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# Intensity-controlled nonreciprocal coupler

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Intensity-controlled nonreciprocal coupler is proposed and studied analytically and numerically. The structure consists of two planar waveguides – the dielectric waveguide and the asymmetric magnetooptic waveguide – coupled through a nonlinear Kerr-type medium. The operation of the device utilises the nonreciprocal phase shift for TM modes in magnetooptic waveguide. The coupled mode equations are solved analytically in terms of Jacobian elliptic function. The transmission characteristics and the coupling efficiency for forward and backward direction propagation have been analysed depending on the input power. The transmission characteristics of the coupler are more sensitive to the input power for this direction propagation for that the linear propagation mismatch between the waveguides is larger. Isolation of the device increases with increasing input power.

## 1. Introduction

The magnetooptical waveguides have been receiving a great deal of attention because they have the application as basic elements for the nonreciprocal devices realized in planar form [1]–[6]. The nonreciprocal devices, such as isolators and circulators, are needed to stabilize the semiconductor–laser operation to avoid self-oscillation of the laser due to reflection in the communication system. Most proposals for waveguide type isolators are based on nonreciprocal TE–TM mode conversion [7]–[9]. In order to achieve efficient TE–TM conversion, the precise phase matching between the interacting modes is necessary by orienting the optic axis of the materials precisely and by controlling the thickness of the film accurately. These requirements are difficult to fulfil in practice. The other concept for an optical isolator is based on the nonreciprocal phase shift for TM modes in magnetooptical waveguide in the so-called Voigt (or equatorial) configuration [5], [6]. It was shown that propagation constant of the TM mode is different for the propagation in the opposite direction in the presence of dc magnetic field applied perpendicular to the direction of propagation. This concept has the advantages because it involves only one polarization, namely TM and therefore no need of phase matching. New possibilities for the realization of integrated optical isolators and circulators offer nonreciprocal couplers. A general coupled mode theory for the structure containing a gyrotropic (ferrite) waveguide was presented by AVAI *et al.* [10]. Based on qualitative and numerical analysis, the feasibility of such coupling structures as the new type of nonreciprocal devices for millimeter-wave applications was studied. The coupling between the two parallel gyrotropic rib waveguides was studied by

ERDMANN *et al.* [11]. Using the method of coupled modes as well as normal mode theory, the basic properties of the nonreciprocal coupler was analyzed.

In this paper, the nonreciprocal coupler with nonlinear Kerr-type coupling medium is studied analytically and numerically. The possibility of using nonlinear couplers for ultrahigh-speed data processing has aroused great interest into the investigation of its characteristics. The first analytical model making use of the coupled-mode theory was proposed by JENSEN [12]. This work was followed by other theoretical models, which improved on the Jansen's analysis [13]–[18]. The introducing of the nonlinear material into coupler structure gives additional "degree of freedom", namely power flow down the coupler. The power dependence of the coupling coefficient allows good control of the switching characteristics of a nonlinear coupler. The inclusion of the nonlinear material into magnetooptic (gyrotropic) coupling structure allows changing of the efficiency of the coupling coefficients with power and therefore the controlling of the transmission characteristics for the forward and the backward direction propagation. This shows the potential for application of the magnetooptic-nonlinear waveguide in the design and the realization of a new class of integrated devices the operation of which combines the magnetooptic and nonlinear effects [19].

This paper is organized as follows. Section 2 briefly reviews the basic properties of the magnetooptic waveguide which utilizes the nonreciprocity TM modes only. In Section 3, the nonreciprocal nonlinear coupler is analyzed theoretically by means of the coupled mode theory and nonlinear coupled-mode equations are solved analytically. The numerical results are presented in Section 4, and effect of nonlinear coupling on the transmission of the device is discussed. Finally, in Section 5 the conclusions are given.

