Optica Applicata VIII/1, 1978

Jiří Keprt*, Miroslav Hrabovský**, Peter Vejbor*

Usage of materials with low resolving power in holographic interferometry

By means of a suitable arrangement of a holographic set-up, e.g. by using the lensless Fourier transform holography, it is possible to obtain holographic recording on materials with relatively low resolving power. Reconstruction of a hologram is made by means of a convergent beam. Primary and secondary images can be seen on a screen at the focal plane of the optical system. The method described is useful in the holographic interferometry, mainly in the case of wide applications, e.g. in nondestructive testing, pedagogical praxis etc.

1. Introduction

The holographic interferometry appears to be a very important experimental method in studies on any displacement or deformation of diffusely reflecting objects. By using a suitable holographic set-up during the recording and apply the quasi-Fourier lensless holography [1, 2, 3], it is possible to record an interferential field of generated by diffusely reflected objects and a reference band on recording materials with relatively low resolving power [4], e.g. on commercially available (ORWO NP 15, DK-5) photographic materials. The use of materials with low resolving power brings some technical advantages as, for example, radical increase of sensitivity in comparison with materials used in conventional holographic practice. Their relatively low prices and easy availability may be also of some value.

2. Recording and reconstruction of a hologram

In any quasi-Fourier arrangements of holographic set-ups the positions of the object point P and of the reference point R are assumed to be in the same plane which has to be parallel to the plane of the hologram (fig. 1).

We consider Carthesian coordinate system u, v, 0 of the hologram plane and Carthesian coordinate system x, y, z_0 of the plane, where the points P, R are located. The distance between these planes is denoted by z_0.

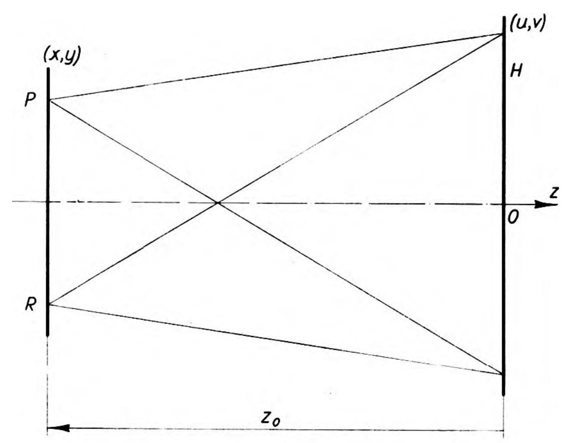

Fig. 1. The principle of a hologram recording

The complex amplitude of an object wave according to [1] is given by the relation

$$a_1 = A_1 \exp\left\{ i \frac{\pi}{\lambda z_0} [(u-x_1)^2+(v-y_1)^2] \right\}, \quad (1)$$

and the complex amplitude of a reference wave by the relation

$$a_0 = A_0 \exp\left\{ i \frac{\pi}{\lambda z_0} [(u-x_0)^2+(v-y_0)^2] \right\}, \quad (2)$$

where A_1, A_0 are the real amplitudes of an object and the reference wave, respectively. Let λ be the wavelength of the coherent light, and (x_1, y_1, z_1) and (x_0, y_0, z_0) be the respective coordinates of the points P and R.

*) Laboratory of Optics, Palacký University, Olomouc, Czechoslovakia.

**) SIGMA-Research Institute, Olomouc, Czechoslovakia.

The intensity at any of the given points of a hologram is determined by relation

$$I = (a_1+a_0)\cdot(a_1^*+a_0^*)$$
$$= A_1^2+A_0^2+A_1A_0\{\exp[i(2\pi\nu_x u+2\pi\nu_y v+\Theta)]+$$
$$+\exp[-i(2\pi\nu_x u+2\pi\nu_y v+\Theta]\}, \quad (3)$$

where

$$\nu_x = \frac{x_1-x_0}{\lambda z_0};$$

$$\nu_y = \frac{y_1-y_0}{\lambda z_0} \quad (4)$$

are spatial frequencies of the interference field in the hologram plane. These frequencies do not depend on coordinates of the hologram u, v, being dependent only on the distance of both points P, R to the hologram plane. The phase member Θ, corresponding to the position of a given object point, can be written as

$$\Theta = \frac{\pi}{\lambda z_0}(x_1^2-x_0^2+y_1^2-y_0^2). \quad (5)$$

If the total spatial frequency

$$\nu = \frac{\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}}{\lambda z_0} \quad (6)$$

is smaller than the cut-off frequency of the recording material then the interference field in the form of a linear fringe system can be recorded on the photosensitive media of low resolving power.

