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# ESTIMATION OF THE DIVERSIFICATION EFFECT IN SOLVENCY II UNDER DEPENDENCE UNCERTAINTY

# SZACOWANIE EFEKTU DYWERSYFIKACJI W SOLVENCY II PRZY NIEPEWNEJ STRUKTURZE ZALEŻNOŚCI

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**Summary:** The paper focuses on the problem of the accurate estimation of the diversification effect in the process of determining Solvency Capital Requirements (SCRs) in Solvency II. First, the method of determining SCRs in Solvency II is briefly characterised, the role of dependences for the correct specification of the diversification effect is presented, and a diversification ratio is defined. This is followed by an analysis of the use of this ratio of the diversification effect sensitivity to the dependence structure based on an example of life and health underwriting risk. The cases of lacking knowledge, partial knowledge (of a correlation coefficient only) and full knowledge are considered (it has been assumed that variables are independent and comonotonic). Dependence structures are modelled using copulas. The article shows that without identifying the actual dependence structure, the application of the standard formula in accordance with Commission Delegated Regulation (EU) 2015/35 may lead to the incorrect estimation of the diversification effect, and it indicates (based on a simulation analysis) the size of possible errors.

**Keywords:** Solvency Capital Requirements, risk aggregation, VaR bounds, diversification effect.

**Streszczenie:** W pracy podjęto problem oszacowania na „rzeczywistym poziomie" efektu dywersyfikacji w procesie wyznaczania kapitałowych wymogów wypłacalności (SCR) w Solvency II. Przedstawiono w nim standardową procedurę wyznaczania kapitałowego wymogu wypłacalności, zaprezentowano i omówiono wybrane wyniki badania QIS5 dotyczące efektu dywersyfikacji ryzyka, który uzyskano w wyniku jej stosowania. Następnie zdefiniowano współczynnik dywersyfikacji i za jego pomocą analizowano wrażliwość efektu dywersyfikacji na strukturę zależności na przykładzie ryzyka aktuarialnego w ubezpieczeniach na życie i ryzyka aktuarialnego w ubezpieczeniach zdrowotnych. Rozważano przypadek braku wiedzy o tej strukturze, przypadek częściowej wiedzy w postaci współczynnika korelacji oraz przypadek całkowitej wiedzy (założono, że zmienne są niezależne i współmonotoniczne). Struktury zależności modelowano za pomocą kopuli. Artykuł pokazuje, że stosowanie formuły standardowej zgodnie z Rozporządzeniem Delegowanym Komisji (UE) 2015/35 bez identyfikacji rzeczywistej struktury zależności może prowadzić do błędnego oszacowania efektu dywersyfikacji oraz wskazuje (bazując na analizie symulacyjnej), jak duże to mogą być błędy.

**Słowa kluczowe:** kapitałowe wymogi wypłacalnosci, agregacja ryzyka, ograniczenia VaR, efekt dywersyfikacji.

## 1. Introduction

In January 2016, in all the European Union countries, the Solvency II Directive came into force. Due to the number of new proposals it introduces and the broad scope it covers, the Directive is considered the most important regulatory change in the EU insurance market for over 30 years. The Directive has introduced new uniform requirements for all the EU countries as regards risk management, reporting and determination of the financial standing and solvency of insurance and reinsurance companies. Similarly to the Basel II Accord, it has been based on the structure of three interrelated pillars to which separate risk categories have been assigned, relating to an insurer's operations. The First Pillar includes the quantifiable risks to which an insurance company is exposed. Within its framework, the Minimum Capital Requirement (MCR) and the Solvency Capital Requirement (SCR) are determined, as well as the manner of considering dependences when determining the SCR, rules for estimating technical provisions, the structure of capital endowment and the principles for the investment activity of an insurance company. It also covers the scope and principles of application of the so-called internal risk assessment models of an insurance company. The Second Pillar, in turn, focuses on those risk types of an insurance company that are not included in the First Pillar and contains standard supervisory procedures. It includes tools for the effective monitoring and control of an insurer's risk, both internal and applied by the supervisory authorities. According to the assumptions of the Second Pillar, a solvency assessment should take into account the individual characteristics of a particular insurance company, including those of a qualitative nature. In the area of interest of the Second Pillar, there are, among other things, quality of company management, internal control and audit as well as rules for oversight and harmonization of supervisory standards, and principles for cooperation between supervisors. On the other hand, the Third Pillar includes tools for market self-regulation by creating conditions for its transparency, defining disclosure requirements and developing accounting standards.

