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Computation of the modulation transfer function of photographic materials by the edge function method

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The most important problems concerning the determination of Modulation Transfer Function (MTF) attained by the measurement and numerical transformation of the optical density spatial distribution function on the border between the image and the background are discussed. Apart from the basic sensitometric characterization of light sensitive materials designed for the information recording, MTF is one of the most important methods of the estimation and diagnostic of these materials. From among numerous, known and practically applied schemes of the MTF determination, the method based on the spatial distribution of the optical density measurement is the most important one. The above method is the most useful one because of the relatively simple manner of its practical application, high precision and short time of obtaining the results. In spite of these advantages and generally known theoretical base of the MTF estimation, there are still theoretical and practical problems connected with the measurements. Therefore, the analysis of algorithms of MTF estimation has been carried out and some changes and modifications are proposed. Moreover, the application of the automatic algorithm for the separation of computing and measurement disturbances is proposed. This algorithm permits further simplification of MTF estimation.

1. Introduction

Modulation transfer function is one of the most important methods for estimation of the quality of images and investigation of structurometric properties of photographic materials. This method has been especially developed for silver halide photographic materials prepared on the transparent base and is currently used as a standard characterization of photographic industry products.

1.1. Modulation transfer function in photography

The definition, theoretical principles, as well as properties of MTF were originated in the information transfer theory known in electronics, acoustics and optics. The first application of this theory comes from Duffieux, who derived the equation for MTF in the photographic process on the grounds of the Fourier transformation [1]–[3]. In this case, MTF expresses the change of the effective exposure distribution inside the light-sensitive photographic layer in relation to luminance contrast of object, expressed as a function of spatial frequency. If the luminance distribution of an object E recorded on photographic materials has a sine-wave response described by the equation

$$E(x) = E_0[1+M\sin(2\pi\nu x)], \tag{1}$$

and assuming isotropic active light diffusion inside the light-sensitive layer, effective distribution of an exposure E' in this material is expressed by the equation

$$E'(x) = E_0[1+M'\sin(2\pi\nu x)]. \tag{2}$$

The modulation of object luminance

$$M = (E_{max}-E_{min})/(E_{max}+E_{min}) \tag{3}$$

and it is constant in the whole range of spatial frequency represented in this object. The changes of modulation for certain spatial frequencies are expressed by relationship determining the coefficient of the modulation transfer

$$T = M'/M. \tag{4}$$

The frequency relationship of this coefficient

$$\mathrm{MTF} = T(\nu) = M'(\nu)/M(\nu) \tag{5}$$

expresses MTF.

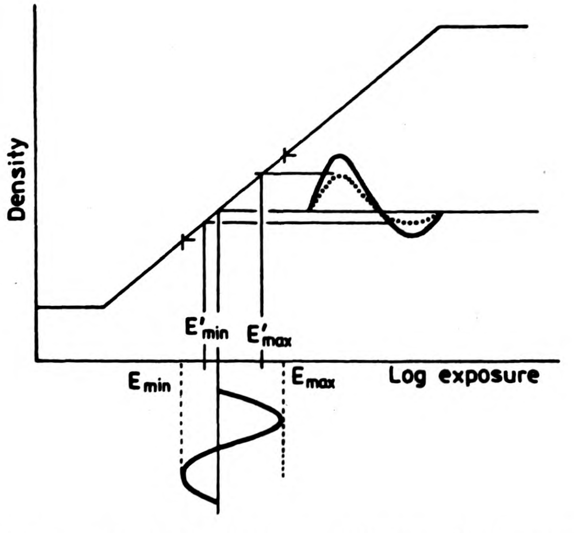

Fig. 1. Scheme of the sine-wave modulated signal recorded on the hypothetical light-sensitive material with the theoretical rectilinear characteristic curve. Solid and dotted lines express the function of optical density chages resulting from exposure without and with light diffusion, respectively

The spatial distribution of an effective exposure inside the light-sensitive layer depends on the optical properties and granularity of these layers being a heterogeneous system. The experimental determination of exposure distribution function by means of direct measurement methods is unattainable. However, using characteristic curve of the material under investigation, which expresses the relationship of the optical density of the image created to the logarithm of its exposure, one can determine approximate value of this function. Figure 1 shows the diagram of sine-wave response of sine-wave modulated luminance record on the hypothetical material with the theoretical characteristic curve

$$D = D_0 + \gamma \log(E/E_0). \tag{6}$$

The process of recording images on photographic materials is nonlinear and depends on the properties of the material as well as on its chemical processing. This nonlinearity comes from the shape of the characteristic curve, but not from the light diffusion inside the layer. This conclusion is essential in determination of MTF and is taken into consideration by the use of the equation

$$D(x) = D_0 + \gamma \log(E/E_0) + \gamma \log[1 + M' \sin(2\pi v x)] \tag{7}$$

derived from (2) and (6).