## 2. Nonreciprocal waveguide

This section briefly reviews the basic properties of magnetooptic waveguide to be used in nonreciprocal coupling structure analysed in the next section. The guiding structure to be considered consists of three layers – the magnetooptic substrate, the dielectric film and the dielectric top layer. The external magnetic field $H_0$ is applied parallel to film plane and perpendicular to the direction of propagation which is along the $z$-axis (Voigt configuration). The magnetooptic (gyrotropic) material is characterized by a Hermitian tensor $\varepsilon$ of the form

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{1s} & 0 & i\delta \\ 0 & \varepsilon_{1s} & 0 \\ -i\delta & 0 & \varepsilon_{1s} \end{bmatrix}. \tag{1}$$

The off-diagonal element $\delta$ is the result of the applied magnetic field and is related to the specific Faraday rotation $\Theta_F$ by $\delta = n_0\Theta_F\lambda/\pi$, where $\lambda$ denotes the wavelength and $n_0$ – refractive index. The sign of $\delta$ changes if the direction of the magnetic field

$\mathbf{H}_0$ is reversed. The film and cover region are assumed isotropic with the dielectric permittivity $\varepsilon_f$ and $\varepsilon_c$, respectively.

In Voigt configuration TE modes are not disturbed by dc magnetic field and no TE−TM mode conversion occurs [8], [9]. TM modes, on the other hand, are elliptically polarized in the plane transverse to H and have nonreciprocal character (*i.e.*, the dependence of the propagation constant on the direction propagation). The magnetic field for TM waves propagating along $z$-axis with angular frequency $\omega$ and propagation constant $\beta$ is written as

$$\mathbf{H}(x,z) = H(x)\mathbf{i}_y \exp[i(\beta z - \omega t)]. \tag{2}$$

In the magnetooptic medium $H(x)$ obeys the equation

$$\frac{d^2H}{dx^2} + (k_0^2 s - \beta^2)H = 0, \tag{3}$$

here $s = (\varepsilon_s^2 - \delta^2)/\varepsilon_s^2$, and $k$ is the free-space wave number. Once Equation (3) is solved the components of the electric fields $E_x$ and $E_z$ can be obtained directly from Maxwell's equations. Assuming decaying field in the substrate and in the cover and applying the boundary condition at the film-substrate and film-cover interfaces one obtains the relation

$$d\sqrt{k_0^2\varepsilon_f - \beta^2} = \tan^{-1}\left[\frac{\varepsilon_f}{\varepsilon_c}\sqrt{\frac{\beta^2 - k_0^2\varepsilon_c}{k_0^2\varepsilon_f - \beta^2}}\right] + \tan^{-1}\left[\frac{\varepsilon_f(\sqrt{\beta^2 - k_0^2 s} - (\beta\delta/k_0\varepsilon_{1s}))}{s\sqrt{k_0^2\varepsilon_f - \beta^2}}\right]. \tag{4}$$

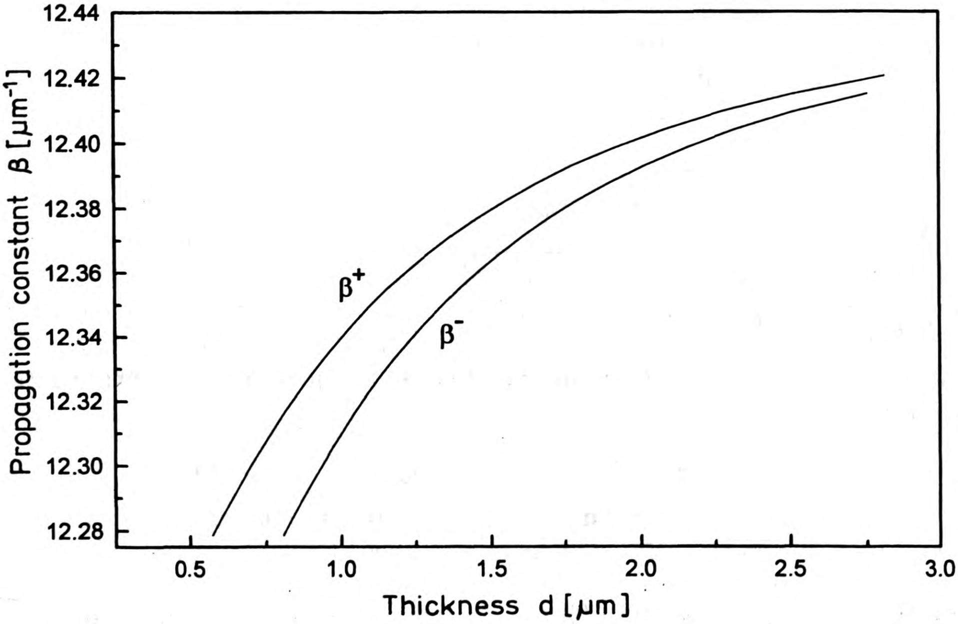


**Fig. 1. Propagation constant of forward (+) and backward (−) $TM_0$ mode in a planar magnetooptical waveguide versus the film thickness**