The new regulations will cause significant changes in the operations of insurance companies, including in the area of risk management, the financial economy and the manner of determining solvency. The possible effects of these changes for the insurance companies, as well as the indirect consequences for policyholders, the insured, beneficiaries and persons eligible under insurance contracts are discussed by both practitioners and researchers (e.g. [Biełasiewicz-Fuszara, Wnęk 2014; Florczak 2014; Czerwińska 2013; Kurek 2011; Lament 2011; Molęda 2011]). In a further part of the article, attention is focused exclusively on:

- the problem of assessing the diversification effect in the process of determining solvency capital requirements (SCRs), and
- closely related methods for the modelling of dependences.

In Solvency II, solvency capital requirements for an insurance company are determined by the aggregation of capital requirements on account of the identified risk factors that pose a threat to the company. Since not all the risk factors are at play at the same time, the total capital required to hedge against those risk factors, determined as a result of such aggregation, is not generally higher than the sum of capitals required to hedge against each of them separately. The diversification benefits (effects) so produced are a key element in the risk management process of an insurance company[1]. In practice, their assessment closely relates to determining the structure of dependences between risks whose capital requirements are aggregated. Dependence between risks has long been recognized as an integral factor affecting the aggregation process. However, in the past, few attempts were made to introduce a dependence structure into this process. The dependence was ignored by adding capital requirements on account of aggregated risks, which led to overestimated total capital requirements (the diversification effect was not taken into account), or it was assumed that the risks are independent, which, in turn, led to their underestimation (the maximum diversification effect was assumed[2]). In the standard Solvency II formula, the variance-covariance method is used to aggregate capital requirements, in which dependence is modelled only with the use of linear correlation coefficients. The subject matter of the article focuses on the indication that this approach is insufficient to estimate the diversification effect at an appropriate level. It expands the existing literature on the subject in terms of quantitative, multidimensional risk modelling in the process of determining solvency capital requirements for life and health underwriting risk. The article shows that without identifying the actual dependence structure, the application of the standard formula in accordance with the

[1] "'Diversification effects' means the reduction in the risk exposure of insurance and reinsurance undertakings and groups related to the diversification of their business, resulting from the fact that the adverse outcome from one risk can be offset by a more favourable outcome from another risk, where those risks are not fully correlated. The Basic Solvency Capital Requirement shall comprise individual risk modules, which are aggregated […] The correlation coefficients for the aggregation of the risk modules […], shall result in an overall Solvency Capital Requirement […] Where appropriate, diversification effects shall be taken into account in the design of each risk module." [Directive 2009/138/EC…, (37) p. 24; Article 104, p. 52].

[2] Assuming that the risks are not characterized by a negative dependence.

Commission Delegated Regulation (EU) 2015/35 may lead to an incorrect estimation of the diversification effect, and it indicates (based on a simulation analysis) the size of possible errors.

The remaining part of the article is organized as follows. The next chapter reviews the literature and formulates the objective of the paper. Chapter 3 briefly characterizes the standard risk aggregation method used in Solvency II and a related method for the estimation of the diversification effect. Chapter 4 discusses the relationship between the diversification effect and the dependence structure. Chapter 5, in turn, contains the assumptions and results of a simulation study, while the final chapter 6 includes the conclusions.

## 2. Literature review

The problem of modelling dependences in the context of setting capital solvency requirements is very important and widely discussed in literature on the subject. Donnelly and Embrechts [2010] indicate that the financial crisis of 2008-2009 has proven once again that a comprehensive understanding of the dependence between and within insurance and financial risks is necessary for prudent risk aggregation. In general, papers show that the use of linear correlation coefficients for this purpose, which enables one to model Gaussian dependence structures exclusively, is highly inadequate (e.g. [Tang, Valdez 2009; Embrechts, Puccetti 2010; Schmeiser, Siegel, Wagner 2012; Embrechts, Wang, Wang 2015]). Focusing their attention only on Solvency II, Pfeifer and Strassburger [2008], Clemente and Savelli [2017], Bermúdez, Ferri, Guillén [2013] and Alm [2015], investigate the consequences for the accuracy of the standard formula and propose other ways to proceed. Devineau and Loisel [2009], in turn, show that the standard formula can be considered a first-order approximation of the result of the internal model. Therefore they propose an alternative method of aggregation that enables one to capture satisfactorily the diversification among the various risks that are considered, and to converge the internal models and the standard formula. It is also possible to mention works discussing the shortcomings of the standard formula, e.g. [Sandström 2007; Filipović 2009; Savelli, Clemente 2011]. Among them, Filipović [2009] seems to be particularly interesting, where the author analyzes the implications of using correlation matrices at two levels and shows that in general, only parameters set at the base level lead to unequivocally comparable solvency capital requirements.