In the case when a larger plane object is to be used we may employ a divergent lens in order to increase the viewing field. The influence of a divergent lens on the spatial frequency spectrum can be seen in fig. 2. Having some spatial object we may use a converging objective and to achieve the „transformation" to the almost planary object, as it is common in photography. The principle of this method is shown in figs 3 and 4.

By reconstructing the hologram of the form of the transparence linear fringe system by means of a plane wave, we get two plane waves which can be

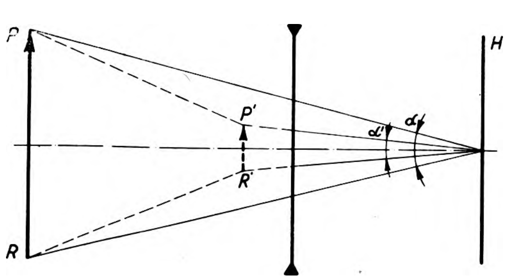

Fig. 2. The set-up for a hologram recording with diverging lens

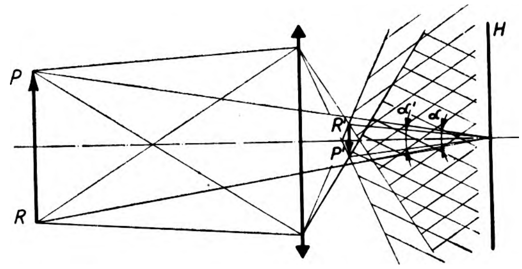

Fig. 3. The set-up for hologram recording with converging lens

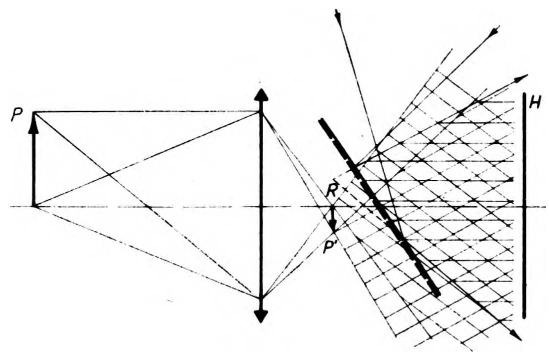

Fig. 4. The set-up for hologram recording with converging lens, the reference beam being obtained by means of half discharge plate

observed in a far field as two images of the point P. Using a converging lens placed in front or behind the hologram, we get in the focal plane of the optical system diffraction images P_1 and P_2 of the point P as well as the maximum of the zero-order F as shown in fig. 5.

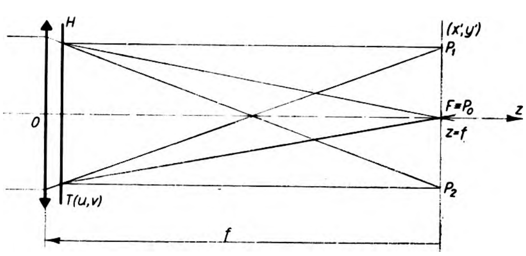

Fig. 5. The principle of hologram reconstruction

The complex amplitude of a convergent wave passing through a developed photographic plate can, according to [2], be expressed by relation

$$a_t(u, v, 0) = a_k\cdot T(u, v)\exp\left[i\frac{\pi}{\lambda f}(u^2+v^2)\right], \quad (7)$$

and the complex amplitude of a diffracted picture in the focus plane by

$$b(x', y', f) = \frac{i}{\lambda f}\int_{\infty}^{\infty}\int a_t\exp\left\{-i\frac{\pi}{\lambda f}[(x'-u)^2+ +(y'-v)^2]\right\}du\,dv, \quad (8)$$

where $T(u, v)$ denotes the amplitude transmittance of a developed photographic plate proportional [1, 2] to the intensity (3); f is the focal distance of the convergent objective O; a_k is the complex amplitude of the plane wave before the objective O; x', y', f are the Carthesian coordinates of the focal plane.

