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Nonlinear magneto-light-induced phenomena in a resonant medium

V. M. Arutunyan, G. G. Adonts, E. G. Kanetsian, S. T. Hovsepian

Research Institute "Lazerayin Technika", 21 Shopron str., 375090 Yerevan, Armenia.

A general theory of magneto-light-induced rotation of the plane of polarization of the probe signal in resonant medium is developed. The propagation of an intensive elliptically polarized electromagnetic wave through a resonant medium with arbitrary angular momenta of the levels is investigated in the adiabatic following approximation. Then the propagation of counter-propagating polarized waves: an intensive pump wave and a probe light signal, through the resonant medium in the presence of the longitudinal magnetic field is studied. It is shown that the induced anisotropy of the medium is determined both by magneto-optical and polarization light effects and by their interference. This allowed us to determine the local value of the external spatially-inhomogeneous magnetic field.

1. Introduction

An intensive elliptically polarized wave induces optical anisotropy in a resonant medium, which leads to rotation of the plane of polarization of the probe signal [1]–[3], as well as to formation of an atom magnetic momentum [4]. The effect of the induced rotation of the plane of polarization has attracted attention of investigators in connection with its application to spectroscopy, plasma diagnostics, rotation of the wave front, *etc.* Optical anisotropy in the medium can also be induced by the external magnetic field (Faraday effect, Cotton–Mouton effect).

The method of quasi-energetic states applied to investigation of polarization plane rotation was used to study resonant Raman scattering in the magnetic field of radiation, when the mix of atomic sublevels becomes significant.

It is necessary to consider theoretically the effect of degeneracy of the energy levels of atoms, which significantly complicates the problem and makes it difficult to find exact solutions. The adiabatic following approximation is quite promising in finding exact analytic solutions on condition that the pulse duration is shorter than the longitudinal relaxation time and that detuning of the resonance exceeds the spectral widths of the lines.

2. General formalism

Let us consider the propagation of two plane monochromatic waves $\mathbf{E}_1$ (strong) and $\mathbf{E}_2$ (probe) in opposite directions through the medium. We represent the electric vector of the field in the form

$$\mathbf{E} = \frac{1}{2}\{\mathbf{E}_1(\mathbf{r})\exp[i(\mathbf{k}_1\mathbf{r}-\omega_1 t)]+\mathbf{E}_2(\mathbf{r})\exp[-i(\mathbf{k}_2\mathbf{r}+\omega_2 t)]\}+\text{c.c.} \tag{1}$$

where $|\mathbf{k}_i| = \omega_i/c$ is the wave vector in vacuum. The electric-field vector given by (1) obeys the following Maxwell equation:

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial^2 \mathbf{P}}{\partial t^2} \tag{2}$$

where $\mathbf{P}$ is the induced macroscopic polarization vector and c is the velocity of light in vacuum. We assume the complex amplitude $\mathbf{E}_1$ to be slowly varying function of $\mathbf{r}$ in the following sense:

$$\frac{c}{\omega_1}\left|\frac{\partial \mathbf{E}}{\partial t}\right| \ll |\mathbf{E}_1|.$$

The resonant medium consists of identical two-level atoms with energy $\mathscr{E}_1$ and momentum j_1 on the ground level 1, and energy $\mathscr{E}_2$ and momentum j_2 on the excited level 2. The Hamiltonian operator $\hat{\mathscr{H}}$ of the two-level atomic system in the presence of the electric field (1) and external magnetic field $\mathbf{H}$ is

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 - \hat{\mathbf{d}}\mathbf{E} - \hat{\mu}\mathbf{H} \tag{3}$$

where $\hat{\mathscr{H}}_0$ denotes the Hamiltonian operator of the atom in the absence of applied field, $\hat{\mathbf{d}}$ is the electric-dipole-moment operator, $\hat{\mu}$ is the magnetic-field operator. We find the solution of the Schrödinger equation

