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The effect of ball size distribution on power draw, charge motion and breakage mechanism of tumbling ball mill by discrete element method (DEM) simulation

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Abstract: In this research, the effect of ball size distribution on the mill power draw, charge motion regime and breakage mechanism in a laboratory ball mill was studied using the discrete element method (DEM) simulation. The mill shell and crushing balls were made of Plexiglas® and compressed glass, respectively. Modeling was performed using Particle Flow Code 3D (PFC3D). Model parameters were back-calculated by comparing the power draws and images obtained from simulation and laboratory test works. After determining the model parameters, the mill was simulated in mill fillings of 15, 20, 25, 30, 35 and 40% with ball media of 2 and 2.5 cm in diameter. For every mill filling, the numbers of big and small balls were changed and 11 scenarios were chosen. The results showed that at a constant mill filling, the power draw was changed with changing the ball size distribution and for all mill fillings the maximum power draw occurred when the fraction of small balls was between 30-40%. The effect of ball size distribution increased with increasing mill filling and for the mill filling of 35%, the ball size distribution had the maximum effect on the power draw. When the mill charge contained mono-sized balls, the ball flow regime inside the mill transited to the cataracting and impact breakage was the main breakage mechanism. Increasing the fraction of big balls from 0 to 70% led the flow of balls into the cascading regime and breakage mechanism to attrition.

Keywords: *discrete element method, ball mill, ball size distribution, mill power, breakage mechanism*

1. Introduction

The power draw and grinding efficiency of tumbling mills depend solely on motion of the grinding charge and the ensuing ball collisions that utilize the input power to cause particle breakage. Empirical correlations developed to calculate the mill power draw from design and operating parameters (Guerrero and Arbiter, 1960; Bond, 1961; Hogg and Fuerstenau, 1972; Morrell, 1992; Hlungwani et al., 2003). All of these models were not able to respond to change in lifter geometry and ball size distribution. The solution to this problem is the use of numerical tools, such as the discrete element method (DEM) that can predict the behavior of granular materials. DEM can simulate motion of the charge and predict the power draw accurately. In this technique, coordinates and velocities of each individual ball are computed from the knowledge of forces arising when balls collide with each other and with mill shell and lifter walls. The unique feature of this method is that the mill power draw can be predicted for a mixture of ball sizes, different lifter geometry and lifter spacing, which is absent in other methods (Data et al., 2013).

DEM in its entirety was first published in the civil engineering literature (Cundall and Strack, 1979) from where it spread rapidly into many engineering disciplines including the mineral processing industry. For the first time, two dimensional numerical methods were used for improving the deficiency of ball mills during 1990s (Mishra and Rajamani, 1992; Inoue and Okaya, 1995).

DEM modeling is now well established and described in review articles (Walton, 1993; Barker, 1994; Mishra, 2003a, 2003b). It has been used successfully in modeling of ball and AG/SAG mills, that

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is simulation of charge motion (Mishra and Rajamani, 1994; Cleary, 1998; Radiszewski, 1999; Rajamani, 2000; Mishra and Murty, 2001; Venugopal and Rajamani, 2001; Monama and Moys, 2002; Powell and Mcbride, 2004; Sun et al., 2009; Kiangi et al., 2013), power draw (Cleary, 2001; El-rahman, 2001; Nierop et al., 2001;Djordjevic, 2005, 2003; Farzanegan et al., 2012; Data et al., 2013; Weerasekara et al., 2016), segregation (Cleary, 1998), grinding actions of ball mills (Inoue and Okaya, 1995), dynamics of ball and rock charge (Rajamani and Mishra, 1996), liner profile and liner wear (Djordjevic et al., 2004; Kalala and Moys, 2004; Kalala et al., 2008, 2005; Makokha et al., 2007; Powell et al., 2011; Franke et al., 2015), predictions of flow patterns (Cleary et al., 2006a, 2003; Jonsén et al., 2015, 2014; Cleary, 2015; Geng et al., 2016), breakage mechanism (Khanal and Morrison, 2008; Cleary and Morrison, 2012, 2011; Wang et al., 2012; Delaney et al., 2013), experimental validation (Cleary and Hoyer, 2000; Cleary et al., 2003; Delgadillo, 2012). DEM has also been used to model stirred mills (Cleary et al., 2006b; Sinnott et al., 2006; Santhanam et al., 2013), planetary ball mill (Beinert et al., 2015; Ye et al., 2015), the centrifugal mills (Inoue and Okaya, 1996a), Hicom mills (Hoyer, 1999; Cleary and Owen, 2016; Ghayour et al., 2016) and Wiley Mill (Naik et al., 2013). Also, modeling of the tumbling mills with combination of CFD and DEM (Jonsén et al., 2014; Mayank et al., 2015; Zhong et al., 2016) and simulation of mill charge in 3D using the BLAZE-DEM GPU framework (Govender et al., 2015) was performed by researchers. A review of the current status of DEM usage in comminution is described (Weerasekara et al., 2013) .