The influence of the standard formula on the level of solvency capital requirements for insurers in European Union countries was assessed in the fifth Quantitative Impact Study (QIS5). This research shows that (see: [EIOPA Report... 2011]) the diversification effect obtained as a result of applying the aggregation method influences considerably the reduction of solvency capital requirements. In total, such requirements for individual insurers and groups of insurance companies participating in the study were lower by 35.1% (EUR 466 billion).

From a methodological point of view, the variance-covariance method is correct when capital requirements are determined for risk factors subject to multivariate normal (elliptical) distribution. When analysing an insurer's risk, this assumption is rarely met (and the creators of the proposed solutions are aware of this fact). It means that in the analysed standard Solvency II model, the diversification effect is estimated using dependence structures that can describe relations between risk factors in an incorrect way. An obvious question arises as to what extent is the diversification effect so estimated reliable? To what extent does the assumption of linear dependence affect its height, i.e. how much can it differ from the effect estimated when taking into account the correct structure of the relationship between the risks?

The main purpose of this article is to try to answer the above mentioned questions. It presents the standard procedure for determining the Solvency Capital Requirement and selected results of QIS5 for the risk diversification effect that arise from its application. This is followed by an analysis of the diversification effect sensitivity to the dependence structure based on an example of life and health underwriting risk. The diversification ratio was estimated applying the standard Solvency II approach, i.e. using the variance-covariance method with the life and health underwriting risk correlation coefficient of 0.5. By using copulas, a number of examples were provided to demonstrate that the same correlation coefficient value may correspond to various dependence structures, and therefore various diversification effects. Then the value of the diversification ratio was calculated for the cases when the considered risks are independent and comonotonic, and the minimum and maximum ratio values were calculated assuming no information about the dependence structure between these risks.

## 3. Risk aggregation and diversification effect in Solvency II

In Solvency II, the principal role in the process of evaluating the solvency of an insurer is played by the solvency capital requirement (SCR). This capital is considered a cushion against significant deviations from an expected loss, whereas coverage for the expected loss is provided through provisions. Hence it is defined as economic capital[3], which should guarantee security for the insured if unpredictable losses occur. It is calculated at least once a year and when a considerable change has occurred in the risk profile of the insurer. It is assumed that the SCR should guarantee

[3] "In order to promote good risk management and align regulatory capital requirements with industry practices, the Solvency Capital Requirement should be determined as the economic capital to be held by insurance and reinsurance undertakings in order to ensure that ruin occurs no more often than once in every 200 cases or, alternatively, that those undertakings will still be in a position, with a probability of at least 99.5%, to meet their obligations to policy holders and beneficiaries over the following 12 months. That economic capital should be calculated on the basis of the true risk profile of those undertakings, taking account of the impact of possible risk-mitigation techniques, as well as diversification effects." [Directive 2009/138/EC…, Note 64, p. 7].

with a 0.995 probability that the insurer will be able to meet its obligations within 12 months. In other words, it is assumed that when the Basic Own Funds are equal to the SCR, the probability of insolvency in the following year is 0.005. The capital solvency requirement of an insurance company can be calculated using:

- the standard formula,
- internal models, full or partial,
- its own parameters (for selected modules),
- the standard form with simplifications.

It must include all measurable risk types to which the insurer is exposed.

In the standard Solvency II approach, the overall Solvency Capital Requirement for an insurer is calculated with the use of the following formula:

$$SCR = BSCR + Adj + SCR_{Op}, \quad (1)$$

where: *BSCR* – Basic Solvency Capital Requirement; *Adj* – adjustment for the risk absorbing effect of technical provisions and deferred taxes; $SCR_{Op}$ – capital requirement for operational risk.

The *BSCR* value is determined when aggregating SCRs designated for main risk modules (i.e. market risk, counterparty default risk, life underwriting risk, non-life underwriting risk, health underwriting risk, intangible assets risk); the SCRs for the modules are determined by aggregating SCRs for sub-modules, whereas the later result, from the aggregation of SCRs for risk drivers[4]. Thus, the process involves three aggregation levels which are presented in detail, for example, in [QIS5 Technical ... 2010; Wanat 2014a, 2014b]. It is assumed in the process that not all risks occur simultaneously; therefore, a SCR determined for a specific tier (e.g. module) is generally not greater than the sum of SCRs set at the -1 level (e.g. for sub-modules). The resulting difference is referred to as the diversification effect (benefit) and it is a key element in the risk management process of an insurer.