As a result of the photographic material exposure by means of the radiation with the luminance distribution according to Eq. (1), and then by chemical processing, an image is obtained in which the optical density fluctuations are described by Eq. (7). These fluctuations are a consequence of the spatial distribution function of the exposure inside the photographic layer according to Eq. (2). The luminance distribution as well as the optical density distribution can be determined a simple manner, indicating the possibility of experimentally estimating the modulation value M'. Additionally, it is possible to measure, in the same way, the MTF which characterizes the system of the information recording under investigation.

The MTF measurement by the method described is difficult, and seldom used. Nevertheless, this method has a great theoretical meaning, being the base for the creation of hypothetical models of light-sensitive layer, and for the design of technology of real systems.

1.2. Edge function

The edge curve expresses the spatial distribution of the photographic image optical density obtained by the exposure of the light-sensitive layer (later on, chemically processed) under an opaque half-plane with the perfect straight edge. Figure 2 presents the scheme of formation of such a curve for a hypothetical registration system. The shape of the edge curve depends on the light diffusion in the photographic layer and on the gradient of its characteristic curve. This relationship was developed by GOLDBERG [4]. During his study of the photographic image acutance, Goldberg concluded that the maximal gradient of the edge curve (dD/dX) correlates with the subjective visible image acutance A. Taking into account the results obtained by MESS [5], the approximate equation

$$A = dD/dX = [dD/d(\log H)] \times [d(\log H)/dX] \tag{8}$$

has been found describing the correlation between the physical properties of photographic layer and the quality of image registered on it (A – acutance,

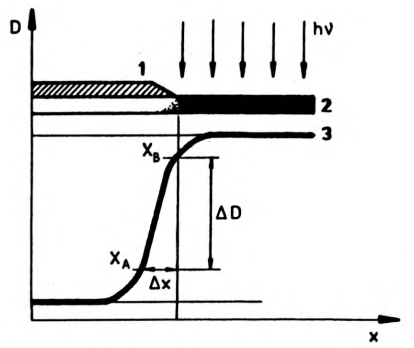

Fig. 2. Formation of the edge curve (3) during the exposure of the light-sensitive layer (2) under the opaque half-plane with the straight edge (1)

D – optical density, X – distance, dD/dX – gradient of the edge function, $dD/d(\log H)$ – gradient of the characteristic curve, $d(\log H)/dX$ – turbidity factor, $H = Et$ – exposure of photographic material, E – light intensity, t – time).

Practically, the formation of the edge curve runs in many steps. The most important three steps are illustrated in Fig. 3. The use of the opaque edge allows us to get the ditribution of the exposure described by step function presented in Fig. 3**a**. As a result of such distribution of the exposure on light-sensitive layer and partial diffusion of this radiation in it, the real distribution of effective exposure inside the layer is obtained (illustrated in Fig. 3**b**). After the photographic development, in which exposed silver halide crystals are reduced to metallic silver, there occurs the formation of spatial optical density distribution (Fig. 3**c**). The shape of the edge curve is determined by the spatial distribution function of an effective exposure inside photographic layer as well as by the characteristic curve. Such a simple description of this relationship is seldom used in practice, because during the photographic develompent, the so-called "border effect" appears, known as the "Eberhard effect" or as the effect of chemical spread function [6]. As a result of these effects, a local