The grinding process within a tumbling ball mill includes coarse and fine grinding, which are mainly provided by big balls and small balls separately. A suitably chosen ball size distribution could affect not only the output, but also the quality of product (Zhang et al., 2014). Many researchers have studied the effect of either ball size diameter or ball size distribution on the mill performance, power consumption, mill throughput, specific rate of the breakage and product fineness using empirical methods such as the Bond, Austin and population balance models (Austin et al., 1976; Fuerstenau et al., 1999; Katubilwa and Moys, 2009; Magdalinovic et al., 2012; Cho et al., 2013; Kabezya and Motjotji, 2015).The influence of ball size distribution on the grinding effect in a horizontal planetary ball mill was studied by DEM. The results showed that the maximum impact energy could be acquired when filling rate was 24%. The mean contact force increased with the proportion increasing of the large balls, meaning that the ball size distribution had some effect on crushing and grinding (Zhang et al., 2014). However, the horizontal planetary ball mill is different than the tumbling ball mill in many aspects.

Djordjevic (2005) studied the influence of ball charge size distribution on the net-power draw of tumbling mill based on DEM modeling. Charge of the mill was composed from the spherical balls whose size (diameter) varied in the range of 10–80 mm. Five modelling runs were performed for charges composed from 5 size fractions, 4, 3, 2 and 1 size fraction. The total mass of balls in the model remained almost exactly the same, although the number of balls gradually decreased as the average size of balls increased. The mill was run with the same velocity (70% of critical) and average power was monitored as a function of time. The specific power showed minimal change when the smallest ball size fraction (10 mm) was eliminated. The shape of the charge also remained practically the same. The power also remained almost constant when balls with diameter of 20 mm were eliminated and their mass transferred to the next coarser size fraction (diameter 40 mm) (Djordjevic, 2005). In this research just 4 runs were simulated and the mass of mill load was kept constant. The results and discussions were limited to power draw and there was no discussion about the charge motion regime.

Despite the importance of modeling the effect of ball size distribution, few papers related to the effect of ball size distribution have been found in the DEM literatures. In this regard, this paper reports on using DEM simulation to study the effect of ball size distribution on the mill power draw and charge motion regime and breakage mechanism of a laboratory ball mill.

2. Materials and methods

In the discrete element method, assemblies of discs in two- dimensional methods or balls in three dimensional methods are influenced by stresses. Therefore, displacements and contact forces are found through a series of calculations. These calculations trace the movements of the individual particles (Cundall and Strack, 1979). During these calculations, some parameters for the mill shell and balls are necessary. Normal stiffness (K_n), shear stiffness (K_s) and friction coefficient (μ) for the mill shell and balls should be used in numerical modeling. These parameters should be validated with experimental observation. For this purpose, a transparent laboratory ball mill was made from the Plexiglas shell and rubber liner and lifters. The balls were made of compressed glass. The transparent ball mill made it possible to view the charge motion inside the mill and capture the necessary images by a high-resolution camera. The shaft of the mill was attached to a load beam to measure the torque. The driving motor of the mill had a digital control unit that was used to set a mill speed up to 120% of the critical speed. The properties of the transparent mill are presented in Table 1. A digital camera (Canon SX710 HS) was used to take pictures of the mill charge with the shutter speed of 1/500 s (about 5 fps) to observe the charge motion.

