition

The formula of Strehl definition for striae causing sinusoidal wavefront deformation has been derived in a way similar to that for the triangle striae. Deformation of this type may be defined as follows:

$$V_i(x,y)=\frac{V_{0_i}}{2}\left[1+\cos\frac{\pi}{b}(x-x_{0_i})\right], \tag{12}$$

$$x_{0_i}-b\leqslant x\leqslant x_{0_i}+b.$$

Basing on (3), Strehl definition for the case of many arbitrarily positioned sinusoidal striae is given by the formula:

$$I=\left(1-\sum_{i=1}^{N}A_i'\right)^2+\left(\sum_{i=1}^{N}B_i'\right)^2, \tag{13}$$

$$A_1'=\frac{\sigma_{S_i}}{S}\left[1-J_0\left(\frac{kV_{0_i}}{2}\right)\cos\left(\frac{kV_{0_i}}{2}\right)\right], \tag{14}$$

$$B_i'=\frac{\sigma_{S_i}}{S}J_0\left(\frac{kV_{0_i}}{2}\right)\sin\left(\frac{kV_{0_i}}{2}\right), \tag{15}$$

where

$J_0\left(\frac{kV_0}{2}\right)$ — Bessel function of the first kind and zero order.

Like for the triangle striae the relations

$$I=f(V_0)_{\frac{\sigma_S}{S}=\text{const}}$$

and

$$I=f\left(\frac{\sigma_S}{S}\right)_{V_0=\text{const}}$$

have been examinated. The results are presented in figs. 4,5.

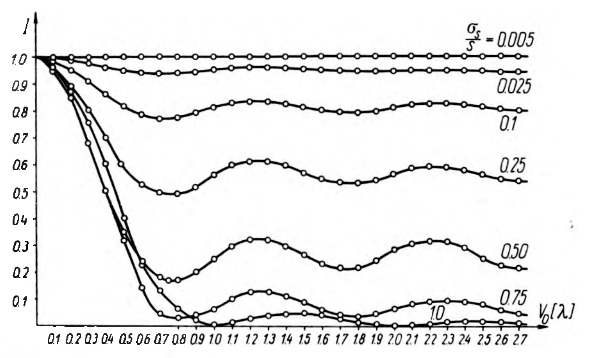

Fig. 4. Strehl definition vs. the relative striae area in the pupil at fixed wavefront deformation value, for sinusoidal striae

It has been stated that the sinusoidal striae are the most harmful, if they occupy about 75% of the pupil area (for arbitrary value of V_0), and the maximal wavefront deformation amounts to about 0.7 λ.

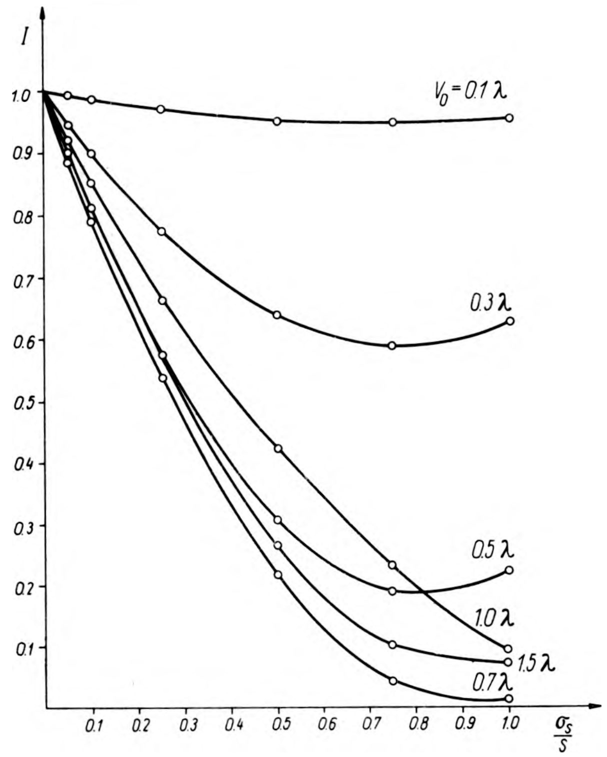

Fig. 5. Strehl definition vs. the maximal wavefront deformation on striae at fixed relative striae area, for sinusoidal striae

When comparing the graphs presented in figs. 2,3, and 4,5 for triangle and sinusoidal striae, respectively, it may be noted that their effects are very similar.