$$i\hbar\frac{\partial \Psi}{\partial t} = \hat{\mathscr{H}}\Psi \tag{4}$$

in the form

$$\Psi = \sum_{m_1}\alpha_{m_1}\varphi_{1j_1m_1}\exp\left[-\left(\frac{i}{\hbar}\right)\mathscr{E}_1 t - \left(\frac{i}{\hbar}\right)\lambda t\right] + \sum_{m_2}\beta_{m_2}\varphi_{2j_2m_2}$$
$$\times\exp\left[-\left(\frac{i}{\hbar}\right)\mathscr{E}_2 t - \left(\frac{i}{\hbar}\right)\lambda t - i\varepsilon_1 t\right] \tag{5}$$

where $\varphi_{ij_im_i}$ denotes the spectrum of the wave function of the atom, $\varepsilon_1 = \omega_1 - \omega_{01}$ is the resonance detuning between intensive wave frequency ω_1 and atomic frequency $\omega_{01} = \frac{\mathscr{E}_2 - \mathscr{E}_1}{\hbar}$, λ is the Stark shift of atomic levels in the field of the intensive monochromatic wave.

3. One strong elliptically polarized wave (H = 0)

In the general case of the elliptically polarized wave in the absence of magnetic field (H = 0) we can obtain the following system of recurrence equations for α_{m_1}

$$\left[4\lambda(\lambda+\hbar\varepsilon_1)-\frac{1}{4}\left|D^{1j_1m_1}_{+2j_2m_1-1}\right|^2|E_{1+}|^2-\frac{1}{4}\left|D^{1j_1m_1}_{-2j_2m_1+1}\right|^2|E_{1-}|^2-\right.$$
$$\left.-\left|D^{1j_1m_1}_{z2j_2m_1}\right|^2|E_{1z}|^2\right]\alpha_{m_1}=\frac{1}{4}\left(D^{1j_1m_1}_{+2j_2m_1-1}D^{1j_1m_1-2*}_{-2j_2m_1-1}\right.$$
$$\left.\times E^*_{1+}E_{1-}\alpha_{m_1-2}+D^{1j_1m_1}_{-2j_2m_1+1}D^{1j_1m_1+2*}_{+2j_2m_1+1}E^*_{1-}E_{1+}\alpha_{m_1+2}\right) \tag{6}$$

where we introduce the circular components $E_\pm = E_x \pm iE_y$ and $d_\pm = d_x \pm id_y$ of the vectors **E** and **d**, respectively, and matrix elements $D^{1j_lm_l}_{\substack{\pm 2j_km_k\\ z}}$ are:

$$D^{1j_1m_1}_{+2j_2m_2}=d^{1j_1}_{2j_2}\left\{\left[\frac{(j_1-m_1+1)(j_1+m_1)}{j_1(j_1+1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2}+\right.$$
$$+\left[\frac{(j_2-m_1+1)(j_2-m_1)}{j_2(2j_2-1)(2j_2+1)}\right]^{1/2}\delta_{j_2,j_1+1}-$$
$$\left.-\left[\frac{(j_1+m_1-1)(j_1+m_1)}{j_1(2j_1-1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2+1}\right\}\delta_{m_1,m_2+1}, \tag{6a}$$

$$D^{1j_1m_1}_{-2j_2m_2}=d^{1j_1}_{2j_2}\left\{\left[\frac{(j_1+m_1+1)(j_1-m_1)}{j_1(j_1+1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2}-\right.$$
$$-\left[\frac{(j_2+m_1+1)(j_2+m_1)}{j_2(2j_2-1)(2j_2+1)}\right]^{1/2}\delta_{j_2,j_1+1}+$$
$$\left.+\left[\frac{(j_1-m_1-1)(j_1-m_1)}{j_1(2j_1-1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2+1}\right\}\delta_{m_1,m_2-1}, \tag{6b}$$

$$D^{1j_1m_1}_{z2j_2m_2}=d^{1j_1}_{2j_2}\left\{\left[\frac{m_1^2}{j_1(j_1+1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2}+\right.$$
$$+\left[\frac{j_2^2-m_1^2}{j_2(2j_2-1)(2j_2+1)}\right]^{1/2}\delta_{j_2,j_1+1}+$$
$$\left.+\left[\frac{j_1^2-m_1^2}{j_1(2j_1-1)(2j_1+1)}\right]^{1/2}\delta_{j_1,j_2+1}\right\}\delta_{m_1,m_2} \tag{6c}$$

where $d^{1j_1}_{2j_2}$ is the reduced matrix element of the dipole transition.