If it is formally assumed that at the *l*-th ($l = 1, \ldots, 3$) aggregation level, the capital requirement for the *j*-th risk $Y_j^{(l)}$ (insurer[5], module, sub-module) dependent on *k* risks $X_{j1}^{(l-1)}, \ldots, X_{jk}^{(l-1)}$ from the $l - 1$ level (modules, sub-modules, drivers) is determined, the diversification effect can be measured with the use of a diversification ratio (see: e.g. [Embrechts, Wang, Wang 2015]):

$$d_j^{(l)} = \frac{\kappa\left(Y_j^{(l)}\right)}{\sum_{i=1}^{k} \kappa\left(X_{ji}^{(l-1)}\right)}, \quad (2)$$

where: $\kappa\left(X_{ji}^{(l-1)}\right)$ – capital requirement for risk $X_{ji}^{(l-1)}$; $\kappa\left(Y_j^{(l)}\right)$ – capital requirement for the aggregate risk $Y_j^{(l)}$.

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[4] The manner of determining *Adj* and $SCR_{Op}$ values is presented in [QIS5 Technical... 2010].

[5] It is obviously BSCR.

The above formula (2) suggests that the diversification effect depends on the manner of determining capital requirements (still, for the purpose of preserving the clarity of notation, superscripts $l$ and subscripts $j$ will be omitted) $\kappa(X_i)$ for individual risks $X_i$ and capital requirement $\kappa(Y)$ for aggregated risk $Y$. As already mentioned, these requirements should correspond to the economic capital determined for one year at the confidence level of 0.995. Therefore, in accordance with its definition (cf. e.g. [Lelyveld 2006] ), $\kappa(X_i)$ and $\kappa(Y)$ should be equal:

$$\kappa(X_i) = VaR_{0.995}(L_i) - \mu_i, \tag{3}$$

$$\kappa(Y) = VaR_{0.995}(L) - \mu, \tag{4}$$

where: $L_i$ i $\mu_i$ – loss distributions for $X_i$ risks and their expected values, respectively; $L$ i $\mu$ – loss distribution for aggregated risk $Y$ and its expected value, respectively; $VaR_{0.995}(\cdot)$ – Value-at-Risk at the confidence level of 0.995.

Hence it is clear that the procedure for estimating the capital requirement for aggregated risk Y, which mainly depends on the modelling of the dependence structure among variables $L_1,\dots, L_k$ (so $X_1,\dots, X_k$ risks), is of key importance for the correct evaluation of the diversification ratio. In the standard Solvency II solution, in the case of the aggregation of solvency capital requirements at the second and third tier, the variance-covariance method is proposed. The method involves:

- Determining capital requirements for individual risks: $\kappa(X_i), \dots, \kappa(X_k)$.
- Determining capital requirement $\kappa(Y)$ for aggregated risk $Y$ based on the correlation matrix $\boldsymbol{R}$ between $X_1,\dots, X_k$ , in accordance with the following formula:

$$\kappa^{(solv)}(Y) = \sqrt{\boldsymbol{WRW^T}} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k}\rho_{ij}\kappa(X_i)\kappa(X_j)}, \tag{5}$$

where:

$$\boldsymbol{W} = \left[\kappa(X_1),\dots,\kappa(X_k)\right],$$

$$\boldsymbol{R} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1k} \\ \rho_{21} & 1 & \cdots & \rho_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{k1} & \rho_{k2} & \cdots & 1 \end{bmatrix}.$$

The assessment of the feasibility and effects of the above standardized method on setting capital adequacy requirements and its impact on the assessment of the diversification effect was carried out from July to November 2010, under the fifth Quantitative Impact Study (QIS5)[6]. Nearly 70% of insurance and reinsurance companies covered by the Solvency II Directive took part in this study. There were

[6] Quantitative research QIS (Quantitative Impact Study) has been proposed by CEIOPS – Committee of European Insurance and Occupational Pensions Supervisors (from 1 January 2011, EIOPA – European Insurance and Occupational Pensions Authority). They are to investigate the impact of Solvency II on insurers' operations and the entire insurance market.