3. The influence of striae on the transfer function for a perfect optical system

The transfer function for an optical system is given by a well known formula:

$$d_n(\tilde{x}, \tilde{y}) = \frac{1}{S} \iint_{S_w} \exp\big(jkV(x, s_x; y, s_y)\big) dx dy, \quad (16)$$

$$V(x, s_x; y, s_y) = V(x+s_x, y+s_y) - V(x, y),$$

$$s_x = \frac{\lambda r_0 \tilde{x}}{a}; \; s_y = \frac{\lambda r_0 \tilde{y}}{a},$$

where

s_x, s_y — normalized spatial frequencies,
$\tilde{x}, \tilde{y}$ — spatial frequencies,
r_0 — radius of the reference sphere,
a — radius of the exit pupil,
S_w — the integration region.

In the case on N arbitrary striae parallel to y-direction (fig. 6) we may present formula (16) in the form:

$$d_n(\tilde{x}) = d_0(\tilde{x}) \Bigg[1 - \frac{\sigma_S}{S_w} + \frac{1}{S_w} \times$$
$$\times \sum_{i=1}^{N} \iint_{\sigma_{S_i}} \exp\big(jkV_i(x, y; s_x)\big) dx dy \Bigg], \quad (17)$$

where $d_n(\tilde{x}) = d_n(\tilde{x}, 0)$.

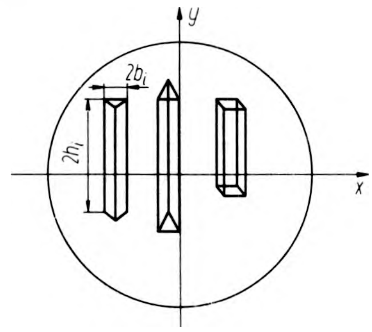

Fig. 6. An auxilliary figure showing striae position in the exit pupil of the optical system, helpful in deriving the formulae for optical transfer function

The influence of the single rectangular stria on the modulation transfer function has been examined in the papers [3, 4].

3.1. The effect of striae causing triangle wavefront deformation on the optical transfer function

Since the wavefront deformation due to real striae is of triangle type (4) the expression (17) has been evaluated for this case. It has been obtained the formula:

$$d_n(\tilde{x}) = d_0(\tilde{x}) - \sum_{i=1}^{N} A_i + j \sum_{i=1}^{N} B_i, \quad (18)$$

where:

$$A_i = \frac{\sigma_{S_i}}{S} [1 - \operatorname{sinc} kV_{0_i}], \text{ and}$$

$$B_i = \frac{\sigma_{S_i}}{S} \frac{\delta_i k V_{0_i}}{2} \operatorname{sinc}^2 \frac{kV_{0_i}}{2},$$

$\delta_i = +1$ for striae attributed to the fixed pupil,
$\delta_i = -1$ for striae attributed to the shifted pupil,
$\delta_i = 0$ for striae located outside the integration region.

This formula is justified for many triangle striae which are separated from anothers within the integration region.

The formula (18) may be written in the form:

$$d_n(\tilde{x}) = |d_n(\tilde{x})| e^{j\theta}, \quad (19)$$

where $|d_n(\tilde{x})|$ — the modulation transfer function — is equal to

$$|d_n(\tilde{x})| = \sqrt{\left[d_0(\tilde{x}) - \sum_{i=1}^{N} A_i\right]^2 + \left(\sum_{i=1}^{N} B_i\right)^2}, \quad (20)$$

Θ — phase of the optical transfer function — is equal to

$$\Theta = \arctan \frac{\sum_{i=1}^{N} B_i}{d_0(\tilde{x}) - \sum_{i=1}^{N} A_i}; \quad (21)$$

in these formulae (18), (20), (21) $d_0(\tilde{x})$ — ideal optical transfer function, A_i, B_i — expressions determined by the formula (18).