Particle Flow Code 3D (PFC3D) was used for discrete element modeling of the ball mill. PFC3D modeling is based on the assumption that the individual particles (balls) can be treated as rigid bodies. At contacts, rigid particles are allowed to overlap. The magnitude of the overlap is related to the contact force. There overlaps are small relative to the size of the particles. PFC3D can simulate the grinding media motion in the ball mill and calculate the power draw (Djordjevic et al., 2004). The power draw calculated using PFC3D refers to the total power draw accumulated for each particle and ball in the mill (Djordjevic, 2003).

Table 1. Mill specifications

Property	Value
Effective mill diameter (cm)	25
Effective mill length (cm)	25
Diameter of small balls (cm)	2
Diameter of big balls (cm)	2.5
Numbers of lifters	4
Dimension of lifters (cm)	1×1×25

3. Determination of simulation parameters

To determine simulation parameters, first a single ball (2 or 2.5 cm) was used as the mill charge and its entire trajectory at 70% of critical speed was traced by snapshots. Afterward, the rotation of mill was modeled by PFC3D. For the back-calculation of mechanical parameters, the parameters were changed in numerical modeling and, simultaneously, the visual results of numerical modeling were compared with images that were taken by the digital camera and also the power draws compared with the power draws that were measured by torque meter. The initial values of stiffness and frictional coefficients for the Plexiglas and glass were considered the same as the values of these parameters published in rock mechanics literature (Jing and Hudson, 2002). However, one single ball may not be enough to represent the total charge behavior due to contact with neighboring balls in the mill charge. Therefore, to ensure the accuracy of simulation parameters, the mill was modeled under three following conditions and the results were compared with experimental images:

- only 100 small balls (8% mill filling),
- only 100 big balls (12% mill filling),
- 150 small and 150 big balls (300 balls in total) (27.5% mill filling).

Table 2. Mechanical parameters obtained for DEM simulations

	mechanical properties		
	normal stiffness K_n (N/m)	shear stiffness K_s (N/m)	friction coefficient μ
wall	5×10^4	5×10^5	0.1
lifters	5×10^4	5×10^5	0.1
small balls (2 cm)	5×10^4	4×10^4	0.05
big balls (2 cm)	5×10^4	5×10^4	0.1

Fig. 1 shows the experimental and simulation results. The best approximate values of normal and shear stiffness and frictional coefficient to result in a proper agreement between experimental and numerical modelings are displayed in Table 2.