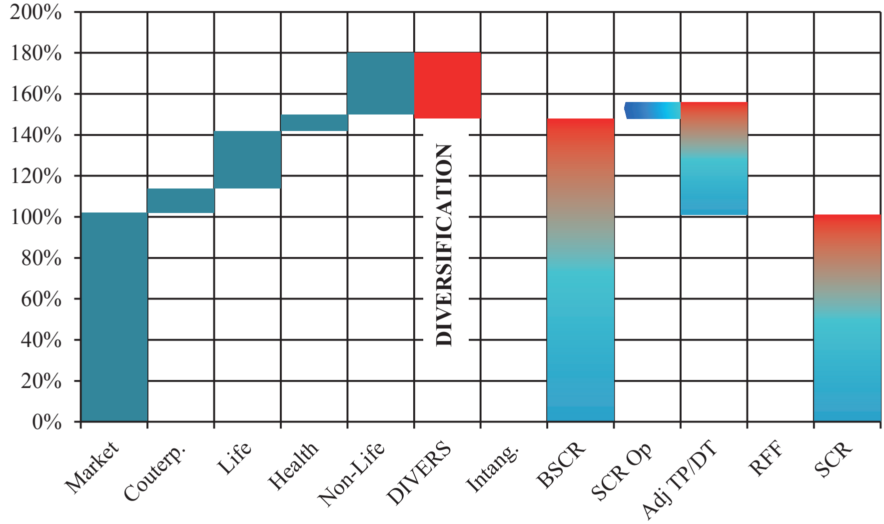


**Fig. 1.** Structure of SCR (in%) – EU solo insurers

Source: own study based on the report [EIOPA Report... 2011].

50 insurers from Poland (24 life insurance companies – 89% market share and 26 insurance companies – division II – 89% market share). The results of the study were published in the report [EIOPA Report ... 2011].

The report shows that as a result of the standard approach, the sum of the capital requirements (total risk) for all risk[7] carriers of the participating companies is € 1328 billion, with the diversification effect estimated at 466 billion (35.1%) and loss coverage from technical provisions and deferred taxes at 314 (23.7%). This resulted in an estimate of the Solvency Capital Requirement (SCR) for the researched institutions at EUR 547 billion, representing approximately 41.2% of the total risk. The SCR structure for individual insurers (without equity groups) is illustrated in Figure 1. As a result of applying the standard variance-covariance method to the aggregation of capital requirements for the main risk modules (i.e. the second level), the diversification effect is 32%.[8]

## 4. Diversification effect and dependency structure

An estimation of the diversification effect at the "actual level" is affected by the proper modelling of dependences among risk factors. A misidentified dependence structure leads to an estimation of an incorrect level of the diversification effect,

[7] The standard methods for determining capital requirements for individual risk carriers include the technical specification for QIS5 (see: [QIS5 Technical... 2010]) with relevant appendices.

[8] For capital groups, the effect of diversification on the second level is 46%.

which may lead to an overestimation or underestimation of capital requirements and may have a considerable impact on an insurer's operations and solvency.

As stated above, $\kappa^{(solv)}(Y)$ should correspond to the economic capital necessary for hedging against potential losses (higher than expected) relating to risk $Y$ over an annual time horizon and at the security level of 0.995. Thus, value $\kappa^{(solv)}(Y)$ obtained as a result of applying the standard procedure (5) should be equal to the $\kappa(Y)$ value obtained with the use of formula (4). With the application of formulas (4) and (5), we obtain the same value of the solvency capital requirement (i.e. $\kappa^{(solv)}(Y) = \kappa(Y)$) only when (cf. e.g. [Dhaene et al. 2005]):

i. Capital requirements for individual risks $\kappa(X_1),\ldots, \kappa(X_k)$ are determined in accordance with formula (3).
ii. $L = L_1 +\ldots + L_k$ and $L_1,\ldots, L_k$ have multivariate normal (elliptical) distribution, which means, in particular, that each variable $L_i$ has normal distribution.

In the process of estimating the solvency capital requirements of an insurer, those risks are aggregated. Due to their essence, they are modelled with the use of different distributions and different methods. Therefore the assumption that they are subject to multivariate normal distribution is ill-founded. In addition, the solvency capital requirements for these risks are not determined according to formula (5).[9] Consequently, it can be stated that the variance-covariance method indicated in Solvency II applies even if neither (i) nor (ii) is satisfied. Therefore the question is about the possible consequences of such a procedure. The answer to this question requires estimating the boundaries of the diversification coefficient in the absence of any knowledge of the multidimensional distribution of random vector $\boldsymbol{L} = L_1,\ldots, L_k)$. This type of estimation can be obtained using, for example, the lower and upper Fréchet bounds, but the difference between the maximum and minimum diversification coefficients is so big that it is useless from a practical point of view.