Equation (4) is the dispersion relation of the TM modes which determines the propagation constant $\beta$. The nonreciprocity of TM modes is evident from (4). Due to off-diagonal elements in dielectric tensor (1), term linear in $\beta$ enters this equation. Thus, the value of the propagation constant is different for forward and backward propagation (*i.e.* $\beta$ replaced by $-\beta$, respectively). We note that changing the sign of $\delta$ (reverse the direction of dc magnetic field) has the same effect as changing the sign of $\beta$. Figure 1 shows the propagation constants of forward and backward $TM_0$ modes in a planar magnetooptic waveguide versus the film thickness $d$. The separation between two curves signifies the nonreciprocity of the TM modes. The value for $\delta$ chosen for the calculation is two orders of magnitude larger than the typical values. This is done to accentuate the effect of $\delta$ on the nonreciprocity. The difference $\Delta\beta = \beta^+ - \beta^-$ increases approximately linearly with $\delta$.

## 3. Nonreciprocal coupler structure

Figure 2 shows the coupler structure to be considered. The nonreciprocal waveguide with magnetooptic substrate (waveguide labelled 1) is placed parallel near a conventional dielectric waveguide (waveguide labelled 2). The nonlinear material covers the region between two waveguides. The propagation is along the $z$-direction. The electric and magnetic fields are proportional to $\exp(-i\omega t)$ and this term is suppressed for simplicity. The structure is analysed using the coupled mode technique expansion in modes of the unperturbed individual waveguides.

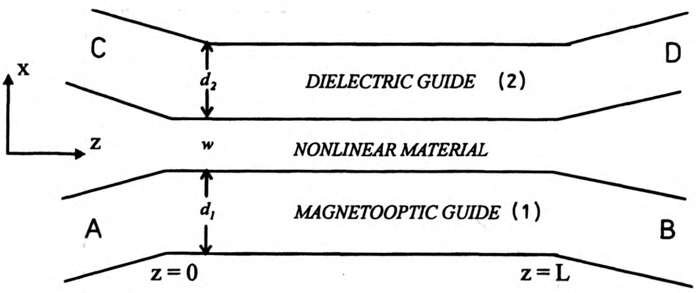


Fig. 2. Geometry of the nonreciprocal coupler

Since the operation of this device involves TM modes only, we express the electric and magnetic field as

$$\mathbf{E}_j(x,z) = (E_{jx}(x)\mathbf{i}_x + E_{jz}(x)\mathbf{i}_z)e^{i\beta_j z}, \quad \mathbf{H}_j(x,z) = H_{jy}(x)\mathbf{i}_y e^{i\beta_j z}, \quad (j = 1, 2). \tag{5}$$

According to the coupled mode theory the total field is written as a linear combination of the mode fields of two unperturbed individual waveguides:

$$\mathbf{E} = a_1\mathbf{E}_1 + a_2\mathbf{E}_2, \quad \mathbf{H} = a_1\mathbf{H}_1 + a_2\mathbf{H}_2 \tag{6}$$

where: $a_1 = a_1(z)$, $a_2 = a_2(z)$ are the complex amplitudes of the modes. We will assume that both waveguides are single-mode waveguides. Substituting (6) into

Maxwell's equations for the coupled structure one may derive the coupled mode equations:

$$\left(A^q_{11}\frac{d}{dz}+iB^q_{11}\right)a^q_1+\left(A^q_{12}\frac{d}{dz}+iB^q_{12}\right)a^q_2=0,$$
$$\left(A^q_{22}\frac{d}{dz}+iB^q_{22}\right)a^q_2+\left(A^q_{21}\frac{d}{dz}+iB^q_{21}\right)a^q_1=0 \tag{7}$$

where:

$$A^q_{jj}=\int(\mathbf{E}^q_j\times\mathbf{H}^{*q}_j+\mathbf{E}^{*q}_j\times\mathbf{H}^q_j)\mathbf{i}_z dx,$$
$$B^q_{jj}=\beta^q_j A^q_{jj}+\omega\mu_0\int[\mathbf{E}^{*q}_j(\varepsilon-\varepsilon_j)\mathbf{E}^q_j]dx,$$
$$A^q_{jk}=\int(\mathbf{E}^q_k\times\mathbf{H}^{*q}_j+\mathbf{E}^{*q}_j\times\mathbf{H}^q_k)\mathbf{i}_z dx,$$
$$B^q_{jk}=\beta^q_k A^q_{jk}+\omega\mu_0\int[\mathbf{E}^{*q}_j(\varepsilon-\varepsilon_k)\mathbf{E}^q_k]dx,\quad (jk=1,\ 2;\ j\neq k),\ (q=+,-), \tag{8}$$

