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Directional coupling for solitary waves in quadratic nonlinearity

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The propagation of temporal solitary waves, formed in the second-order nonlinear medium, is analysed in waveguide directional couplers. The influence of the third-order nonlinearity on the switching effect is also presented.

Recently, the great effort of nonlinear guided-wave phenomena research is focused on the cascaded second-order process which can produce relatively large intensity-dependent phase changes of the wave propagating in optical waveguides [1]–[7]. Among a variety of nonlinear effects, the cascaded second order nonlinearity can be a source of forming temporal or spatial solitons [3]–[7]. The phase mismatched second harmonic generation ($\omega+\omega$) and difference frequency generation ($2\omega-\omega$) lead to effective intensity-dependent phase changes of the pulse at both frequencies. This allows to form the pair of solitary waves at fundamental frequency and second harmonics with amplitude envelopes not changed along the propagation.

In this paper, the propagation of the temporal solitary waves, formed in the second-order nonlinear medium, is investigated in nonlinear directional couplers [8]–[12], Fig. 1. The slowly varying complex amplitudes of fundamental wave U and second harmonic V in the directional coupler structure fulfil equations [13]:

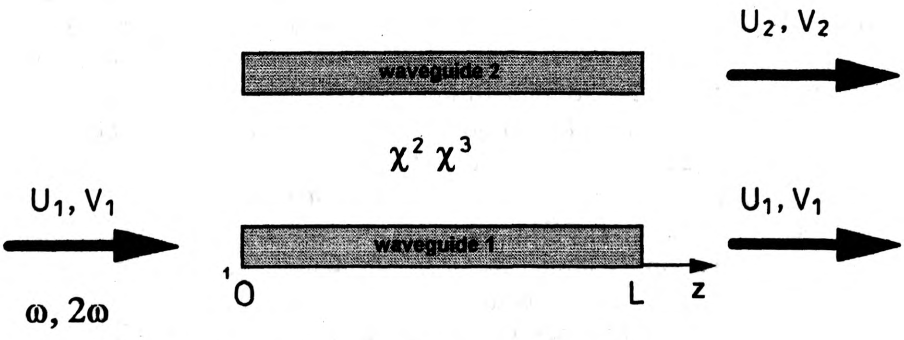

Fig. 1. Schematic drawing of the analysed directional coupler

$$i\left[\frac{\partial}{\partial z}+\frac{1}{v_U}\frac{\partial}{\partial t}\right]U_l+\frac{1}{2}D_U\frac{\partial^2}{\partial t^2}U_l$$
$$=\gamma U_l^* V_l\exp(-i\Delta\beta z)+\left(\frac{1}{2}r_{UU}|U_l|^2+r_{UV}|V_l|^2\right)U_l+\varkappa_U U_{2-l},$$
$$i\left[\frac{\partial}{\partial z}+\frac{1}{v_V}\frac{\partial t}{\partial}\right]V_l+\frac{1}{2}D_V\frac{\partial^2}{\partial t^2}V_l$$
$$=\gamma U_l U_l\exp(i\Delta\beta z)+(r_{VV}|V_l|^2+2r_{UV}|U_l|^2)V_l+\varkappa_V V_{2-l} \tag{1}$$

where $l=1, 2$ describes the waveguide number, $\Delta\beta$ is a difference of propagation constants of both frequencies, $v_a=$ $(a=U,V)$ is a group velocity, D_a is a group velocity dispersion, γ is a second-order nonlinearity coefficient, r_{ab} is a third-order nonlinearity coefficient and $\varkappa_a$ is a coupling coefficient of the directional coupler. In this paper for numerical analysis it was assumed that $\Delta\beta<0$ and $v_U=v_V$, $D_U=D_V$ and $\varkappa_U=\varkappa_V$. It should be pointed out that typically $\varkappa_V>\varkappa_U$ and therefore both pulses are switched at different lengths.