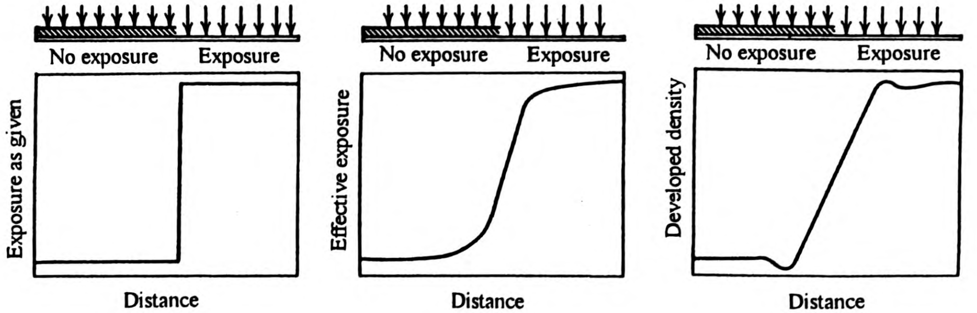

Fig. 3. Illustration of the profile cross-section formation of the knife-edge photographic image: **a** – the distribution of the optical image luminance, **b** – the effective distribution of the exposure inside photographic layer, **c** – the real, spatial distribution of the optical density on the border of an image and background

increase or decrease of optical density on the border of the image and in the background (Fig. 3c) can be observed. The consequence of these processes is a substantial improvement of the photographic image quality due to the presence of the border effects. In contemporary technologies of photographic materials production the border effects are applied to improve the quality of such materials.

1.3. Light spread function

According to the Bouger law, the light beam intensity decreases upon passing through a photographic layer. At the same time, some part of this radiation diffuses in different directions and then further decreases due to turbid environment of photographic emulsion layer. Figure 4 illustrates the scheme of the light propagation in the silver halide photographic layer being an optical heterogeneous environment. The diffusion light intensity in the depth of the layer h and at the distance x from edge is represented by the following equation:

$$dI = \alpha \times \exp(-h/k_2) \times \exp(-x/k_1) dh. \tag{9}$$

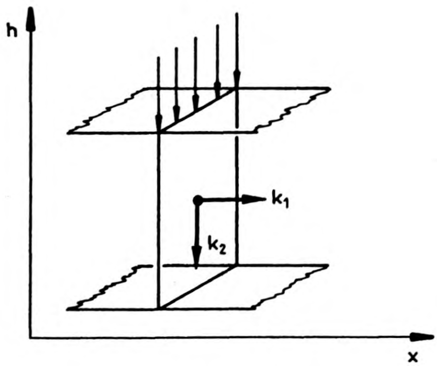

Fig. 4. Scheme of the light diffusion inside photographic layer

The integration of Eq. (9) leads to the relation determining the distribution of the diffusion light intensity, which occurs along the light-sensitive layer cross-section versus its depth

$$L_h = \alpha \exp(-x/k_1) \int_{0}^{-\infty} \exp(-h/k_2) dh. \tag{10}$$

For the strong light-diffusing photographic layers containing large sliver halide crystals the inequality

$$h \gg k_2 \tag{11}$$

is satisfied.

The integral expression in Eq. (10) is practically equal to 1. In this case, the diffusion light intensity inside photographic layer $L(x)$ depends only on an absolute value $|x|$. For weak light-diffusing layers, a part ρ of the light is absorbed by the light receptor without diffusion. This event can be described by Dirac function δ

$$L(x) = (1-\rho)/k_1 \times \exp(-|x|/k_1) + \rho\delta(x). \tag{12}$$

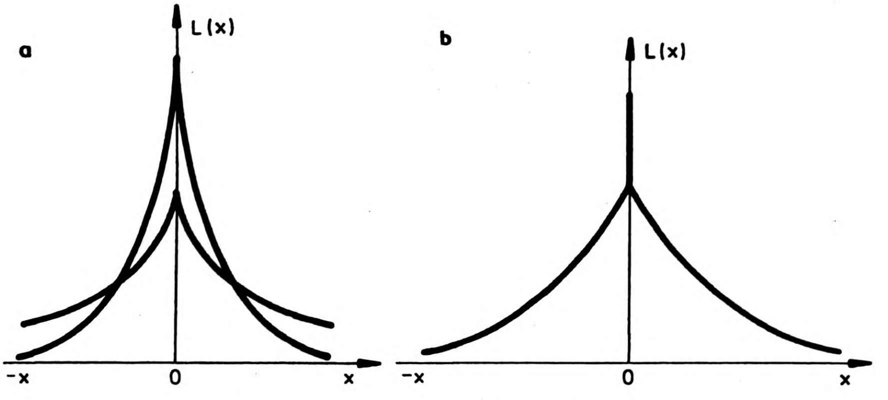

Fig. 5. Light diffusion function of photographic materials: **a** — strong light-diffusing systems differing in the coefficient value k_1, **b** — weak light-diffusing system

The light diffusion function (10) or (12) correlates with MFT following from Fourier transformation

$$\mathrm{MTF}(\nu) = FT[L(x)] \tag{13}$$

and using the reversibility of this transformation the reverse relationship is

$$L(x) = FT^{-1}[\mathrm{MTF}(\nu)]. \tag{14}$$

Equations (13) and (14) have been formulated by Frieser [7] and provide the grounds for the modulation transfer function estimation.