By introducing the relations (3) and (7) into equation (8) we obtain a complex amplitude corresponding to the reconstructed point $P_0 \equiv F$ written as

$$b_0 = i\frac{a_k}{\lambda f}(A_1^2+A_0^2)\exp\left[-i\frac{\pi}{\lambda f}(x'^2+y'^2)\right]\times$$
$$\times\int_{-X_0}^{X_0}\int_{-Y_0}^{Y_0}\exp\left[i\frac{2\pi}{\lambda f}(ux'+vy')\right]du\,dv, \quad (9)$$

and complex amplitudes corresponding to the points P_1 and P_2 in the form

$$b_{1,2} = i\frac{a_k}{\lambda f}A_1A_0\exp\left[-i\frac{\pi}{\lambda f}(x'^2+y'^2)\right]\exp(\pm i\Theta)\times$$
$$\times\int_{-X_0}^{X_0}\int_{-Y_0}^{Y_0}\exp\left\{\pm i\frac{2\pi}{\lambda z_0}\left[(x_1-x_0)u+(y_1-y_0)v\pm\right.\right.$$
$$\left.\left.\pm\frac{z_0}{f}(ux'+vy')\right]\right\}du\,dv, \quad (10)$$

where Θ is the phase member given by relation (5).

The integral region $2X_0\times 2Y_0$ in relations (9), (10) is determined by the size of a hologram which, in the indicated case, has a form of a rectangular. By calculation of integrals in relations (9), (10) we obtain the following expressions

$$\gamma_{1,2} = 4\cdot\frac{\sin c_n X_0}{c_n}\times\frac{\sin d_n Y_0}{d_n}; \quad (11)$$
$$(n = 0, 1, 2),$$

where coefficients c_n and d_n are given by relations

$$c_0 = \frac{2\pi}{\lambda f}x', \quad (12)$$

$$d_0 = \frac{2\pi}{\lambda f}y', \quad (13)$$

$$c_{1,2} = \frac{2\pi}{\lambda f}\left[x'\pm\frac{f}{z_0}(x_1-x_0)\right], \quad (14)$$

$$d_{1,2} = \frac{2\pi}{\lambda f}\left[y'\pm\frac{f}{z_0}(y_1-y_0)\right]. \quad (15)$$

Complex amplitudes b_0, b_1, b_2 represent Fraunhofer's amplitudes of images of the object point P. The intensity distribution of the diffracted images P_0, P_1, P_2 can be written as

$$I_n = K_n^2\left(\frac{\sin c_n X_0}{c_n X_0}\right)^2\cdot\left(\frac{\sin d_n Y_0}{d_n Y_0}\right)^2, \quad (16)$$

which is the well known relation of the classical diffraction theory.

The maximum of zero order (the wave in z-axis direction) will be, as seen from the point of geometrical optics, located into the focus of the optical system. The positive-first-order diffracted wave is focused into the point

$$P_1\left[-\frac{f}{z_0}(x_1-x_0);\ -\frac{f}{z_0}(y_1-y_0); f\right],$$

and analogically, the negative-first-order diffracted wave is focused into the point

$$P_2\left[\frac{f}{z_0}(x_1-x_0);\ \frac{f}{z_0}(y_1-y_0); f\right],$$

provided that $z_0 < 0$, and $f > 0$.

We can see that by choosing the relation f/z_0 it is possible to change lateral magnification and consequently the size of a reconstructed image.

Let us consider now the problem of separation the reconstructed images and diffracted light occurring due to the mutual interference of light coming from individual object points (so-called intermodulation effect).

If the selected points of the object $P(x_1, y_1, z_0)$ and $Q(x_1, y_2, z_0)$ are such that the point P is the nearest point $R(x_0, y_0, z_0)$ and the point Q is the most faraway point related to the reference point, then the complete separation of reconstructed images of an object from the diffracted images, occurring due to the mutual interference of individual object points, can be secured by fulfilling of the condition

$$(x_1-x_0)^2+(y_1-y_0)^2 > (x_2-x_1)^2+(y_2-y_1)^2. \quad (17)$$

From the geometrical point of view this condition says that the distance of the reference point R from the nearest point of the object P must be greater than the mutual distance of points P, Q.

