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Spatial solitons in nonlinear quadratic media: recent theoretical results

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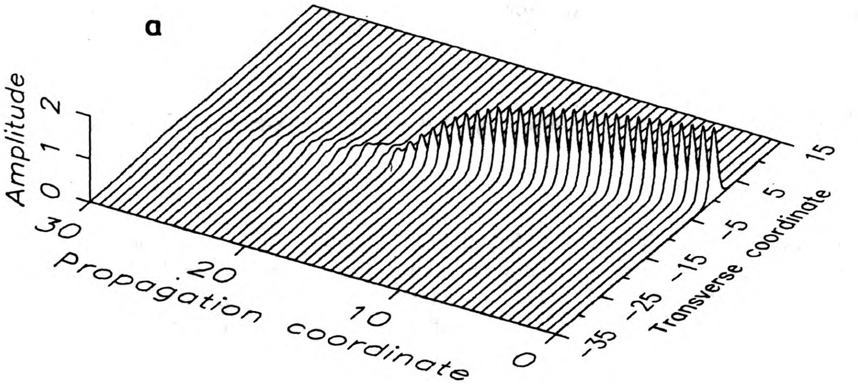
We present a summary of the theoretical understanding of spatial solitons in nonlinear quadratic media. We concentrate on the recently discovered stationary "walking solitons". These solitons exist in the presence of Poynting vector beam walk-off between the interacting waves, for different wave intensities and transverse velocities, and constitute a two parameter family.

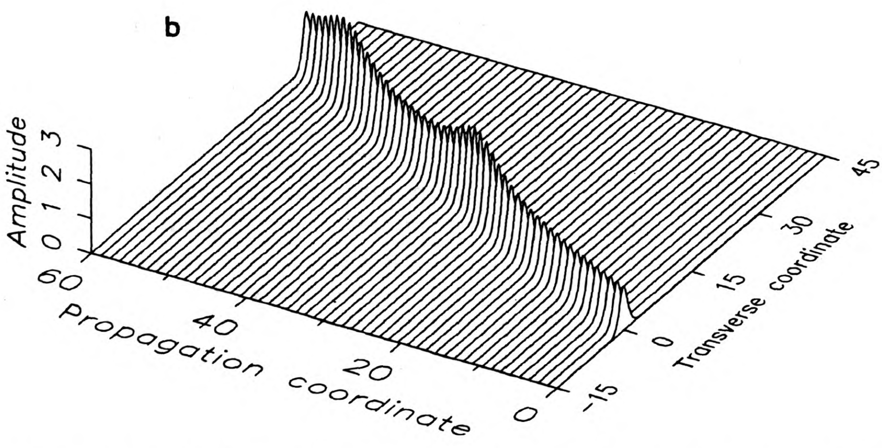
Optical solitons, both temporal and spatial are a subject of constant intense investigation due to their fascinating properties and potential applications to optical communications and different optical devices. Until recently optical solitons have been pursued using the Kerr effect in cubic nonlinear media, and the photorefractive effect.

However, solitons (more properly, solitary waves) also form in quadratic nonlinear media [1]–[12], and they have been observed experimentally in second-harmonic generation settings [13], [14]. Numerical experiments show that spatial solitons in quadratic nonlinear media form for a variety of input and material conditions [7], [15], and families of stationary soliton solutions are known for both $(1+1)$ [8], [9], and $(2+1)$ [16], [17] geometries. In contrast to light beams propagation described by the celebrated nonlinear Schrödinger equation (NLSE), $\chi^{(2)}$ solitons exist and are stable in both $(1+1)$ and $(2+1)$ geometries, and here we concentrate on the propagation of spatial solitons.

One important feature of $\chi^{(2)}$ solitons is that they are made out of two waves. We consider the formation of spatial solitons in a second harmonic generation configuration, hence the solitons exist due to the mutual trapping of the fundamental and second harmonic beams. In general, except under suitable conditions, in the low-power regime the beams propagate along different directions due to Poynting vector walk-off present in anisotropic media. Numerical investigations and experimental observations show that in the presence of linear walk-off, when a soliton is formed the fundamental and second harmonic beams propagate locked together [18]–[20].

We have recently found two-parameter families of stationary, "walking solitons" in the presence of Poynting vector beam walk-off for both (1+1) [21] and (2+1) [22] geometries and we have showed that they exhibit new features in comparison to the corresponding non-walking solitons. The walking solitons do not have simple travelling-wave forms. They exist at different values of material parameters, with different wave intensities and soliton velocities. Three main points follow. First, even in the presence of the walk-off there are zero-velocity soliton solutions. Second, both the amplitudes and the phase-front shapes of the walking solitons depend strongly





Typical decay of the unstable (2+1)-dimensional stationary solutions: **a** – the spreading of the input stationary solitons at negative phase mismatch $\beta = -3$, velocity $v = -0.5$ and power flow $I = 50$, **b** – the reshaping of the input stationary solitons at phase-matching $\beta = 0$, velocity $v = 0.5$ and power flow $I = 36$

on the various parameters involved, and very particularly on the soliton velocity. In general, the walking solitons exhibit complicated phase-front curvatures and oscillating tails. Third, we have verified numerically in selected cases that the stationary solutions with increasing wavenumber for increasing energy flow are stable under propagation. The solutions corresponding to the negatively sloped branches of the nonlinear wavenumber shift-energy flow diagram would be unstable on propagation, so that they either eventually spread, or they decay into a stable walking soliton. In the latter case the beams reshape.

The figure shows the two different decays of the unstable (2+1)-dimensional stationary solutions in some representative cases. The nonlinear wavenumber shift-power flow diagram shows that at phase matching, the new (2+1)- dimensional walking solitons exist only above a certain threshold power [22], in contrast with non-walking solitons in (2+1) geometries which exist for any total wave power [17]. For both positive and negative mismatches, and depending on the sign and magnitude of soliton velocity, the threshold powers of (2+1)-dimensional walking solitons can be either lower or higher than those corresponding to non-walking solitons [22].

The excitation of the walking solitons is an important issue that deserves a comprehensive study that shall be addressed in a separate publication. Recall that excitation of spatial solitons from different inputs has been observed both numerically and experimentally under a variety of conditions [7], [13], [15], [18]–[20], but the existence of two-parameter families of walking solitons might open new possibilities to be explored in detail. Other important issues, such as the rigorous stability analysis of these lowest-order stationary walking solitons and the implications of the existence and properties of the walking solitons to the streering of beams in bulk media, ramain to be addressed elsewhere.

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