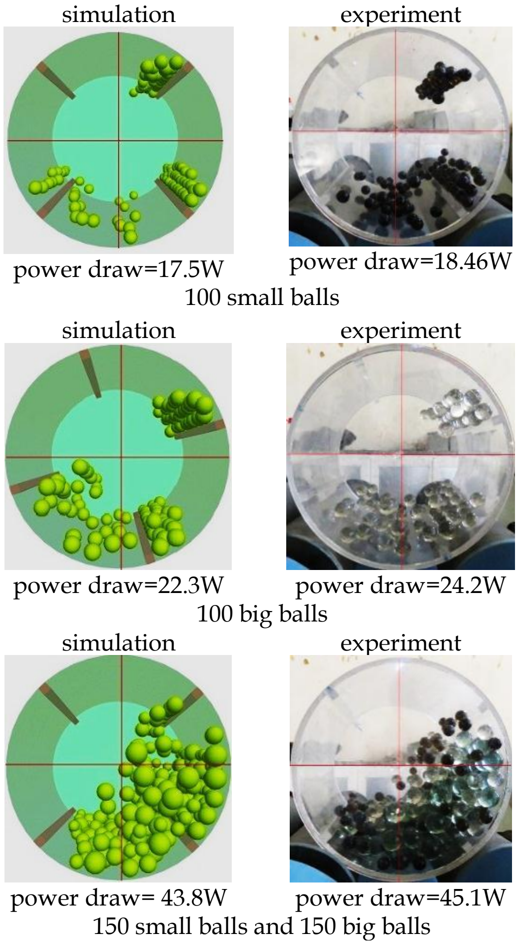

Fig. 1. Comparison between experimental and simulated charge positions and power draws at 60 rpm

4. Simulations

According to the mechanical parameters obtained for DEM simulation (Table 2), the mill was simulated at 60 rpm (70% of critical speed) with 6 different fillings of 15, 20, 25, 30, 35 and 40%. For every mill filling, the number of big and small balls was changed and 11 scenarios were chosen. The mixture of balls was changed from 100% small balls and 0% big balls to 0% small balls and 100% big balls. The volume of small balls decreased in step of 10% and their volume assigned to the big balls. For example, the scenarios for 40% filling are given in Table 3, and the numbers of big and small balls are reported in Table 3. For each scenario, the power draw was determined and images were taken from simulated mill charge motion.

5. Results and discussion

5.1. The effect of mill filling and ball size distribution on the mill power draw

Fig. 2 demonstrates the effect of mill filling and ball size distribution on the power draw. The results showed that the power draw increased with increasing the mill filling. As can be seen, at a constant mill filling, the power draw changed with changing the ball size distribution. For all mill fillings, the maximum power draw occurred when the fraction of small balls was between 30–40%. The minimum power draw occurred when the mill charge contained mono-sized balls.

Fig. 3 shows the comparison between charge position for the small balls fractions of 30 and 100% for different mill fillings. It can be observed for all mill fillings the toe angle increased with decreasing fraction of the small balls from 100 to 30%. With increasing the toe angle, the mill filling in the third quarter (180–270°) decreased and the mill filling in the fourth quarter increased. This led to the increase in the mill power draw because the ball charge in the third quarter (toe region) generated a torque in the direction of mill rotation and reduced the driving torque from the motor, while the ball charge in the fourth region generated the inverse torque on the mill rotation.

The standard deviations of the power draw data for various levels of the mill filling were calculated and the results are given in Fig. 4. Data presented in Fig. 4 indicate that the standard deviation in power draw data increased with increasing mill filling. With the further increase in mill filling from 35 to 40%, the standard deviation decreased. It means that the effect of ball size distribution increased with increasing the mill filling, and at the mill filling of 35%, the ball size distribution had the maximum effect on the power draw.

Fig. 5 shows images of mill charge for the ball mill filling of 35%. It can be seen that when the small ball fraction was between 100-70%, the toe angle was about 180° and there was only one layer of balls in the region 315–45°. With decreasing the fraction of the small balls to 30% (fraction of the big balls to 70%), the toe angle increased to 225° and the layers of balls in the region of 315–45° increased.

Table 3. Simulated scenarios for 40% filling

scenario number	small balls (d=2 cm)		big balls (d=2.5 cm)	
	(%)	number of balls	(%)	number of balls
1	100	1015	0	0
2	90	914	10	52
3	80	812	20	104
4	70	711	30	156
5	60	609	40	208
6	50	508	50	260
7	40	406	60	311
8	30	305	70	363
9	20	203	80	415
10	10	102	90	467
11	0	0	100	519