Mutual interference pattern generated by all the object points is localized in the vicinity of the z-axis. If the condition (17) is fulfilled the diffracted images of the object itself are not influenced by this mutual interference and the perfect separation of diffracted bands can be achieved.

The reconstruction of a hologram recorded on photographic materials with low resolving power ORWO DK-5 can be seen in fig. 6. The reconstructed object is 25 cm high and 5 cm wide. The hologram was recorded without using any optical system. The same object seen in fig. 7, was recorded by means of the holographic set-up in fig. 3. All the reconstructed images as well as those shown in fig. 7, are characterized by a strong mutual interference at focus point.

Fig. 6. The reconstructed object 25 cm high and 5 cm wide
Hologram was reached without using any optical system

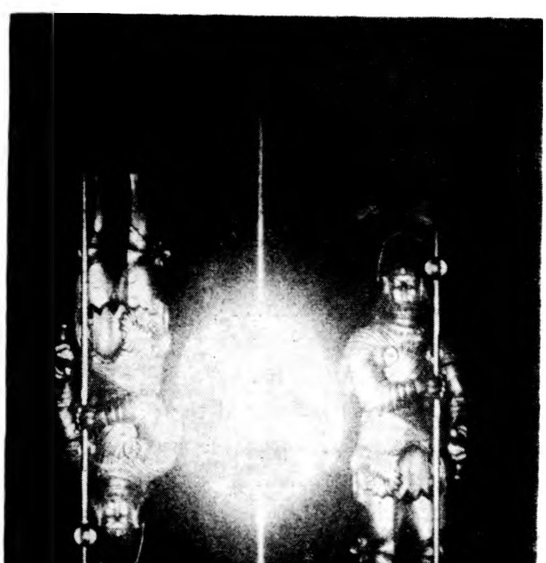

Fig. 7. The same object as in fig. 6. The record was made by holographic set-up as illustrated in fig. 3

The holographic image of the relief of the circle shape (fig. 8) having about 20 cm in diameter was obtained by using the arrangement presented in fig. 2.

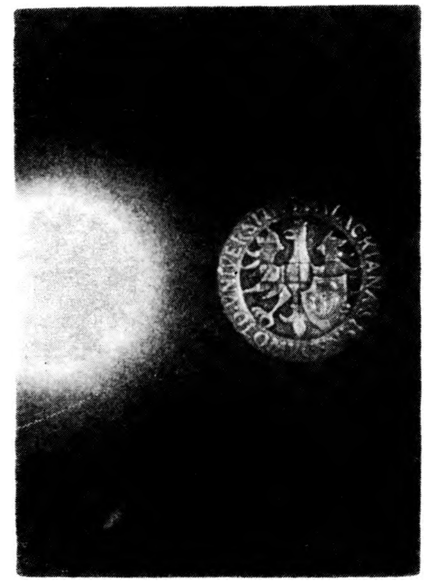

Fig. 8. The relief of the circle shape with 20 cm in diameter obtained by the arrangement shown in fig. 2

3. Holographic interferometry

Let us consider a method of double exposure holography as applied to deformation or displacements measurements of individual points of a diffusely scattering object. Let us assume that after the first exposure the point $P(x_1, y_1, z_0)$ has changed its position into the point $P'(x_1+\Delta x, y_1+\Delta y, z_0+\Delta z)$. Then the complex amplitudes of diffracted waves after the reconstruction are

$$b_{1,2} = i\frac{a_k}{\lambda f} A_0 A_1 \times$$

$$\times \exp\left[-i\frac{\pi}{\lambda f}(x'^2+y'^2)\right] \times$$

$$\times \exp(\pm i\Theta)\gamma_{1,2} = B_0 \exp(\pm i\Theta)\cdot\gamma_{1,2}, \quad (18)$$

where the quantities Θ and $\gamma_{1,2}$ are functions of the independent variables x_1, y_1, z_0. The reconstructed diffracted images of the point P' can be expressed by complex amplitude

$$b'_{1,2} = B_0 \exp(\pm i\Theta')\gamma'_{1,2} \quad (19)$$

with

$$\Theta' = \Theta + \Delta\Theta, \quad (20)$$

where $\Delta\Theta$ is a phase difference caused by infinitesimal changes of Δx, Δy, Δz of the point P.

