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First-order nonlinear optics — aberrationless, nonspecular and bistable effects

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Basics of nonlinear first-order optics in a multidimensional space-time-frequency domain are presented. The linear first-order optics formalism is extended on nonlinear propagation, reflection and transmission phenomena. The extension is accomplished by use of the recently introduced aberrationless effects of nonlinear propagation and nonspecular effects of reflection/transmission at nonlinear planar structures.

One of the useful applications of the ray transfer matrix (RTM) [1] and Wigner distribution function (WDF) [2] formalisms is to compute an optical beam or pulse parameters simultaneously in space-time and frequency domains. In the simplest case of dim $(1+1)$ structure, *i.e.*, with one transverse x and one longitudinal z directions, light propagation in a quadratic medium can be described by the 2×2 *ABCD* (RTM) matrix

$$\begin{bmatrix} x_0 \\ s_0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_i \\ s_i \end{bmatrix}. \tag{1}$$

The conjugate variables x, s in the phase space in the input $z = z_i$ and output $z = z_0$ planes are linearly related by the RTM elements A,B,C,D, which for the free-space propagation takes a particularly simple form: $A = 1$, $B = z_0 - z_i$, $C = 0$ and $D = 1$. Within first-order optics a beam or pulse field travels along trajectories given by the conjugate pair x, s dynamically dependent on the propagation distance z.

Several extensions of the first-order formalism have been made to model the optical field changes in more complex configurations [1]. Systems with focusing elements, misalignment, axis curvature, gain or loss in one or two transverse dimensions were analyzed in this fashion by use of the generalized *ABCD* matrices. Among others, the nonspecular phenomena [3]–[6] of reflection and transmission at the multilayered planar structures were described by the nonspecular 3×3 RTM $(z = z_i = z_0)$ [6]

$$\begin{bmatrix} x_0 \\ s_0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & C & e \\ B & D & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ s_i \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\delta_z + i(1-\mu^2) & z\delta_0 + \delta_z \\ 0 & 1 & \delta_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ s_i \\ 1 \end{bmatrix}. \tag{2}$$

The elements of this matrix are specified by nonspecular distortions of the beam interacting with the reflecting/transmitting structure, *i.e.*, the transverse δ_x and longitudinal δ_z shifts of the beam waist centre, angular shift δ_0 of the beam axis and the beam waist size magnification or contraction μ.

The optical signal processing through the first-order system is frequently described by the WDF [7]

$$W(x, s; z) = \int V(x+x'; z) V^*(x-x'/2; z) \exp(-isx') dx', \tag{3}$$

which expresses simultaneously both spatial (or temporal, or both) and spectral characteristics of the beam or pulse. The crucial feature of the WDF seems to be its invariance with respect to the *ABCD* matrix transformation. For example, in the free-space dim (1+1) propagation case, the WDF invariance reads

$$W(x_0, s_0; z_0) = W(x_0 - z_0 s_0, s_0; 0), \tag{4}$$

as a consequence of that the field envelope $V(x; z)$ is a solution to the parabolic Fock equation

$$[i\partial_z + (1/2)\partial_x^2] V(x; z) = 0. \tag{5}$$

The WDF provides, in spite of an on-axis amplitude term indepenedent of x, all the information necessary to describe the shape and the spectrum of the optical signal, *e.g.*, the beam size, spectral width, phase-front curvature or the pulse chirp, position of the signal axis and centre frequency of its spectrum, and so on. The detailed description of the WDF characteristic features can be found in [7], [8].

In nonlinear optics, the first-order analysis has been partially applied within the so-called aberrationless approach [9], [10]. Several versions of the nonlinear RTM were derived mainly in the context of passive mode locking in laser cavities, Z-scan measurements in nonlinear media, pulse compression and, in general, multidimensional pulse or beam propagation down the samples of medium with the third-order Kerr type nonlinearity [11]–[15]. Recently, a multidimensional analysis of propagation problems of this sort revealed possibility to convert the nonlinear process into the linear one as described by Eqs. (1)–(5) [16], [17]. Such a conversion can be accomplished within the first-order formalism by use of the aberrationless effects, that is the nonlinear changes of the beam or pulse field with respect to its linear low-power propagation.

Un to the second-order (in x) terms the nonlinear propagation of a fundamental mode in a Kerr medium is completely described by four aberrationless effects, namely, the self-shortening of the optical distance, the shift of the beam waist, modification of the beam radius, and on-axis beam phase shift [16], [17]. These effects scale the coordinate frame in the phase domain and the propagation equations into the form specific to the linear process. Therefore, the nonlinear generalizations of the WDF, RTM and the dynamics of the optical system can be reformulated accordingly with a formal analogy to Eqs. (1)–(5). In this way, the nonlinear process can be efficiently traced on the linear low power level. In the context of the well developed first-order optics formalism a generalization into the higher-order nonlinear propagation seems also straightforward although certainly nontrivial [16], [17].

Numerical verification of the method has been recently performed in the context of the beam propagation in thick nonlinear samples, simulations of beam or pulse multidimensional compression and novel modifications of Z-scan measurements [17]. Careful examination of the nonspecular reflection/transmission theory reveals also possibility of analyzing bistable nonspecular phenomena within the nonlinear first-order formalism [18]. Other possible applications of the method are under current investigation.

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