2. Method of computation of the modulation transfer function

The starting point fot the modulation transfer function computation is the experimental measurement of the edge curve and the determination of the general sensitometric properties of the photographic materials under investigation. The scheme of indispensable calculations and transformations is illustrated in Fig. 6.

The function on chart 1 (Fig. 6) is the edge curve, whereas chart 2 illustrates the characteristic curve. On the basis of the characteristic curve and edge curve the distribution of effective exposure is calculated in operation **A**. The distribution function obtained is represented on chart 3 (Fig. 6). Then, the function is smoothed in operation **B** to remove the measurement and calculation disturbances. In operation **C** a differentiation is made, which leads to the light diffusion illustrated on chart 5 (Fig. 6). As a result of Fourier transformation (operation **D**) an unsmoothed MTF is obtained and presented on chart 6 (Fig. 6). The application of the repeated smoothing process in operation **E** allows us to obtain a smoothed MTF of

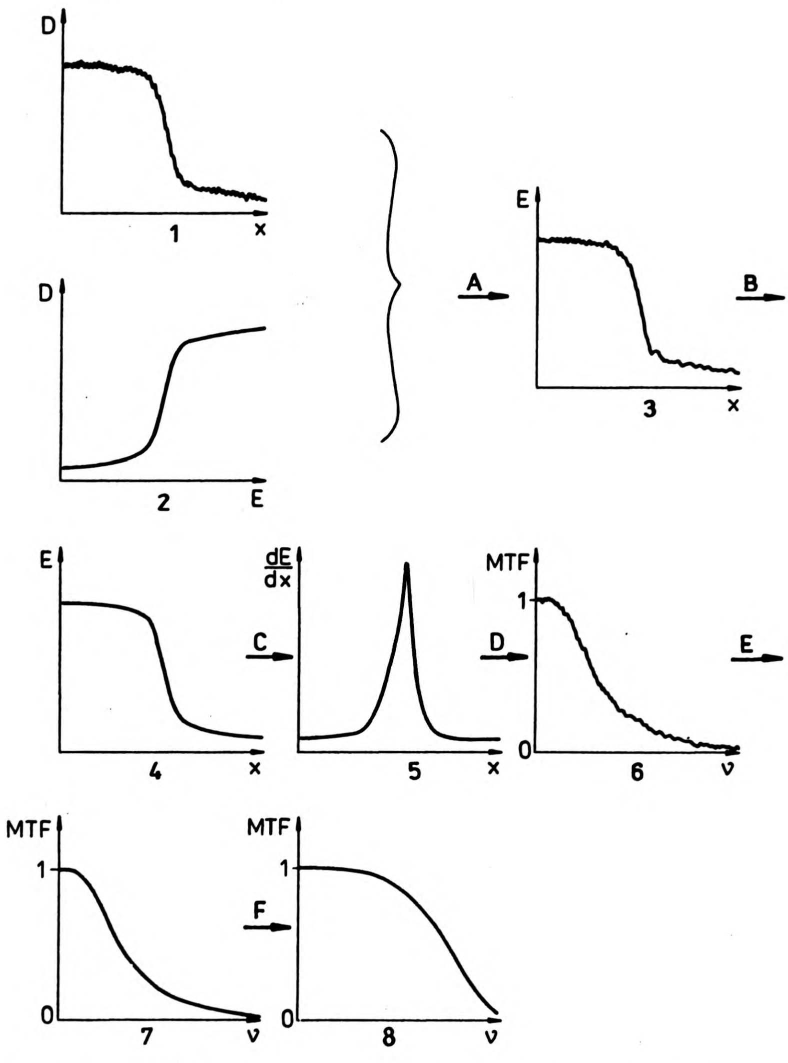

Fig. 6. Scheme of the photographic material MTF measurement from the microdensitometric determination of the edge curve

light-sensitive systems, which, however, is charged with an error resulting from the imposition of the MTF by optical and mechanical systems of microdensitometers. In operation F, the separation of MTF for measurement system takes place, which leads to the MTF of photographic material illustrated on chart 8 (Fig. 6) [8].