In the general case of the elliptically polarized wave the solution of system (6) and determination of the characteristic roots is a very complicated mathematical problem. Physically it is clear. The different components of the wave polarization $E_{\pm}, E_z$ interact with different magnetic sublevels of the ground and excited states and mix them. For pure circular or linear polarizations the chain of equation (6) has broken and we really have $n = \min\{2j_1+1, 2j_2+1\}$ isolated two-level atoms in the field of strong wave.

4. Circular polarization of the strong wave ($E_{1-} \neq 0$, $E_{1+} = E_{1z} = 0$) and external magnetic field ($\mathbf{H} \neq 0$)

Let us consider the propagation of a probe elliptically polarized signal in the presence of the strong polarized wave and constant magnetic field. In the case when the strong wave is circularly polarized it is possible to construct exact wave functions of the two-level atom taking into account both Stark and Zeeman shifts and to solve the problem of the wave propagation. The nonlinear indices $n_{\mp}$ for the circular components of the probe wave are:

$$n_- = 1 + \frac{Q_1}{4}\sum_{m_1} \frac{F^{j_1 m_1}_{-j_2 m_1+1}}{(1+G_{m_1})^2+\xi_{m_1}}\left[\frac{\left(1+G_{m_1}+\sqrt{(1+G_{m_1})^2+\xi_{m_1}}\right)^2}{\omega_1-\omega_2-\varepsilon_1\sqrt{(1+G_{m_1})^2+\xi_{m_1}}} - \frac{\left(1+G_{m_1}-\sqrt{(1+G_{m_1})^2+\xi_{m_1}}\right)^2}{\omega_1-\omega_2+\varepsilon_1\sqrt{(1+G_{m_1})^2+\xi_{m_1}}}\right], \tag{7a}$$

$$n_+ = 1 + \frac{Q_1}{4}\sum_{m_1} \frac{F^{j_1 m_1}_{+j_2 m_1-1}}{\sqrt{(1+G_{m_1})^2+\xi_{m_1}}\sqrt{(1+G_{m_1-2})^2+\xi_{m_1-2}}}$$
$$\times\left[\frac{\left(1+G_{m_1}+\sqrt{(1+G_{m_1})^2+\xi_{m_1}}\right)\left(1+G_{m_1-2}+\sqrt{1+G_{m_1-2})^2+\xi_{m_1-2}}\right)}{\omega_1-\omega_2-\frac{\varepsilon_1}{2}\left(\sqrt{(1+G_{m_1})^2+\xi_{m_1}}+\sqrt{(1+G_{m_1-2})^2+\xi_{m_1-2}}\right)} - \right.$$
$$\left. - \frac{\left(1+G_{m_1}-\sqrt{(1+G_{m_1})^2+\xi_{m_1}}\right)\left(1+G_{m_1-2}-\sqrt{1+G_{m_1-2})^2+\xi_{m_1-2}}\right)}{\omega_1-\omega_2+\frac{\varepsilon_1}{2}\left(\sqrt{(1+G_{m_1})^2+\xi_{m_1}}+\sqrt{(1+G_{m_1-2})^2+\xi_{m_1-2}}\right)}\right] \tag{7b}$$

where:

$$Q_1 = \frac{\pi N_1 |d^{1j_1}_{2j_2}|^2}{(2j_1+1)\hbar}, \quad \xi_{m_1} = \frac{|d^{1j_1}_{2j_2}|^2 |E_{1-}|^2}{4\hbar^2\varepsilon_1^2} F^{j_1 m_1}_{-j_2 m_1+1},$$

$$F^{j_1 m_1}_{-j_2 m_1+1} = \frac{(j_1-m_1)(j_1+m_1+1)}{j_1(2j_1+1)(j_1+1)}\delta_{j_1,j_2} + \frac{(j_2+m_1)(j_2+m_1+1)}{j_2(2j_2-1)(2j_2+1)}\delta_{j_2,j_1+1} + \frac{(j_1-m_1)(j_1-m_1-1)}{j_1(2j_1-1)(2j_1+1)}\delta_{j_1,j_2+1},$$

$$F^{j_1 m_1}_{+j_2 m_1-1} = \frac{(j_1+m_1)(j_1-m_1+1)}{j_1(2j_1+1)(j_1+1)}\delta_{j_1,j_2} + \frac{(j_2-m_1)(j_2-m_1+1)}{j_2(2j_2-1)(2j_2+1)}\delta_{j_2,j_1+1} + \\ + \frac{(j_1+m_1)(j_1+m_1-1)}{j(2j-1)(2j+1)}\delta_{j_1,j_2+1},$$

$$G_{m_1} = -(g_1^{m_1}/\hbar\varepsilon_1)+(g_2^{m_1+1}/\hbar\varepsilon_1), \quad g_i^m = -\mu_0 m\tilde{g}_i H.$$