If by the quantity $\gamma_{1,2}$ we mean an amplitude function, and by $\exp(\pm i\Theta)$ a phase function of the variables x_1, y_1, z_0, then we can observe that with small changes of the values x_1, y_1, z_0 the phase function, varies much quicker than the amplitude function. Hence, for small changes of Δx, Δy, Δz the amplitude function may be considered constant with respect to the phase function. Therefore, the complex amplitude of the point P', seen from this view point, can be written as

$$b'_{1,2} = B_0 \exp[\pm i(\Theta + \Delta\Theta)]\gamma_{1,2}. \qquad (21)$$

The eye as a quadratic detector sees the intensity I at points P_1, P_2 which is determined by relation

$$I = (b_{1,2} + b'_{1,2})(b^*_{1,2} + b'^*_{1,2}) = 2B_0^2\gamma_{1,2}^2(1+\cos\Delta\Theta), \qquad (22)$$

i.e. cosine form of the intensity of diffracted images of the point P depends on the changes of the phase $\Delta\Theta$ or of Δx, Δy, Δz, respectively. Consequently, a fringe pattern of relatively low spatial frequencies appearing in the reconstructed image is due to small displacement of the subject.

As the first example the deformation of a disturbed sample of artificial cloth, the size of which is 12 cm $\times 7$ cm, is shown (figs 9 and 10). Deformation of pump's moving blade can be seen in fig. 11. Interference fringes on the image of the relief obtained by changing the light direction are shown in fig. 12.

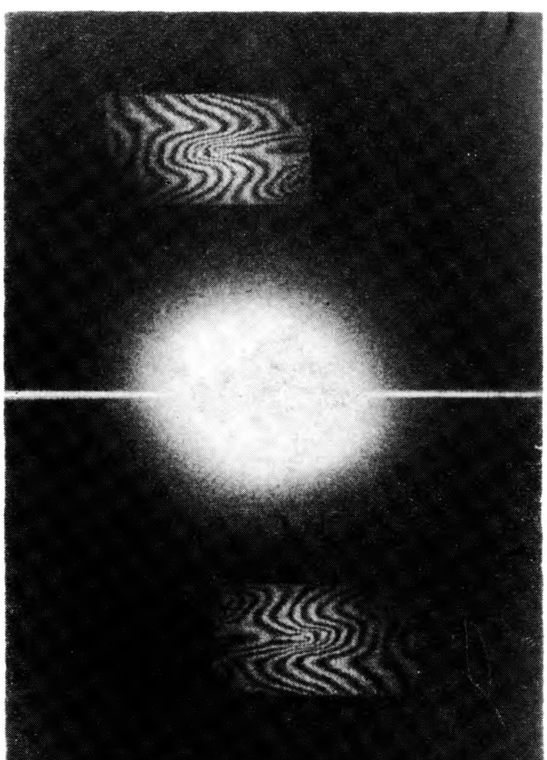

Fig. 9. Deformation of a disturbed artificial cloth

The quality of the reconstructed images from the interferograms are good enough to observe the form and number of the interference fringes. That is while the use of this method is suitable in holographical non-destructive testing as well as in measuring of deformations. To evaluate the deformations from these interferograms the object should be illuminated from different angles and the deformations determined from several holograms [5].

Because of low price of the recording materials they may be easily applied in pedagogical praxis. A current photo-camera without objective or with negative lens can be used as a holder of a hologram and the relative great sensitivity of the materials allows to obtain holograms by using He—Ne lasers of low powers.