3. Scope of investigation

The present method of evaluation of MTF for photographic materials uses the Fourier transform for transformation of the line sperad function [9]–[11]. A typical block diagram of the computational procedure of one particular automatic method using Fourier transform, is shown in Fig. 6. The Fourier transformation can also be used for the computation of convolutin of the smoothing function and the effective exposure distribution. The application of the Fourier transformation at this stage creates, according to the author, very good conditions for the elimination of every noise and disturbance present in calculations and experimental measurements. This noise is characterized by some nonzero spectrum in the range of high frequencies and hence their removal can be made by reducing this part of spectrum to zero. The setting of the frequency range including the proper signal and containing a computing and measuring noise, is difficult and requires good knowledge of the problems of smoothing and fitting.

The use of arbitrarily chosen conditions of this process does not lead to obtaining of the corrected function as a result of computing, so it is necessary to individually settle the practical conditions of the numerical transformation of experimental data. In the present work, an attempt was made to automatize this stage of computing based on the investigation results of the influence of smoothing intensity on the integral from the line spread function and the influence of this process on average and maximal gradient of the edge-density function.

4. Experimental

4.1. Exposure of the knife-edge specimen

The exposure of the sensitometric strip of the knife-edge was estimated from numerical, standardized characteristic curves. The exposure of the bright field H_1 is determined for the transmittance coefficient $\tau = 0.9$, 0.7 or 0.5. However, the exposure of dark field is determined as a multiplicity of the bright field exposure,

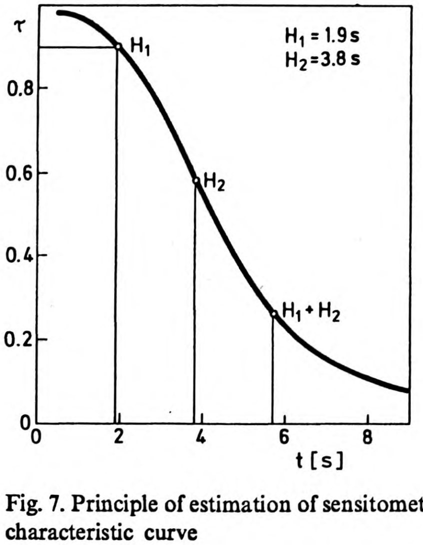

Fig. 7. Principle of estimation of sensitometric strip knife-edge exposure based on numerical, standardized characteristic curve

according to scheme: $H_2 = 2 \times H_1$, $H_2 = 4 \times H_1$ or $H_2 = 8 \times H_1$. Figure 7 explains the principle of determination of parameters. The exposure of sensitometric strips is carried out in two stages. First, the whole area of sensitometric strip is exposed in the time which follows from the accepted value H_1, then one half of the area is covered by "knife-edge" and the sample is exposed once more during the time that follows from value H_2. The intensity of illumination should be settled to attain similar exposure time of sensitometric strip with knife-edge image and sensitometric strip for obtaining numerical characteristic curve. This is necessary to prevent the possibility of the reciprocity law failure.

4.2. Apparatus and method of edge tracing

The edge function was recorded for each density step on strips, using computer-controlled photometric systems based on the Carl Zeiss Jena microdensitometer model MD100 with high-precision scanning stages [12]. The optical configuration of the system and scanning apertures varied according to the parameter being measured. Parameters of the optics system applied are consistent with the proposition included in *American National Standard* [13]. The system is presented in Fig. 8. A block scheme of measuring the apparatus controlled by computer program is presented in Fig. 9.

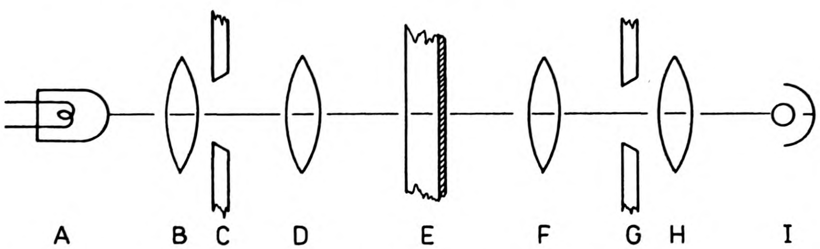

Fig. 8. Typical optical system of the microdensitometer: A – light source (3200 K), B – condenser lens, C – influx aperture, D – influx optics, E – sample, F – efflux optics, G – measuring aperture, H – collecting lens, I – photodetector