Formulae (7a), (7b) are exact expressions for the nonlinear refractive indices of the medium, which allows us to consider the perturbation regime of small nonlinearities ($\xi_{m_1} \ll 1$) as well as saturation regime ($\xi_{m_1} \simeq 1$). The refractive indices $n_\mp$ have a complicated structure: they contain the Zeeman and Stark shifted poles of the atomic absorption (the first term) and the three-photon scattering process (the second term).

The difference in the nonlinear refractive indices n_- and n_+ leads to an induced rotation of the polarization ellipse (plane) of the probe signal by an angle $\varphi = \omega_2(n_+ - n_-)/2c$. Let us note that in the first nonlinear approximation in terms of the pump field one can successfully sum up the series in equations (7a) and (7b) an obtain the expression for the angle φ dependent only on the angular momenta j_1 and j_2

$$\varphi = \frac{\omega_2}{c}\frac{Q_1}{3\varepsilon_2}[A_1\xi_- + A_2 H + A_3\xi_- H] \tag{8}$$

where:

$$\varepsilon_2 = \omega_2 - \omega_{01}, \quad \xi_- = \frac{|d^{1j_1}_{2j_2}|^2 |E_{1-}|^2}{40\hbar^2\varepsilon_1^2},$$

$$A_1 = -5\left(1+\frac{\varepsilon_1}{\varepsilon_2}\right)\left[\frac{1}{j_1(j_1+1)(2j_1+1)}\delta_{j_1,j_2} + \frac{4j_1^3+14j_1^2+14j_1+3}{(j_1+1)(2j_1+1)(2j_1+3)^2}\delta_{j_2,j_1+1} + \right. \\ \left. + \frac{4j_1^3-2j_1^2-2j_1+1}{j_1(2j_1+1)(2j_1-1)^2}\delta_{j_1,j_2+1}\right],$$

$$A_2 = \frac{\mu_0}{\hbar\varepsilon_2}\left\{(\tilde{g}_1+\tilde{g}_2)\delta_{j_1,j_2} + [(j_1+2)\tilde{g}_2 - j_1\tilde{g}_1]\delta_{j_2,j_1+1} + \right. \\ \left. + [(j_1+1)\tilde{g}_1 - (j_1-1)\tilde{g}_2]\delta_{j_2,j_1-1}\right\},$$

$$A_3 = \frac{\mu_0}{\hbar\varepsilon_2}\left\{-\frac{5(\tilde{g}_2+\tilde{g}_1)}{j_1(j_1+1)(2j_1+1)}\left[1+\frac{\varepsilon_1}{\varepsilon_2}(4j_1^2+4j_1+2)+\frac{\varepsilon_1^2}{\varepsilon_2^2}(8j_1^2+8j_1-1)\right]\right\}\delta_{j_1,j_2} + \\ + \frac{\mu_0}{\hbar\varepsilon_1}\frac{1}{(j_1+1)(2j_1+3)(2j_1+1)}\left\{\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)[j_1+2)(2j_1+5)\tilde{g}_2 + j_1(2j_1-1)\tilde{g}_1] - \right. \\ \left. -2\left(1+\frac{\varepsilon_1}{\varepsilon_2}+\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)[4j_1+5)(j_1+2)(2j_1+1)\tilde{g}_2 - (2j_1+3)(4j_1+3)j_1\tilde{g}_1\right\}\delta_{j_2,j_1+1} +$$

$$+\frac{\mu_0}{\hbar\varepsilon_1}\frac{1}{j_1(2j_1-1)(2j_1+1)}\left\{\left(1-\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)[j_1+1)(2j_1+3)\tilde{g}_1+(j_1-1)(2j_1-3)\tilde{g}_2]-\right.$$