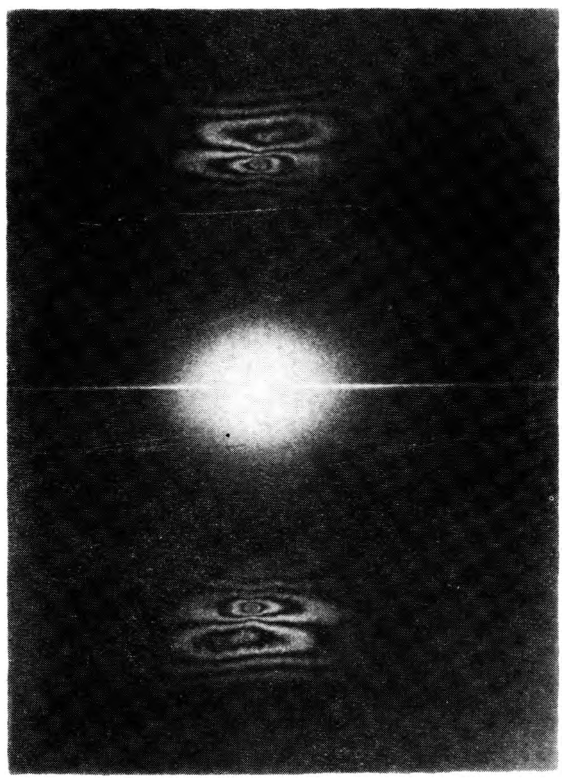

Fig. 10. Deformation of a disturbed artificial cloth

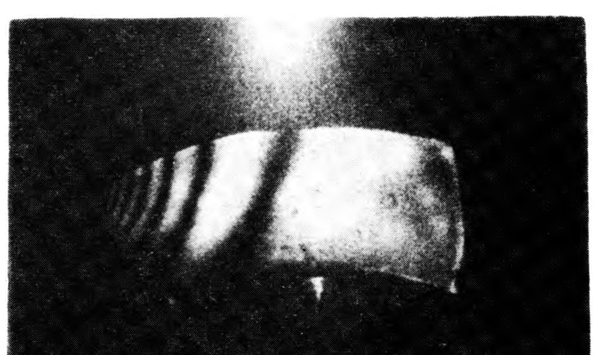

Fig. 11. Deformation of pump's moving blade

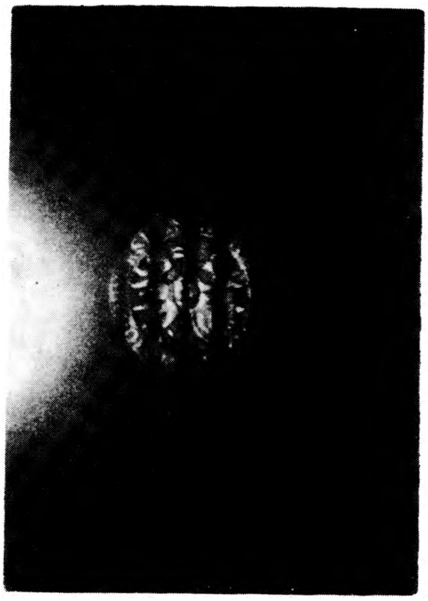

Fig. 12. Interference fringes on the image of a relief obtained by changing the light direction

Применение материалов с низкой разрешающей способностью в интерферометрической голографии

С помощью соответствующего голографического набора, то есть при использовании безлинзовой фурье-голографии, можно получить голографическую запись на материалах с низкой разрешающей способностью. Восстановление голограммы производят конвергентным пучком. Первичное и вторичное изображения могут наблюдаться на экране в плоскости фокусирующей оптической системы. Описанный метод пригоден в голографической интерферометрии, а именно в случае материалоемких испытаний, как, например, при бесконтактных испытаниях, в учебной практике и т.п.

References

[1] Goodman J.W., *Introduction to Fourier Optics*, McGraw--Hill Book Comp., New York 1968.

[2] Collier R.J., Burckhardt Ch.B., Lin L.H., *Optical Holography*, Academic Press, New York-London 1977.
Holography, Academic Press, New York-London 1971.

[3] Miller M., *Holografie*, SNTL, Prague 1974.

[4] Keprt J., Hrabovský M., *Holografická interferometrie při použiti normálnich fotografických materiálů*, JMO **9**, (1975) 255.

[5] Havelka B., Blabla J., *Recent Advantages in Optical Physics*, Proceedings of the ICO-10, Prague 1975, 441.

Received, August 8, 1977