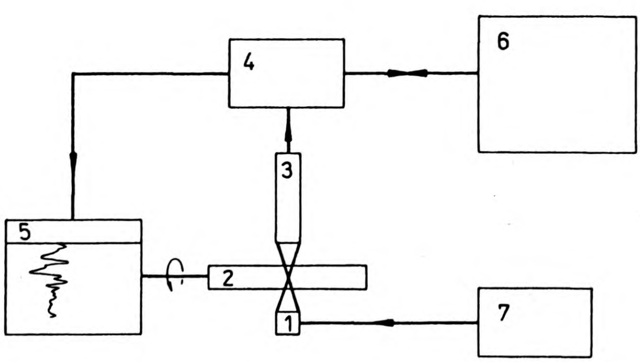

Fig. 9. Block scheme of the microdensitometer: 1 – light source, 2 – sample, 3 – photodetector, 4 – amplifier, 5 – recorder, 6 – computer, 7 – power supply

The specifications of conditions for modulation transfer function measurements on microdensitometer are as follows: scanning distance 25 mm, scanning aperture of area ranging from 50 μm^2 to 7000 μm^2, illumination lens (magnification 10 × and numerical aperture NA = 0.30 and 20 ×, NA = 0.60), scanning lens (10 × and NA = 0.20, 20 × and NA = 0.40), number of sampling points 64768, 12-bit A/C converter, light source which may be: white (colour temperature 2850 K and 5500 K), monochrome (wavelength 463, 546 and 695 nm) or densitometry status filter type A and M. The minimal measurement step equals 0.4 μm. The sensitometry investigation of the photographic materials was carried out at 20°C according to manufacturers recommendations for the nominal exposure indices. All strips of each material were identically and uniformly exposed to tungsten light, appropriately filtered to give simulated daylight of correlated colour temperature of 5500 K using a neutral step wedge and exposure time of 0.1 s. The same procedure was repeated with another step wedge.

The optimal width of measuring aperture has been experimentally estimated by determining the relationship between the maximal gradient of the edge-curve and the measuring aperture value applied. Figure 10 illustrates the edge-curves obtained in

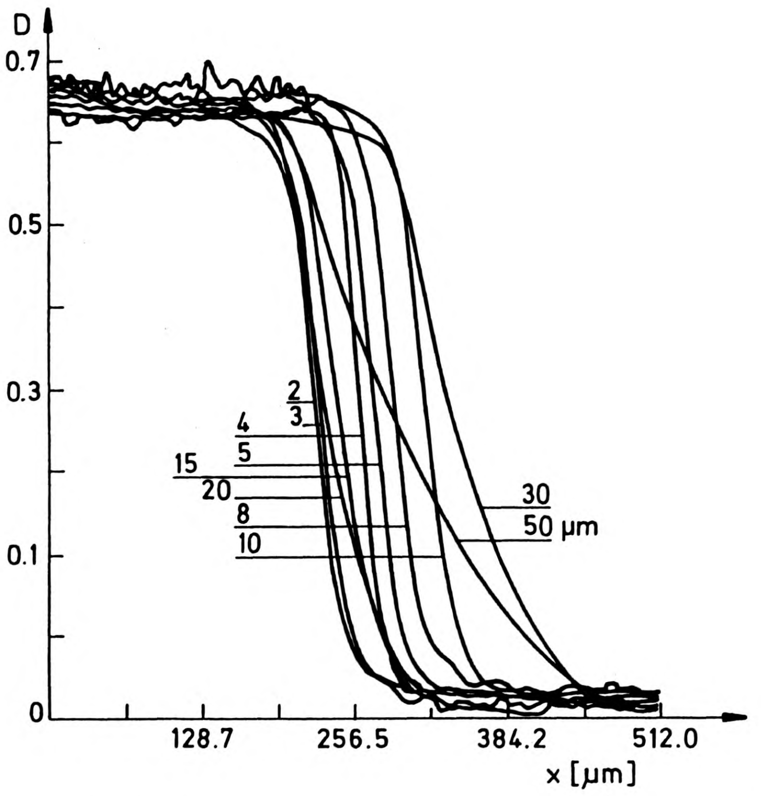

Fig. 10. Edge curves obtained during the scanning of the same edge of image using the measuring aperture of different widths