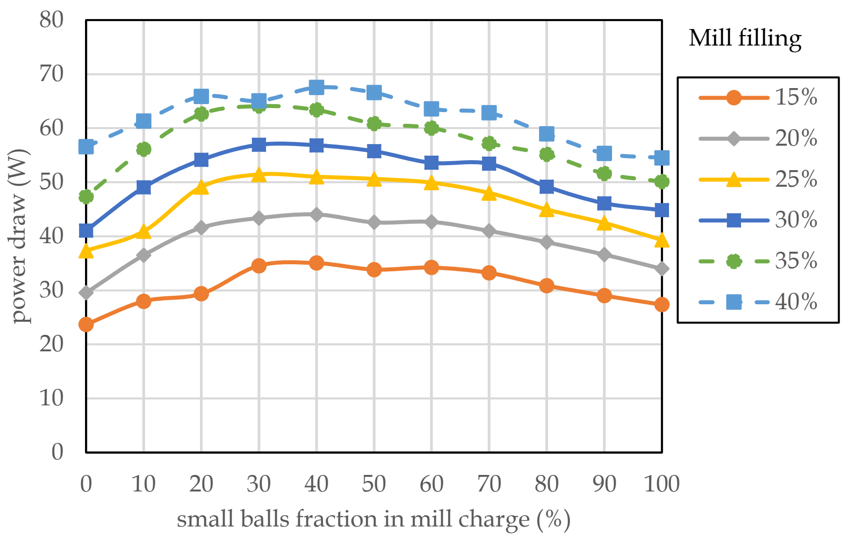

Fig. 2. Variations of power draw with changing mill filling and ball size distribution