$\varepsilon$ is tensor permittivity of the coupled structure and is real function of position $\varepsilon=\varepsilon(x)$. The quantities expressed in (8) with superscript $q$ ($q=+,-$) refer to the modes propagating in opposite direction. The positive sign "+" applies to forward propagation, the negative sign "−" is taken to backward propagation. In order to shorten the notation we will omit in following the superscript $q$. The quantities $A_{11}$ and $A_{22}$ are four times the Poynting flux energy of the unperturbed waveguides. $A_{12}$ and $A_{21}$ represent the cross integrals of the two modes. We will simplify the consideration by assuming that the cross integral $A_{ij}$ is approximately zero due to weak coupling of two basis waveguides. The nonlinear material occupying the region between waveguides is assumed to be Kerr type so that the nonlinear dielectric function is

$$\varepsilon=\varepsilon^L+\alpha(|E|^2),\quad (d_1\leqslant x\leqslant d_1+w) \tag{9}$$

where $\varepsilon^L$ is linear part, $\alpha$ is a nonlinear coefficient.

For two-waveguide configuration with nonlinear dielectric permittivity (9) the coupled mode equations (7) governing the mode amplitudes $a_1$ and $a_2$ can be written as follows:

$$\frac{da_1}{dz}=-iN_{11}a_1-iN_{12}a_2-i(Q_1|a_1|^2+Q_3|a_2|^2)a_1,$$
$$\frac{da_2}{dz}=-iN_{22}a_2-iN_{21}a_1-i(Q_2|a_2|^2+Q_3|a_1|^2)a_2, \tag{10}$$

$N_{11}$, $N_{22}$ and $N_{12}$, $N_{21}$ are the normalized linear self- and cross-coupling coefficients and normalization constant is $P=A_{11}=A_{22}$. The first and second terms in parentheses arise from the nonlinear interaction of the modes and account for the self- and cross-phase modulation, respectively [12]. The nonlinear coupling coefficients are defined as follows:

$$Q_1=\frac{\alpha\omega\varepsilon_0}{P}\int_{d_1}^{d_1+w}|E_1|^4dx,$$

$$Q_2 = \frac{\alpha\omega\varepsilon_0}{P} = \int\limits_{d_1}^{d_1+w} |E_2|^4 dx,$$

$$Q_3 = \frac{\alpha\omega\varepsilon_0}{P} \int\limits_{d_1}^{d_1+w} [|E_1|^2 |E_2|^2 + |E_1 E_2^*|^2] dx. \tag{11}$$

The system of equations (10) can be solved analytically, subject to the initial condition that all the power is launched into one waveguide.

If we assume, $a_2(0) = 0$ and $|a_1(0)|^2 = I_{in}$, then the following equation for the power propagating in the waveguide 2 is obtained [20], [21]:

$$\left(\frac{dI_2}{dz}\right)^2 = r^2 I_2(I_t - I_2)[I_2^2 + (I_t - 2P)I_2 + (I_t - P)^2 + h^2] = 0 \tag{12}$$

where:

$$\begin{aligned} &h^2 = 4N_{12}N_{21}/r^2, \\ &r = (N_{12}/N_{21}q_1 - q_2)/2, \quad q_1 = Q_1 - 2Q_3, \quad q_2 = 2Q_3 - Q_2, \\ &P = (N_{11} - N_{22} + q_1 I_0)/r, \\ &I_0 = |a_1|^2 + N_{12}/N_{21}|a_2|^2 = \text{const}, \end{aligned} \tag{13}$$

and $I_2 = a_2 a_2^*$ is the intensity in waveguide 2. $I_t$ is the power remained in waveguide 2 at the maximum transfer coupling length, and is found from equation

