Optica Applicata, Vol. XXVI, No. 3, 1996

Letter to the Editor

Delusive simplicity of some formulae for birefringence media

F. RATAJCZYK

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

The refractive indices n' and n'' of the eigenwaves travelling along a birefringent medium in the direction of s (Fig.) are usually computed from the formula

$$\frac{s_x^2}{N-N_x^2}+\frac{s_y^2}{N^2-N_y^2}+\frac{s_z^2}{N^2-N_z^2}=0 \tag{1}$$

where: s_x, s_y, s_z – the projections of the versor **s** onto the x, y, z axes of the Cartesian coordinate system,

N_x, N_y, N_z – the reciprocals of n_x, n_y, n_z, respectively,

N – unknown magnitude of Eq. (1) having two values of N' and N'' being, in turn, respective reciprocals of n' and n''.

For the uniaxial media the formula (1) takes a simpler form

$$N^2 = N_e^2\sin^2\vartheta+N_o^2\cos^2\vartheta \tag{2}$$

where: N_e and N_o – are reciprocals of n_e and n_o, while

n_e – refractive index of the extraordinary wave,

n_o – refractive index of the ordinary wave,

ϑ – angle between the versor **s** and the optical axis.

From the Fresnel equation an accurate calculation procedure for the difference of the refractive indices $n' - n''$ of the eigenwaves [1] may be derived.

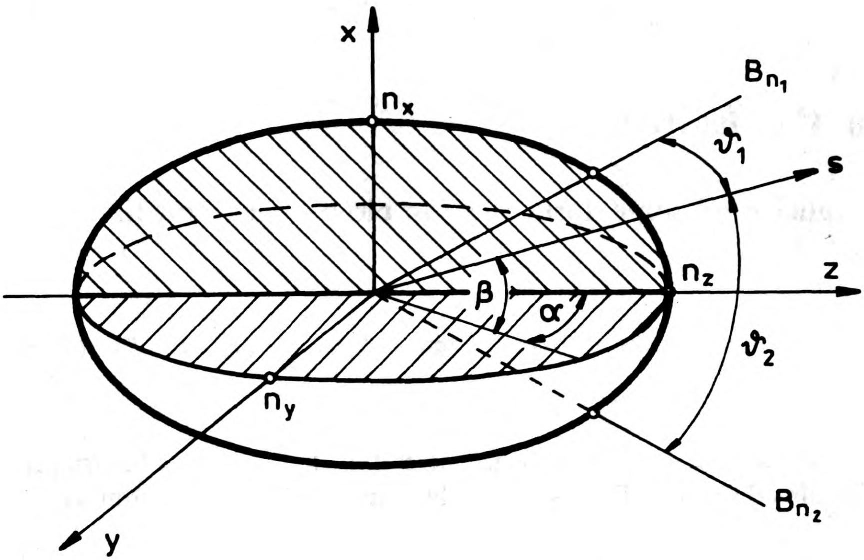
In the literature, the approximate formula is known allowing us to calculate in an easy way the birefringence $n'-n''$ of the medium in the direction of the versor **s** defined by the angles ϑ_1 and ϑ_2 (Fig.) which are created by the versor **s** and the binormal optical axes Bn_1 and Bn_2

$$n'-n'' = (n_z-n_x)\sin\vartheta_1\sin\vartheta_2. \tag{3}$$

For the uniaxial media, the form of this formula becomes even simpler, *i.e.*

$$n'-n'' = (n_o-n_e)\sin^2\vartheta. \tag{4}$$

The derivations of formulae (3) and (4) are given in [2]. These formulae are especially useful when making preliminary calculations with the help of a calculator. However, I felt always uneasy about the degree of approximations of the results



The ellipsoid of normals. Bn_1 and Bn_2 — binormal optical axes, n_x, n_y, n_z — main refractive indices of the birefringent medium, **s** — versor of the direction of the eigenwaves, α and β — polar coordinates of the versor **s**, ϑ_1 and ϑ_2 — angles between the versor **s** and the optical axes Bn_1 and Bn_2

obtained from formulae (3) and (4) as related to the real values obtained directly from the Fresnel formulae. In order to explain the due doubts, I calculated $n'-n''$ for several cross-sections of this ellipsoid of normals (Fig.) using both methods and compared the results obtained. The direction of the versor **s** was determined by the polar coordinates α and β (Fig.).

Uniaxial media:

Angle ϑ 5°	relative error $\Delta(n'-n'')/n'-n''$ is about	$100\times(n_o-n_e)$%	calcite 17%
30°		$75\times(n_o-n_e)$%	13%
45°		$50\times(n_o-n_e)$%	9%
90°		0%	0%

An example for calcite refers to the wavelength of 650 nm.

Biaxial media:

In the biaxial media, the errors cannot be described as simply as for the uniaxial ones. Their values and the distribution depend not only on the difference of the external refractive indices n_x and n_z but also on the value of the intermediate refractive index n_y. For the drastic value of the birefringence $n_x = 1.5$, $n_y = 1.7$, $n_z = 1.8$ in the z,y plane, the errors $\Delta(n'-n'')/n'-n''$ ranged from 0% for the wave travelling along the y axis up to about 11% for the wave travelling along the z axis. In the section zx, the corresponding values are 11% for the wave travelling along the z axis to about 30% for the wave travelling along the x axis. The minimum (3%) appeared in the vicinity of $\beta = 45°$, while the maximum (70%), when the wave travelled very close to the optical axis (α and β — polar coordinates of the versor **s**, see Fig.) For the oblique cross-section from ($\alpha = 0°$, $\beta = 0°$) to ($\alpha = 90°$, $\beta = 45°$) the values of the error in the extreme positions of the versor **s** amounted to about 10%, reading the minimum (0.4%) for $\alpha = 40°$.

Thus, the errors of the appropriate calculations may be significant and, what is interesting, the greatest, when the light travels in the close vicinity of the optical axis.

References

[1] BORN M., WOLF E., *Principles of Optics*, Pergamon Press, Oxford 1968.
[2] BORN M., *Optik*, Springer-Verlag, Berlin, Heidelberg, New York 1965.

Received October 17, 1996