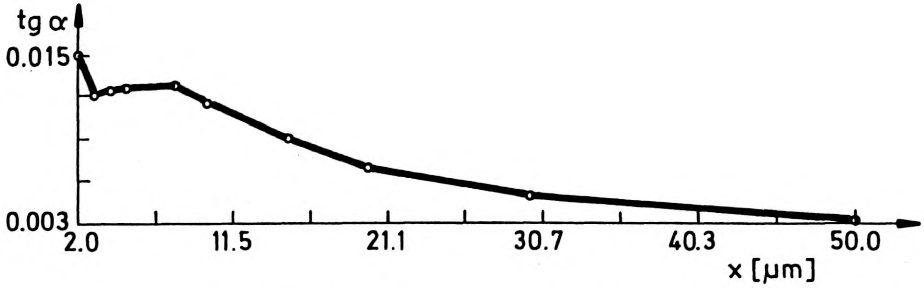

Fig. 11. Relationship between maximal gradient of edge-curve and the width of measuring aperture

this experiment, whereas Fig. 11 presents the relationship between the maximal gradient of the edge-curve and width of measuring aperture. The slit of width 3 μm and height not smaller than 1000 μm was chosen as an optimum.

4.3. Measurement and computation

Several photographic materials of various types, black and white and colour, with medium exposure sensitivity were utilized in this work. Their sensitometric properties were determined by standard methods. In the computing stage of MTF, when the Fourier transformation (operation D in Fig. 6) is carried out, the number of harmonics was changed from 10 to 500. As a result of computing for all photographic materials, the following functions were obtained: smoothed edge -density function, smoothed line spread function and smoothed modulation transfer function. Based on the functions obtained, the maximal gradient and average gradient for edge-density function as well as the integral from line spread function versus the number of harmonics, contained in the Fourier transformation spectrum have been calculated. Figures 12–14 illustrate corresponding curves.

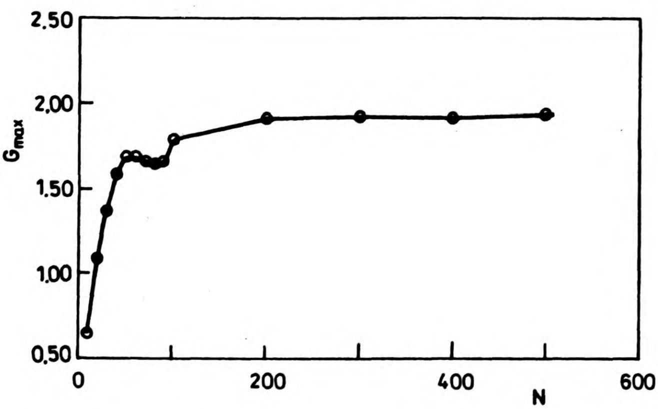

Fig. 12. Dependence of the maximum gradient of the edge-density function on the number of harmonics in the Fourier transformation

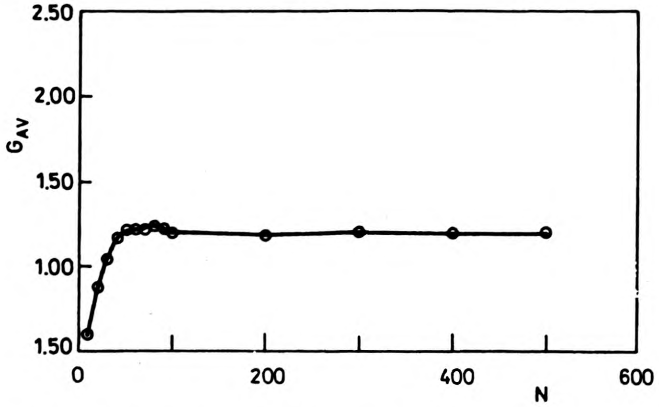

Fig. 13. Dependence of the average gradient of the edge-density function on the number of harmonics in the Fourier transformation