An as input pulse launched into the waveguide 1 (Fig. 1) of the directional coupler the solution in the form of bright solitary waves governed by the quadratic nonlinearity is taken:

$$U_1=\frac{3\sqrt{D_1D_2}}{\gamma\tau_0^2}\cosh^{-2}(\tau/\tau_0),$$
$$V_1=\frac{3|D_1|}{\gamma\tau_0^2}\cosh^{-2}(\tau/\tau_0) \tag{2}$$

where $\tau=(t-z/v)$, $\tau_0^2=2|(2D_U-D_V)/\Delta\beta|$ and as an input in the second waveguide $U_2=V_2=0$ is taken.

First, the coupling phenomenon in the directional coupler without the third-order nonlinearity has been analysed (*i.e.* for $r_{ab}=0$). Figure 2 presents the pulse energies $E_{Vl}=\int|V_l|^2d\tau/\tau_0$ and $E_{Ul}=\int|U_l|^2d\tau/\tau_0$ evolution along the directional coupler length.

The switching of the temporal pulses in the waveguide directional coupler depends on the value of the normalised coupling $K=\varkappa\tau_0^2/D=L_S/L_B$, where $L_B=\pi/\varkappa$ is the half-beat length of the coupler and $L_S=\pi\tau_0^2/D$ is the dispersion length. For analysed solitary waves the second order nonlinearity compensates the effects caused by the dispersion, *i.e.*, the changes of the pulse shape and width. Therefore large values of the dispersion length are equivalent to small values of the nonlinearity terms. It means that for large coupling $K\gg 1$ the second-order nonlinearity terms are small in comparison with the coupling terms and the directional coupler behaves as a linear coupler (dotted lines in Fig. 2).

The different behaviour appears in the case of low coupling K, *i.e.*, for long half-beat lengths of the directional coupler. In this case, the influence of the quadratic nonlinearity is comparable with the directional coupling effect. The nonlinearity destroys the resonant coupling conditions, first for the waves of the second harmonic (for $K\sim 1$), and then also for waves at the fundamental frequency (for smaller