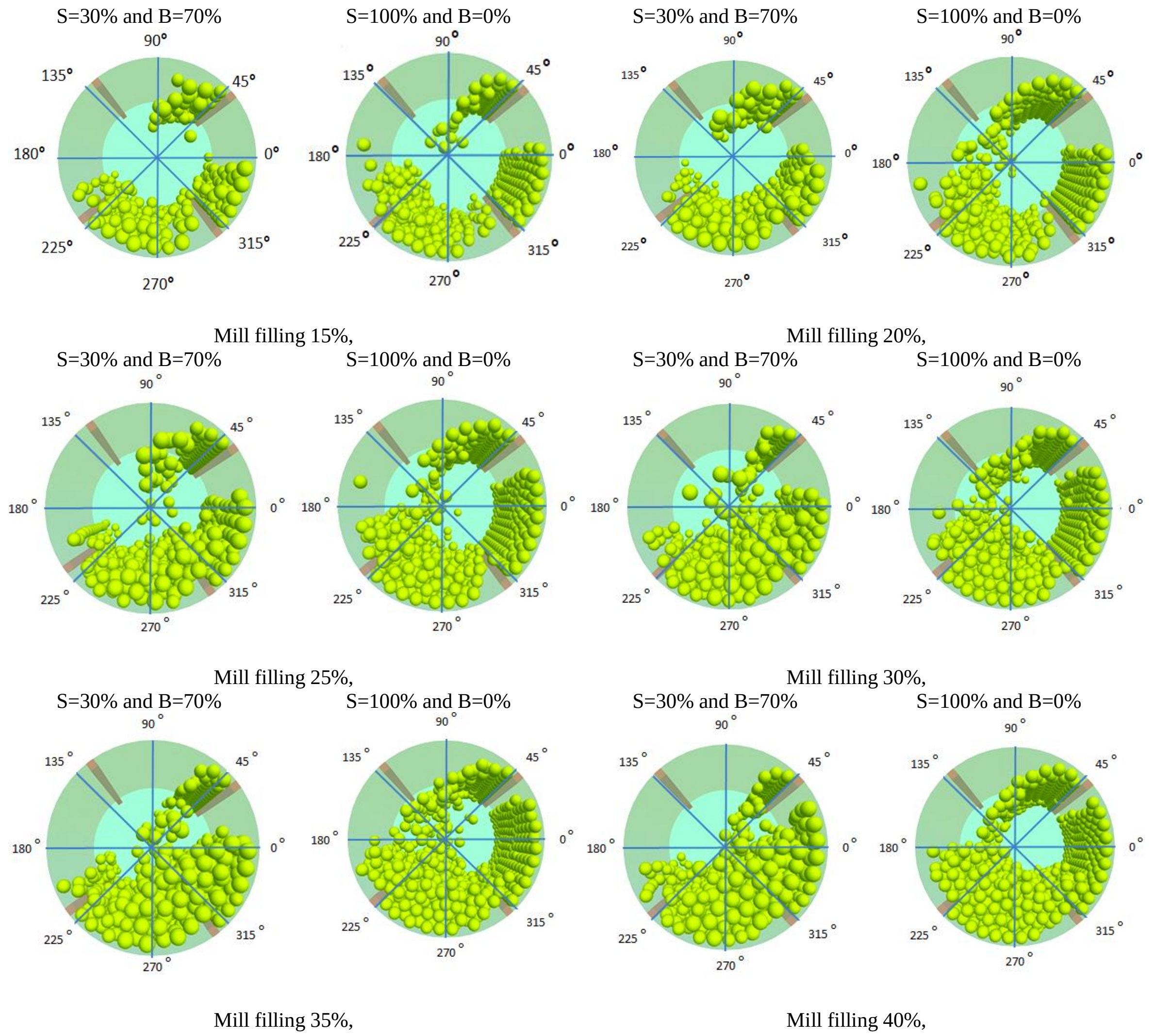

Fig. 3. Comparison between charge position for the small balls fraction 30 and 100% for different mill fillings (S= small balls fraction and B= big balls fraction)