$$\left.-2\left(1+\frac{\varepsilon_1}{\varepsilon_2}+\frac{\varepsilon_1^2}{\varepsilon_2^2}\right)[(2j_1-1)(4j_1+1)(j_1+1)\tilde{g}_1-(2j_1+1)(4j_1-1)(j_1-1)\tilde{g}_2]\right\}$$

$$\times\delta_{j_2,j-1}.$$

Expression (8) contains the term proportional to the magnetic field and describing a purely Faraday rotation in the magnetic field; the term proportional only to the nonlinearity parameter and describing induced rotation in the field of intensive circularly polarized wave, as well as the term proportional to both the magnetic field and the nonlinearity parameter and describing the interference between the magnetic and light induced rotation. The latter may be used to define the local value of the spatially-inhomogeneous magnetic field in systems with arbitrary angular momenta. It is necessary to pass intensive and probe signals through the medium, so that they could cross at a certain point in space. By measuring the difference between the purely Faraday rotation angle (with no intensive wave) and the rotation angle in the presence of an intensive wave we can define the local value of the magnetic field.

5. Laser-induced magnetic momentum of atoms (in the absence of external magnetic field H = 0)

Let us consider the formation of induced magnetic momentum of atoms in the field of an intensive resonant wave. For free atom the ground and excited states are degenerated with respect to the projection of the angular momentum. As these states are incoherent, having the same statistical weight, their contribution in the average value of a magnetic momentum of atom is equal. So the total momentum of atom equals zero. The intensive polarized field, introducing a certain symmetry in the space, takes off the degeneracy of levels. It leads to formation of a total atom magnetic momentum. For simplicity, we consider the transition $j_1=1/2\rightarrow j_2=3/2$. In this case, Eqs. (6) and (4) can be solved exactly. Averaging the magnetic momentum $\bar{\mu}$ by wave functions (5) we have:

$$\bar{\mu}_z=-\frac{\mu_0}{2}\frac{1+\sqrt{1+\xi_{-1/2}}}{2\sqrt{1+\xi_{-1/2}}}\left[-1+\frac{3\xi_{+1/2}-7\xi_{-1/2}}{3(1+\sqrt{1+\xi_{-1/2}})^2}\right]-$$

$$-\frac{\mu_0}{2}\frac{1+\sqrt{1+\xi_{+1/2}}}{2\sqrt{1+\xi_{+1/2}}}\left[1+\frac{7\xi_{+1/2}-3\xi_{+1/2}}{3(1+\sqrt{1+\xi_{+1/2}})^2}\right]. \tag{9}$$

As is obvious from (9), in the case of linearly polarized monochromatic wave, which means for $\xi_{-1/2}=\xi_{+1/2}$, the magnetic momentum is zero. It is explained by the fact that a linearly polarized wave does not take off the degeneracy completely, the sublevels that are distinguished merely by the sign of m_i remain still degenerated among each other. The maximum magnetic momentum is realized in the case of circularly polarized wave.

Let us give the expression for averaging magnetic momentum of atom in the case of circularly polarized wave ($E_{1+} = E_{1z} = 0$, $E_{1-} \neq 0$) for a system with arbitrary angular momenta j_1 and j_2

$$\bar{\mu}_z = \langle \psi | \mu_z | \psi \rangle = -\frac{\mu_0}{2j_1+1} \sum_{m_1} \frac{1+\sqrt{1+\xi_{m_1}}}{2\sqrt{1+\xi_{m_1}}} \left[m_1 g_1 + (m_1+1) g_2 \frac{\xi_{m_1}}{(1+\sqrt{1+\xi_{m_1}})^2} \right]. \tag{10}$$

For small nonlinearities $\xi_{\pm 1/2} \ll 1$, in the case of circularly polarized wave, from (9) we have

$$\bar{\mu}_z = -\frac{7}{12} \mu_0 \xi \tag{11}$$

where

$$\xi = \frac{|d_{2j_2}^{1j_1}|^2 |E_{1-}|^2}{24\hbar^2 \varepsilon_1^2}.$$

So the magnetic momentum is proportional to the intensity parameter ξ and is absent in the linear theory.

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