In the case of many triangle striae, causing maximal wavefront deformations of different sizes, the evaluation of expressions (20) and (21) is tedious.

If, however, the striae in the exit pupil of the optical system are lying together and cause the same maximal wavefront deformation, and if the width of each striae is the same, then for certain frequencies the expression (20) and (21) may be evaluated in a simple way. This may be done when the spatial frequencies are either a) integer multiples of the stria width $s_x = n2b$ or b) an odd multiple of the stria half-width $s_x = (2n+1)b$. In an extreme case, when the striae cover the whole area of the exit pupil the expressions (20) and (21) take the following forms:

— in case a)

$$|d_n(\tilde{x})| = d_0(\tilde{x}), \quad \Theta = 0 \quad (22)$$

— in case b)

$$|d_n(\tilde{x})| = d_0(\tilde{x}) \text{sinc}\, kV_0, \quad \Theta = 0. \quad (23)$$

It appears that the modulation transfer function will range between the values determined by the formulas (22) and (23) (fig. 7).

We have also examined the effect of rotation of stria in the exit pupil plane on the selected spatial frequencies ($s_x = 0.2$; 0.6; 1.2). The transfer function

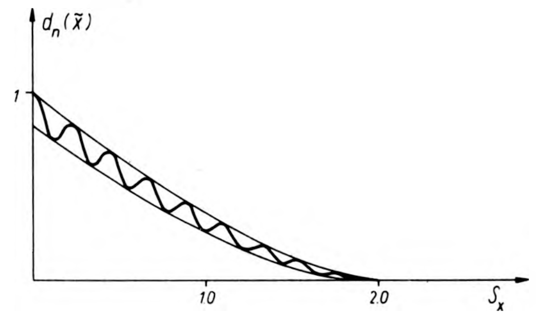

Fig. 7. The graph of the modulation transfer function for a system containing triangle striae of equal width causing equal maximal wavefront deformation and covering the whole area of the exit pupil

was determined numerically. Two following conclusion may be formulated:

1. Within the range small (up to 0.2λ) wavefront deformations caused by the striae the influence of the stria rotation on the selected spatial frequencies is very small and practically may be neglected (fig. 8).

2. For greater wavefront deformation (0.3λ—0.7λ) one observes a distinct effect of the angle of stria rotation upon the transfer of selected frequencies,

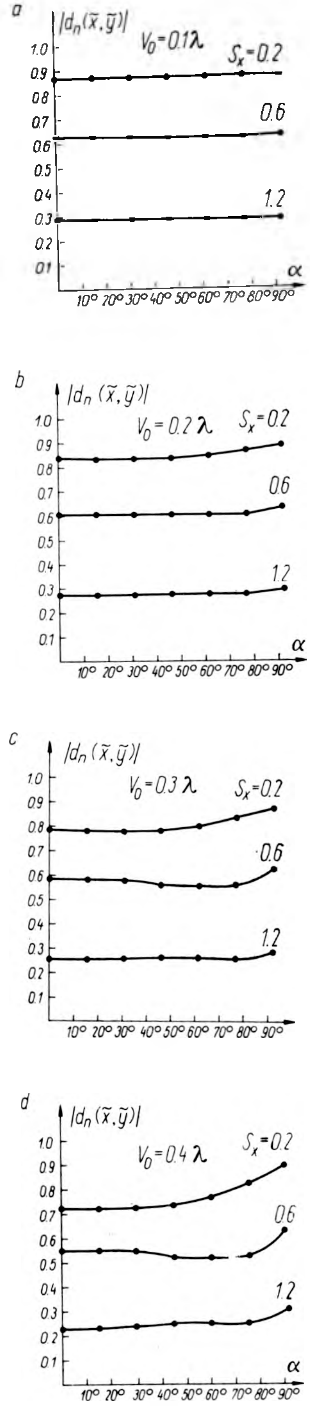

Fig. 8. Modulation transfer function vs. angular position of stria in the exit pupil for maximal 0.1 λ to 0.4 λ wavefront deformations due to stria