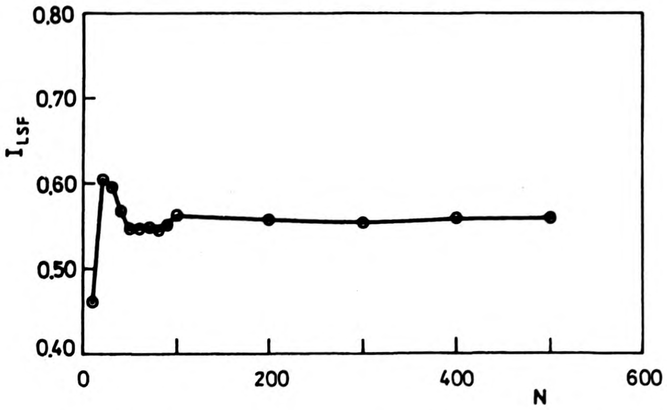

Fig. 14. Dependence of the integral of the line spread function on the number of harmonics in the Fourier transformation

The functions were calculated applying the condition, proposed by JONES and HIGGINS [14], of a minimal luminance gradient of human eye, namely 0.005 unit of the optical density on one micrometer. The edge-density function was calculated by using the reversed sequence of computation in relation to the algorithm presented in Fig. 6. Each of the functions demonstrated in Figs. 12—14 shows some optimal range of values, for which the noise and the process of removing does not influence the properties of smoothed functions. There is no need to estimate the quality of the relationships presented, because the most important task is to determine the optimal harmonic frequency number of Fourier transformation. Figure 15 illustrates MTF obtained for 10, 50, 100 and all harmonics frequency of the Fourier transformation.

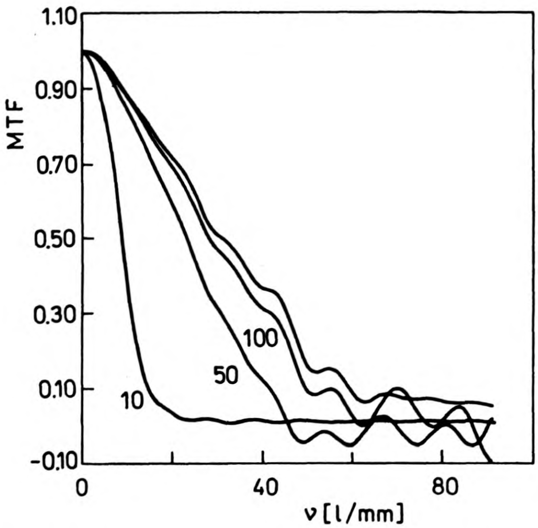

Fig. 15. Modulation transfer function for 10, 50, 100 and all harmonics of the Fourier transformation

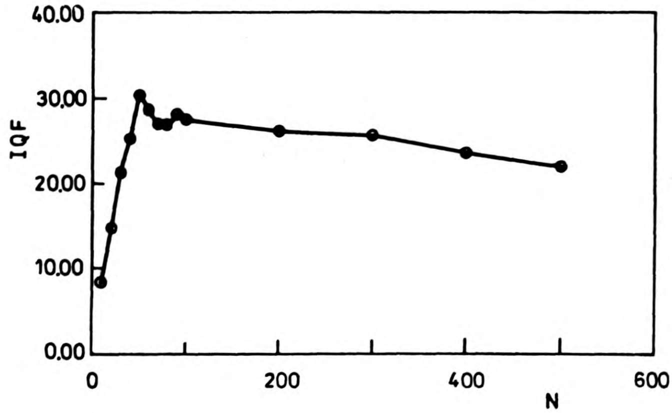

Fig. 16. Dependence of the integral quality factor, for 30% of MTF, on the number of harmonics of the Fourier transformation

5. Conclusions

Taking into account the accuracy of calculations and shapes of the functions, it can be concluded that the optimal signal-to-noise ratio (SNR) can be obtained by using 50–100 first harmonics of the Fourier transformation. Figure 16 illustrates the relationship of the integral quality factor [15], calculated in the range from 30 to 100% of MTF value, as a function of Fourier transformation harmonics number. From MTF (Fig. 15) and from the curve presented in Fig. 16 it follows that above certain number of harmonics of Fourier transformation its increment does not influence the modulation transfer function being calculated. Based on the results obtained we can conclude that the optimization of the number of Fourier transformation harmonics, which eliminates the noise and smoothes the functions calculated, can be entirely automated. A suitable computer algorithm was constructed which was included in the program for calculation of MTF of the photographic materials by the edge-density functions method. By applying this algorithm the estimation of MTF is considerably simplified and users do not need to get acquainted with difficult problems of fitting and smoothing processes.

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