$$(I_t - P)^2 I_t + h^2(I_t - N_{21}/N_{12}I_{in}) = 0. \tag{14}$$

The intensity in the magnetooptic guide 1 is given by

$$I_1 = I_{in} - N_{12}/N_{21}I_2. \tag{15}$$

Equation (12) can be integrated in terms of elliptic function [22]. For the case $\Delta < 0$, where $\Delta$ is determinant of trinomial in the parentheses in Eq. (12) one obtains the solution

$$I_2(z) = \frac{I_t[1 - \mathrm{cn}(pz;m)]}{[(1+w) - (1-w)\mathrm{cn}(pz;m)]} \tag{16}$$

where:

$$\begin{aligned} &p = r\sqrt{fg}, \\ &f = \sqrt{g^2 + 2I_t(I_t - P)}, \\ &g = \sqrt{(I_t - P)^2 - h^2}, \\ &w = f/g, \\ &m[I_t^2 - (f-g)^2]/(4fg), \end{aligned} \tag{17}$$

and $\mathrm{cn}(pz;m)$ is a Jacobian elliptic function. In the limit of very small input intensities ($m \to 0$ and $t \to 0$) one obtains solutions for the linear magnetooptical coupler.

## 4. Numerical results and discussion

In this section, we present numerical results. All numerical results presented here were calculated for the following values: free-space wavelength $\lambda = 1.15$ µm, film permittivity $\varepsilon_{1f} = \varepsilon_{2f} = 5.198$, cover permittivity $\varepsilon_{1c} = \varepsilon_{2c} = 4.796$, substrate permittivity $\varepsilon_{1s} = 5.033$, $\varepsilon_{2s} = 4.796$, gyrotropic parameter $\delta = 0.25$, nonlinear coefficient $\alpha = 6.4 \cdot 10^{-12}$ $m^2V^{-2}$, film thickness $d_1 = 1.3$ µm, $d_2 = 1.4$ µm, distance between waveguides $w = 0.5$ µm. The propagation constants in the magnetooptic guide 1 are different for forward and backward propagating modes and are $\beta_1^+ = 12.3664\ \mu m^{-1}$ and $\beta_1^- = 12.3460\ \mu m^{-1}$, respectively. The parameters of the dielectric guide 2 are chosen to obtain a good matching between the backward propagating modes in two-waveguide structure. The determined propagation constant in the waveguide 2 is $\beta_2 = 12.3466\ \mu m^{-1} \approx \beta_1^-$.

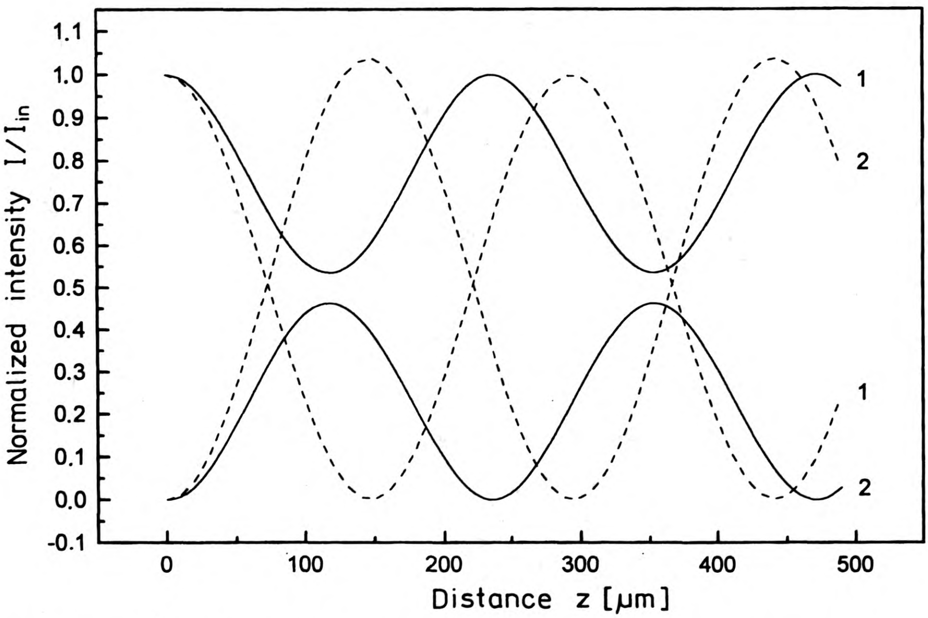


Fig. 3. Intensity of the magnetooptic (1) and the dielectric (2) guide versus propagating distance in the linear regime. Solid and broken lines are for the forward and the backward propagation, respectively

Figure 3 shows the power carried in linear regime by each waveguide with propagating distance for the forward and the backward propagation. The energy exchange rate depends strongly on $|\beta_1 - \beta_2|$. Nearly complete coupling takes place for the backward propagation as the phase velocity mismatch between the both guides is small for this direction propagation. Conversely, for the opposite direction the propagation constant mismatch is larger and the transfer energy between waveguides is small.