An insurer may estimate quite precisely the distribution of losses related to individual risks $X_1,\ldots, X_k$, where are marginals of vector $\boldsymbol{L}$. Let us attempt to answer the question by assuming that distributions of variables $L_1,\ldots, L_k$ are known whereas the dependence structure among them is unknown. With this assumption, the diversification effect will only depend on the dependence structure among variables $L_1,\ldots, L_k$ that determines $VaR_{0,995}(L)$. Based on Sklar's Theorem (see: Theorem 5.3 in [McNeil et al. 2005]), all information on this dependence is contained in copula C. Thus, the *k-th* dimensional random vector with fixed marginals $F_1,\ldots, F_k$ and the dependence structure in the form of copula $C$ will be designated by $(L_1^C,\ldots,L_k^C)$. If the dependence structure between $L_1,\ldots, L_k$ is unknown, we are unable to determine the exact value $VaR_\alpha(L)$, and it can only be assumed that it fulfils the following inequalities:

$$\underline{VaR_\alpha}(L) \le VaR_\alpha(L_1^C + \ldots + L_k^C) \le \overline{VaR_\alpha}(L), \tag{6}$$

[9] The procedure for estimating SCRs that are later aggregated using the variance-covariance method is presented in detail in [QIS5 Technical... 2010].

where:

$$\underline{VaR_\alpha}(L) \le \inf_{C \in \mathcal{C}_k} \left\{ VaR_\alpha \left( L_1^C + \ldots + L_k^C \right) \right\}, \tag{7}$$

$$\overline{VaR_\alpha}(L) \le \sup_{C \in \mathcal{C}_k} \left\{ VaR_\alpha \left( L_1^C + \ldots + L_k^C \right) \right\}, \tag{8}$$

whereas $\mathcal{C}_k$ denotes the class of all *k*-dimensional copulas. Then the diversification ratio d:

$$\underline{d} \le d \le \overline{d}, \tag{9}$$

where:

$$\underline{d} = \frac{\underline{VaR_\alpha}(L) - \mu}{\sum_{i=1}^{k} \left( VaR_\alpha(L_i) - \mu_i \right)}, \tag{10}$$

$$\overline{d} = \frac{\overline{VaR_\alpha}(L) - \mu}{\sum_{i=1}^{k} \left( VaR_\alpha(L_i) - \mu_i \right)}, \tag{11}$$

The calculation of the lower (10) and upper (11) bounds of the diversification ratio requires the estimation of values (7) and (8). The issue of seeking bounds (7) and (8) is extremely important from the perspective of risk management and has a long history. The first results in this respect for the sum of two random variables were presented in papers by [Makarov 1981] and [Frank, Nelsen, Schweizer 1987], and independently in [Rüschendorf 1982]. In recent years it was discussed, for example, in [Puccetti, Rüschendorf 2012a, 2012b, 2014; Embrechts, Puccetti, Rüschendorf 2013; Puccetti, Wang, Wang 2013; Bernard, Rüschendorf, Vanduffel 2015].

The natural outcome of research on respecting the *VaR* boundaries was the creation of an RA algorithm (*Rearrangement Algorithm*), proposed in [Puccetti, Rüschendorf 2012a] and [Embrechts, Puccetti, Rüschendorf 2013]. It allows for designating the *VaR* boundaries for known but not necessarily identical marginal distributions. Research conducted so far indicates quite good accuracy of the algorithm but the estimation ranges of *VaR* obtained in various cases are quite broad. In brief, the RA algorithm is about constructing dependence functions between random variables $L_i$ by properly regrouping columns made of random variables so that the distribution of the sum of random variables in convex order is as low as possible.

A modification of the RA algorithm, also known as ARA (*Adaptive Rearrangement Algorithm*), was proposed in [Hofert et al. 2015]. The main impulse for the creation of the algorithm was the huge (even if the use of computers is taken into account) number of operations which needed to be performed with the use of the RA algorithm in the case of numerous variables $L_i$. Bernard et al. [2015], constructed an ERA algorithm (*Extended Rearrangement Algorithm)* also on the basis of the RA algorithm. In relation to the RA algorithm, the extension involves determining the minimum elements of the distribution of a sum of random variables in the sense of convex order in the upper and lower part of the distribution separated by the set significance level $\alpha$ with a limitation of variance taken into account. The ERA algorithm is used (through relevant regrouping of elements in the sphere of variable $L$) for the upper and lower part of distribution $L$, respectively. The examples presented by the authors prove that the ERA algorithm works well and the additional condition of a limited variance leads to better (compared to the RA algorithm) estimations of the *VaR* boundaries. On the basis of the ERA algorithm, the authors prove that models used by the participants in, and the regulators of the capital market can underestimate *VaR* whereas the values-at-risk designated in this manner may be incomparable. They additionally claim that the determination of capital requirements at a high confidence level, e.g. 99.5%, is justified.