however, the rotation angles, at which the striae are the most harmful, are different for different frequencies (fig. 9).

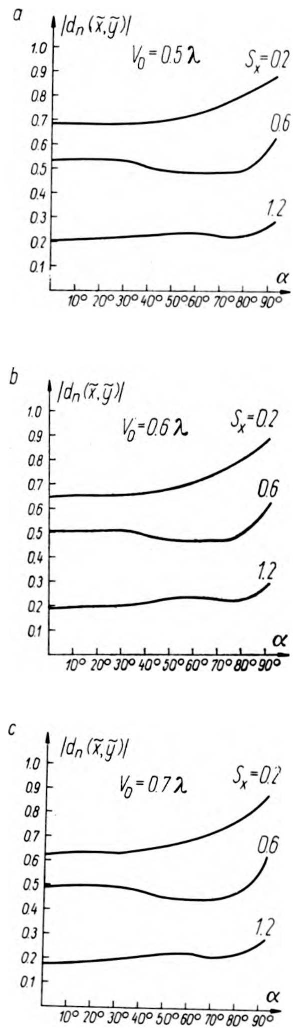

Fig. 9. Modulation transfer function vs. the angular position of stria in the exit pupil for maximal 0.5λ to 0.7λ wavefront deformations due to stria

Therefore it is impossible to give an unique answer to the question at which rotation angle of the stria in the exit pupil these occur the greatest reduction of the modulation transfer function.

3.2. The influence of sinusoidal striae on the transfer function of an ideal system

The effect of sinusoidal striae on the optical transfer function has also been examined. It has been assumed, that there exist N sinusoidal striae (defined by formula (12)) causing different maximal wavefront deformations and that each of them occupies another area.

For the modulation transfer function the following general formulae have been obtained:

$$|d_n(\tilde{x})| = \sqrt{\left[d_0(\tilde{x}) - \sum_{i=1}^{N} A_i'\right]^2 + \left(\sum_{i=1}^{N} B_i'\right)^2}, \tag{24}$$

$$\Theta = \arctan \frac{\sum_{i=1}^{N} B_i'}{d_0(\tilde{x}) - \sum_{i=1}^{N} A_i'}, \tag{25}$$

where

$$A_i' = \frac{\sigma_{S_i}}{S}\left[1 - J_0\left(\frac{kV_{0_i}}{2}\right)\cos\left(\frac{kV_{0_i}}{2}\right)\right],$$

$$B_i' = \frac{\delta_i \sigma_{S_i}}{S} J_0\left(\frac{kV_{0_i}}{2}\right)\sin\left(\frac{\delta_i kV_{0_i}}{2}\right).$$

4. Concluding remarks

The relations obtained are valid for the ideal system. It has been stated [1] that in some real cases ideal and real systems differ considerably.

From our last research it follows that for real systems containing small aberrations and striae both the Strehl definition and optical transfer function may be presented as algebraic sums of several terms, of which one is identical with the respective Strehl definition and optical transfer function for aberration-free optical systems with striae.

This gives the practical meaning to the relations presented in this paper, as they may be useful in determining the tolerances of striae in optical systems of small aberrations.

Влияние свили на качество отображения в совершенной оптической системе

Исследовано влияние треугольных и синусоидальных полос, находящихся в выходном зрачке идеальной оптической системы на яркость по Штрелю и оптическую функцию передачи. Некоторые из полученных зависимостей представлены на графиках.

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