The dependence of the coupler transmission on input power for the forward and

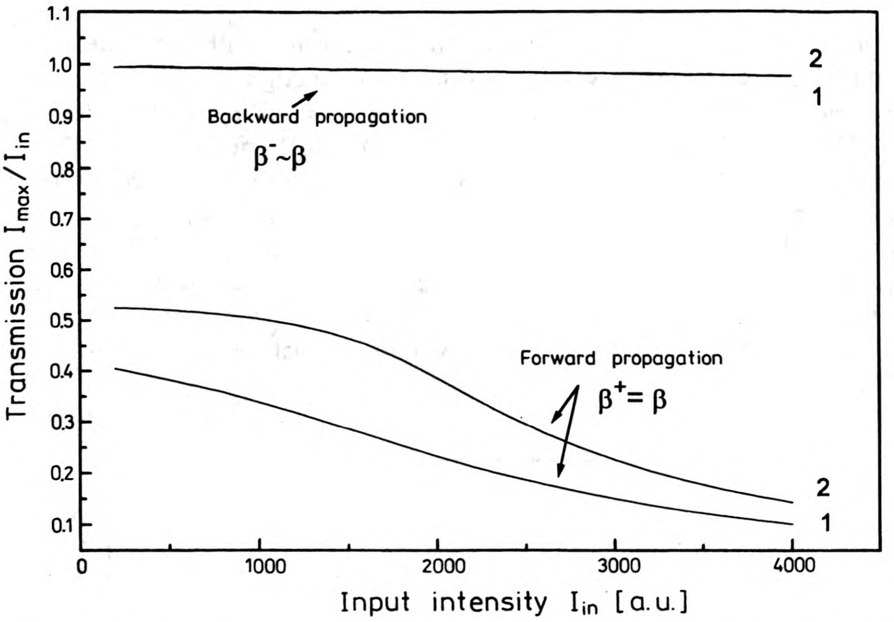


Fig. 4. Transmission (1) of the dielectric guide as function input intensity in magnetooptic guide, (2) of the magnetooptic guide as function input intensity in dielectric guide