## 5. Diversification effect for life and health underwriting risk – an empirical example

This section presents the results of an analysis of the impact of selected dependence structures on the diversification effect in the case of aggregation of capital requirements for life and health underwriting risk.

### 5.1. Research method

In the study it was assumed that losses (in million euros) related to life underwriting risk and health underwriting risk, are modelled with the use of random variables of normal distribution[10]: $L_1 \sim N(0;\ 392)$ and $L_2 \sim N(0;\ 248)$, respectively. Capital requirements $\kappa(Y)$ and diversification ratio $d$ have been determined:

- in accordance with the standard procedure proposed in Solvency II, i.e. with the use of formula (5) with correlation coefficient[11] $\rho_{12} = 0.25$;
- with the assumption that the dependence structure between $L_1$ and $L_2$ is modelled with the use of the Student copula (df = 2), the Student copula (df = 5), the Gumbel copula, the Frank copula, the Clayton copula and the Galambos copula. Copula parameters are determined in such a way that the linear correlation coefficient $\rho$ between marginal distributions should be equal to 0.25 in each of the analysed structures. The values of these parameters and additional information on dependences in the lower ($\lambda_L$) and upper ($\lambda_U$) tail are given in the first column in Table 1;
- with the assumption that $L_1$ and $L_2$ are independent;
- with the assumption that $L_1$ and $L_2$ are comonotonic.

Then, it was assumed that there was no information on the dependence structure between $L_1$ and $L_2$, and lower and upper estimations $\kappa(Y)$ and $d$ were determined. The

[10] The parameters adopted were the same as in [Bernard, Denuit, Vanduffel 2016].

[11] See: [Directive 2009/138/EC…, ANNEX IV, p. 124].

$\underline{VaR}_{0.995}(L)$ and $\overline{VaR}_{0.995}(L)$ values necessary for this purpose were obtained with the use of the ARA algorithm. The results are given in the second and third column of Table 1 and in Figure 2.

### 5.2. Results

The conducted study indicates that in the process of determining solvency capital requirements in Solvency II, knowledge of only distributions of aggregated risks $X_i$, without knowledge of the dependence structure between them, is insufficient. The range of possible values $\kappa(Y)$ from EUR 274.7 to 1791.5 million obtained in this way is useless from a practical point of view. Considering the above, in the process of aggregating capital requirements, one should take into consideration the dependence between risks. The standard Solvency II solution proposes the use of linear correlation coefficients exclusively. However, as the results of the analysis indicate, the method does not guarantee that the capital will be determined unequivocally. The same correlation coefficients between marginal distributions may correspond to different dependence structures. This results in an estimation of capital $\kappa(Y)$ and the corresponding diversification ratio at different levels. However it seems natural to expect that additional information on the dependence structure in the form of correlation coefficients between risks should largely narrow down the range of potential values for $\kappa(Y)$ and $d$. The presumption was confirmed in the studies only in the case of several selected dependence structures (i.e. the Student copula (df = 2), the Student copula (df = 5), the Gumbel copula, the Frank copula, the Clayton copula and the Galambos copula). Capital values $\kappa(Y)$ from 1234.4 to 1468.1 million were obtained for them, which corresponded to the diversification ratio from the range (74.9; 89.1). Generally it can be stated that the greater the dependence in the upper