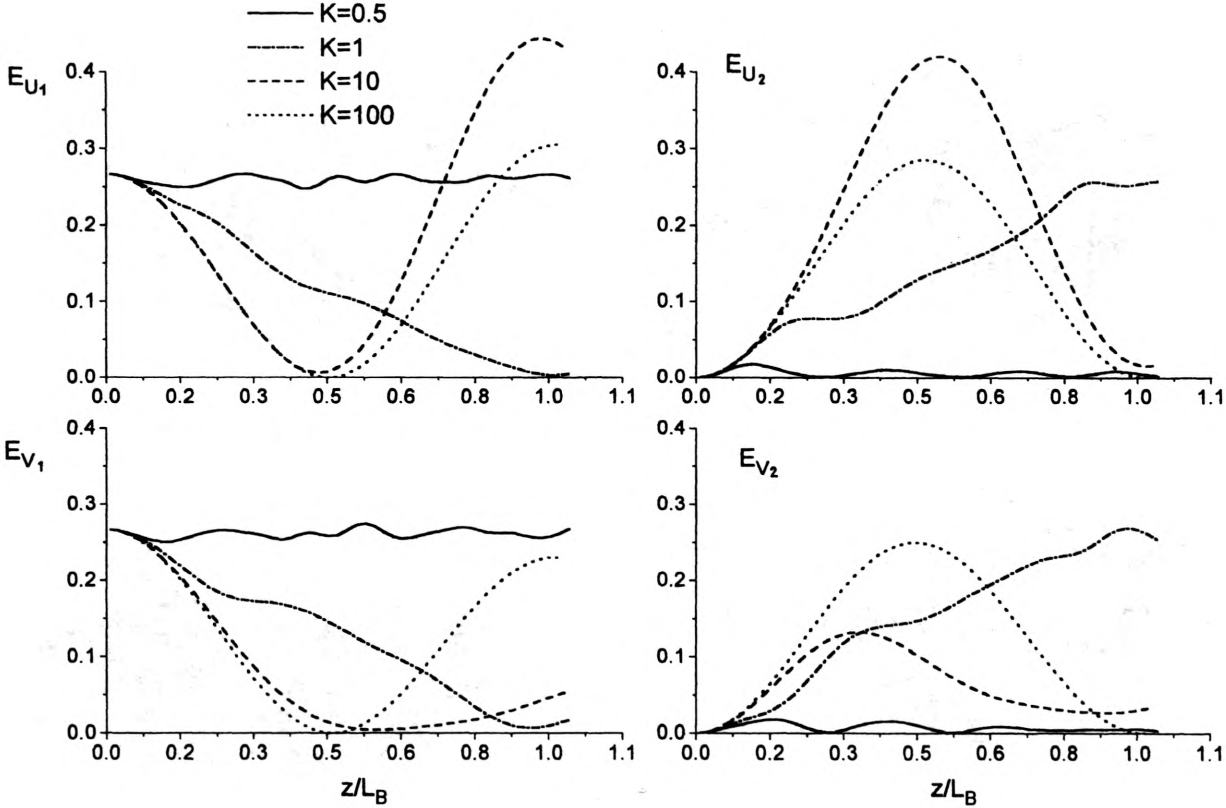

Fig. 2. Pulse energy evolution along the coupler length for the fundamental frequency (top) and second harmonic (bottom) in the input waveguide 1 (left) and the output waveguide 2 (right) for different values of the normalised coupling coefficient *K*