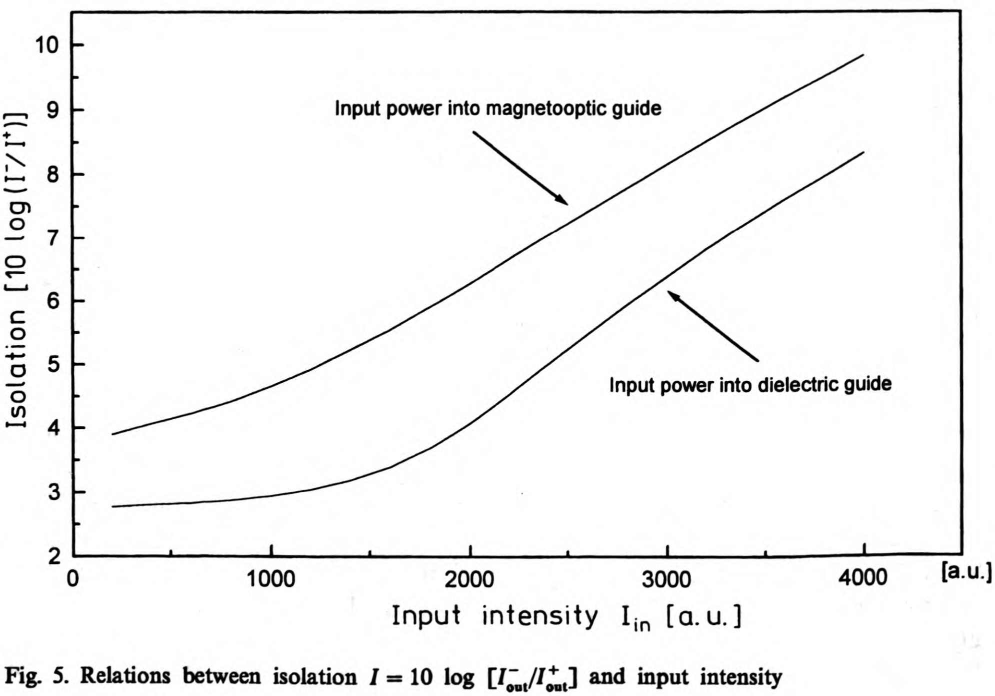


Fig. 5. Relations between isolation $I = 10 \log [I^{-}_{out}/I^{+}_{out}]$ and input intensity

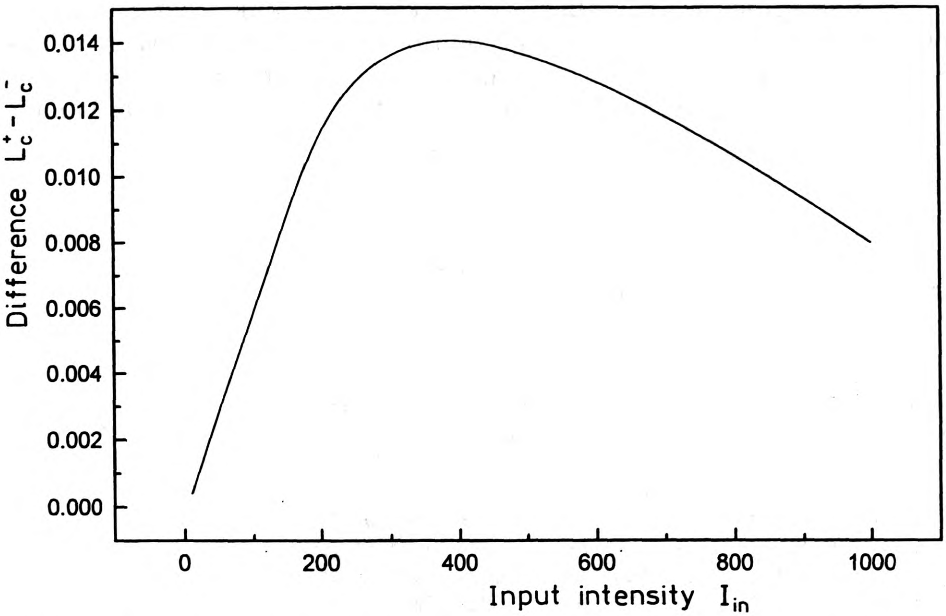


Fig. 6. Difference of the normalized coupling lengths for the forward ($L_c^+$) and backward ($L_c^-$) direction propagation in dielectric guide versus input intensity in magnetooptic guide

the backward propagation is shown in Fig. 4. For the backward propagation (the small linear mismatch) the transmission changes slightly with the input power. For forward propagation (larger deviation between propagation constants $\beta_1$ and $\beta_2$) the transmission characteristics are more sensitive to the input intensity. The change is stronger if all of the power is initially launched into dielectric waveguide. Figure 5 presents the relation between the isolation $I$ of the coupler structure and input intensity. It is shown that isolation increases with increasing of the input intensity. We note that when the input energy is directed into magnetooptic guide, the isolation becomes larger. Difference of the normalized coupling length for the forward and backward propagation in dielectric guide versus input intensity in magnetooptic guide is shown in Fig. 6.

## 5. Conclusions

The nonreciprocal coupler with the nolinear Kerr-type coupling medium has been investigated by the coupled mode theory. The operation of the device makes use of the nonreciprocal phase shift for TM modes in magnetooptical waveguide. Analytical expressions in terms of Jacobian elliptic function for the power exchange between two waveguides have been presented. The transmission characteristics and the coupling efficiency for opposite direction propagation have been analysed depending on the input power. It was shown that the nonlinear coupling modifies the nonreciprocal coupling in two-waveguide structure in different way depending on the direction of propagation. The transmission characteristics of the coupler are more sensitive to the input power for this direction of propagation for that the linear propagation constants mismatch between the waveguides is smaller. The changes are stronger if all of the power is initially launched into dielectric waveguide. Isolation of the device increases with increasing input power.

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