**Table 1.** The research results

| Dependence structure | Capital requirement $\kappa(Y)$ | Diversification ratio in % |
|---|---|---|
| Solvency II standard formula | 1322.9 | 80.2 |
| Student copula (df = 2, $\rho$ = 0.265, $\lambda_L = \lambda_U$ = 0.278) | 1468.1 | 89.1 |
| Student copula (df = 5, $\rho$ = 0.253, $\lambda_L = \lambda_U$ = 0.107) | 1395.9 | 84.7 |
| Gumbel copula ($\theta$ = 1.186, $\lambda_L$ = 0, $\lambda_U$ = 0.206) | 1445.1 | 87.7 |
| Frank copula ($\theta$ = 1.631, $\lambda_L = \lambda_U$ = 0) | 1279.1 | 77.6 |
| Clayton copula ($\theta$ = 0.370, $\lambda_L$ = 0.154, $\lambda_U$ = 0) | 1234.4 | 74.9 |
| Galambos copula ($\theta$ = 0.426, $\lambda_L$ = 0, $\lambda_U$ = 0.196) | 1450.6 | 88.0 |
| Independence structure | 1194.8 | 72.5 |
| Comonotonic structure | 1648.5 | 100.0 |
| Unknown dependence structure | (274.7; 1791.5) | (16.6; 108.7) |

Source: own calculations.

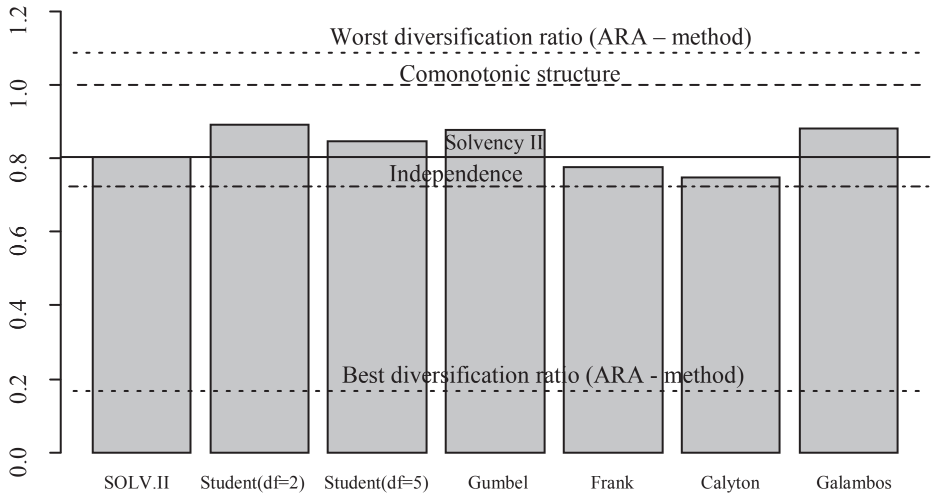


**Fig. 2.** Diversification ratio for the analysed dependence structures

Source: own calculations.

tail, the greater capital requirement $\kappa(Y)$, and thus the lower the diversification effect. It is just the opposite in the case of dependences in the lower tail – the stronger the dependence, the lower value $\kappa(Y)$ and the greater the diversification effect. The purpose of further research to be undertaken by the authors, will be to determine the lower and upper limit for $\kappa(Y)$ and $d$ for any dependence structures.

## 6. Conclusions

The results of QIS5 presented in [EIOPA Report... 2011] show that the diversification effect may have a considerable impact on the decrease in the solvency capital requirement of an insurer. On the one hand, the solution proposed as part of Solvency II, where the requirement is taken into account when determining the SCR, belongs to elements awarding good risk management systems but, on the other hand, it requires that managers develop the right risk aggregation methods.

It should be emphasised here that the diversification effect is closely related to (or results from) the dependence structure between risks for which capital requirements are aggregated. Therefore in order to estimate correctly the diversification effect the structure must be identified properly. The covariance-variance method used in standard solutions has several advantages, including:

- it is relatively simple and intuitive;
- it facilitates consensus on modelling of typical relationships between diversified types of risk;
- it enables one to add new types of risk (e.g. new media, a new business line, a new subsidiary);

- correlation is a well-known dependence method which facilitates communication with non-professionals.

However, the example presented in the previous chapter as well as the results of other studies on the impact of dependence structures on solvency capital requirements indicate that the applied variance-covariance method (which takes into consideration only linear dependencies) may lead to an erroneous evaluation of the diversification effect. Dependence structures between aggregated types of risk can be so complex that a correlation matrix is insufficient (they may be nonlinear or may have stronger dependences in distribution tails, etc.). In addition, due to the lack of sufficient reliable data for many types of risk, the correlation coefficients used in standard formulas in most cases are determined by an expert method (depending on an individual expert opinion).

Therefore there is a need to carry out research that will focus on seeking new, more precise methods of recognising and modelling dependence structures as well as ways of including them in solvency models. In internal models (full or partial) or own parameters, the Solvency II Directive allows for or even encourages such research and introduction of non-standard solutions into solvency models. New methods must be accepted by the market regulator.

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