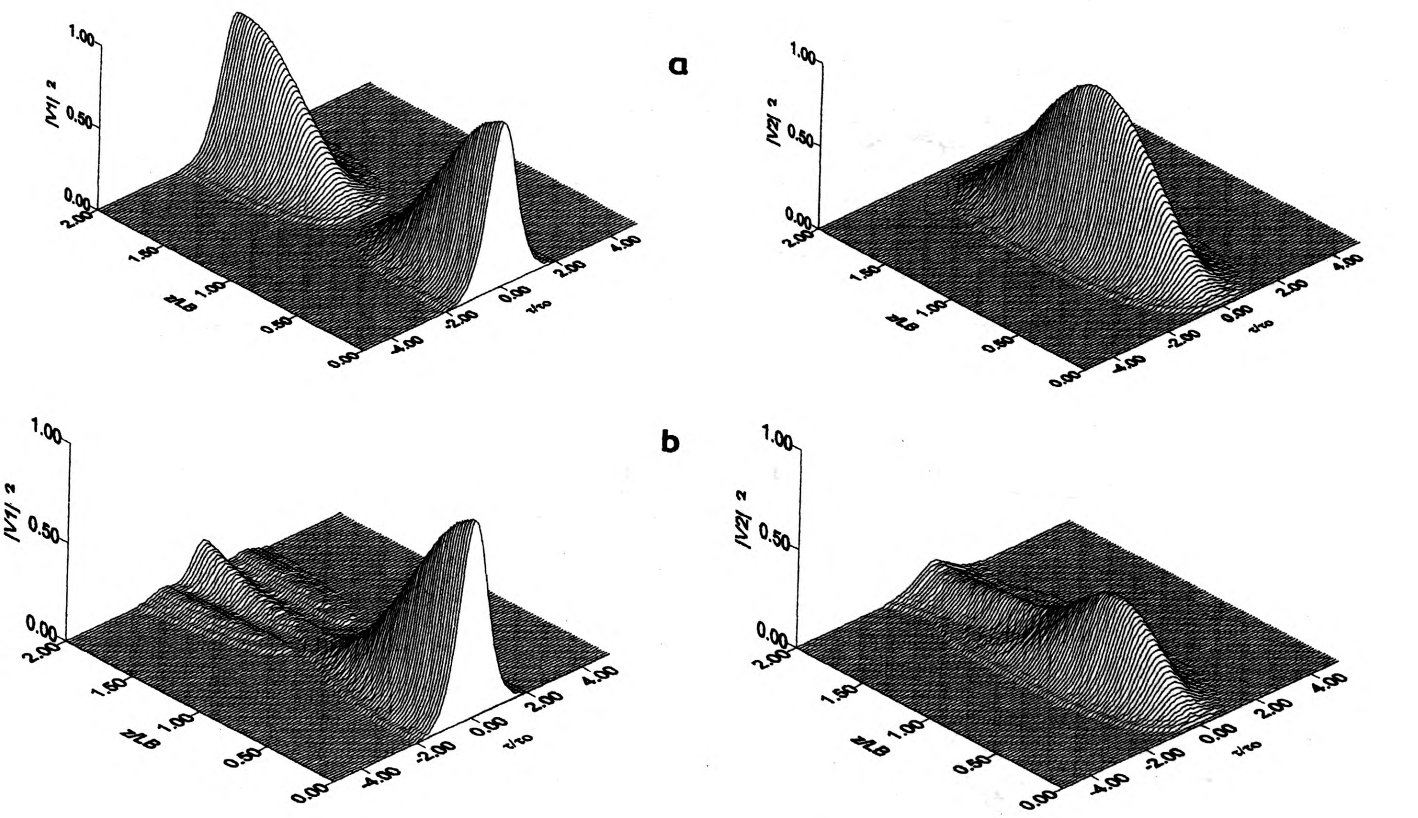

Fig. 3. Second harmonic pulse envelope $|V_i|^2$ in the input waveguide 1 (left) and output waveguide 2 (right) for two values of the normalised coupling: **a** – $K = 100$ and **b** – $K = 10$

coupling coefficients K). The effect of diminishing of coupling due to the nonlinearity is similar to the effects in directional coupler with the third-order nonlinearity (Kerr-type) [10] – [12]. In the Kerr-type nonlinear directional couplers the effect of increasing the coupling length (like in analysed coupler for $K = 1$) is also observed.

The changes in coupling cause that pulse envelope shapes are also modified. This is presented in Fig. 3, where the second harmonic pulse envelopes evolution in both waveguides is plotted for two values of the effective coupling coefficient K.

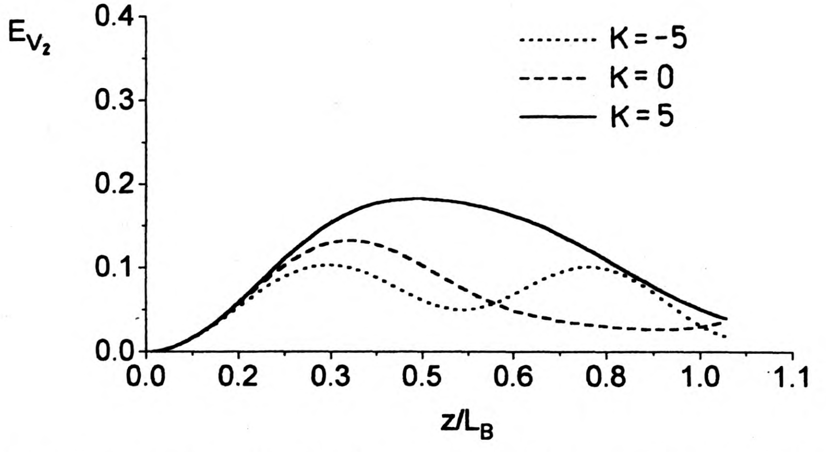

Fig. 4. Pulse energy evolution along the coupler length for second harmonic in the output waveguide 2 for the normalised coupling coefficient $K = 10$ and for different values of the third-order nonlinearity coefficient R

The effect caused by the second-order nonlinearity can be compensated by the third-order nonlinearity existing in the directional coupler. The sign of the r_{ab} coefficients (connected with the third-order nonlinearity) can be the same or opposite to the sign of the γ coefficient (connected with the second-order nonlinearity). Therefore the third-order nonlinearity can amplify or diminish the second-order nonlinearity effect. This phenomenon is shown in Fig. 4. The third-order nonlinearity coefficients were taken as follows: $2r_{UU} = 6r_{VV} = 12r_{UV}$ and the dimensionless coefficient $R = 2r_{UU}D/\tau_0^2$ was introduced. The third-order nonlinearity causes the focusing of the output pulses for the $R < 0$ and defocusing for $R > 0$. However, for $R < 0$ the energies in both output pulses are different.

In conclusion, the directional coupler for the solitary waves governed by the second order nonlinearity has been analysed. It has been shown that the third-order nonlinearity allows to modify the switching phenomena and it seems to be useful especially for long couplers (for small coupling coefficients). Presented results can be applied to spatial case of solitary waves coupled between two planar waveguides of directional coupler.

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