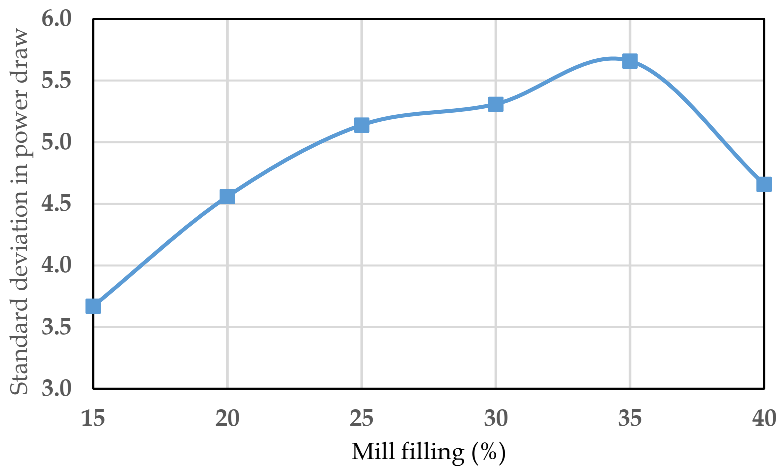

Fig. 4. Standard deviations of the power draw data

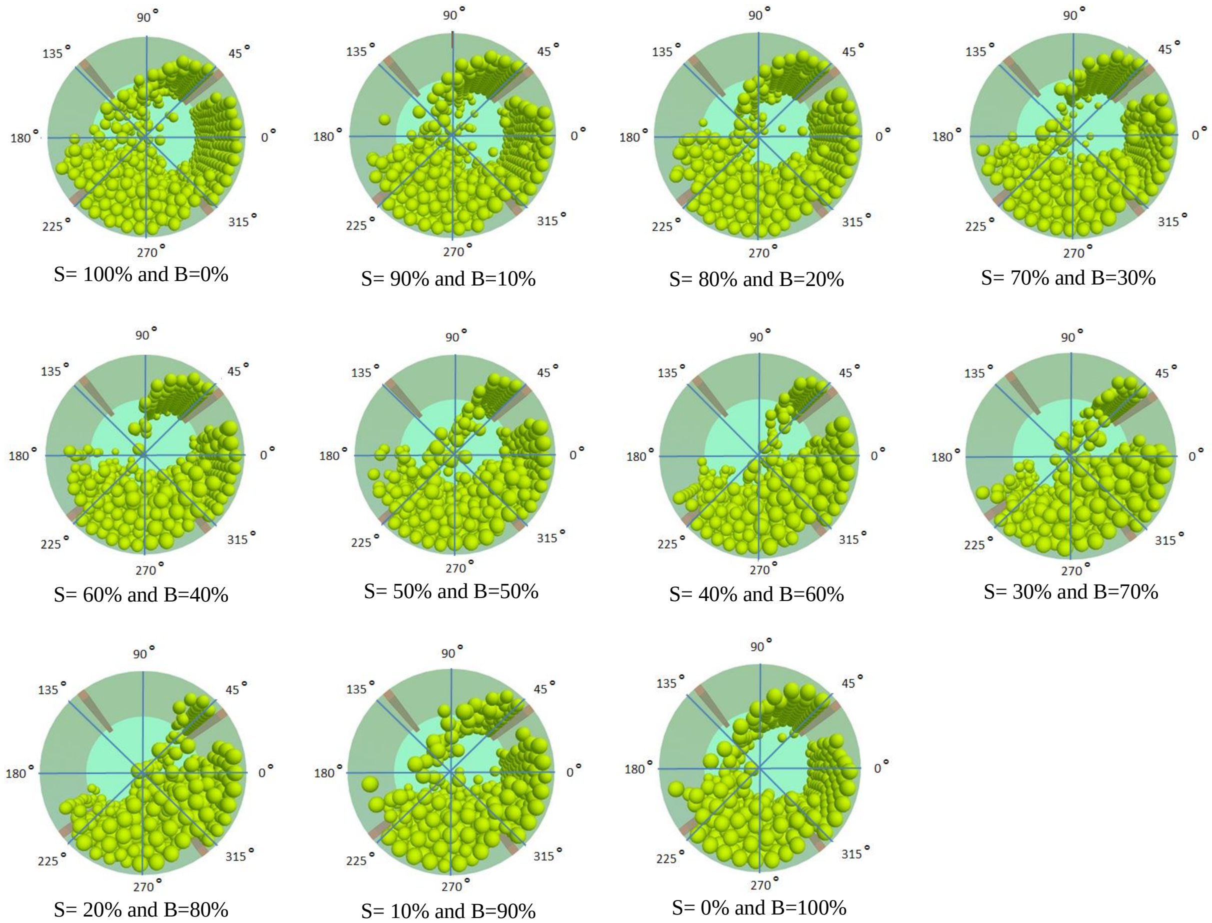

Fig. 5. Change in mill charge shape and position due to changing ball size distribution at ball mill filling 35% (S= small balls fraction and B= big balls fraction)

5.2. The effect of mill filling and ball size distribution on charge motion regime and breakage mechanism

The motion of grinding media and the energy distribution have a profound influence on comminution of particles in tumbling mills (Weerasekara et al., 2016). It has been established that the charge motion in a tumbling grinding mill is characterized as cascading and cataracting (Powell and Nurick, 1996). Cataracting and cascading regimes are supposed to play the major role in comminution (Inoue and Okaya, 1996b). Spherical balls are carried along the mill shell to a point where a component of the gravitational force overcomes the centrifugal force, letting the charge fall in a parabolic path till it hits the mill shell. The process of balls rolling down the surface is called cascading, while that of projected out stream is called cataracting (Venugopal and Rajamani, 2001). The cataracting regime enhances the grinding efficiency by producing a fraction of charge for a high energy impact breakage. In the cascading regime, breakage is governed by contact mechanics, where the grinding action takes place through collision and attrition (Hlungwani et al., 2003). Fig. 6 shows cascading and cataracting regimes in a tumbling mill.

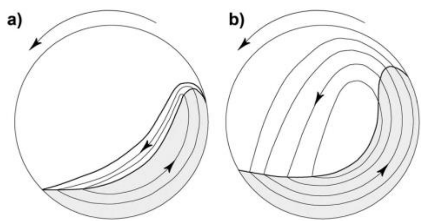

Fig. 6. Cascading (a) and cataracting (b) regimes in a tumbling mill

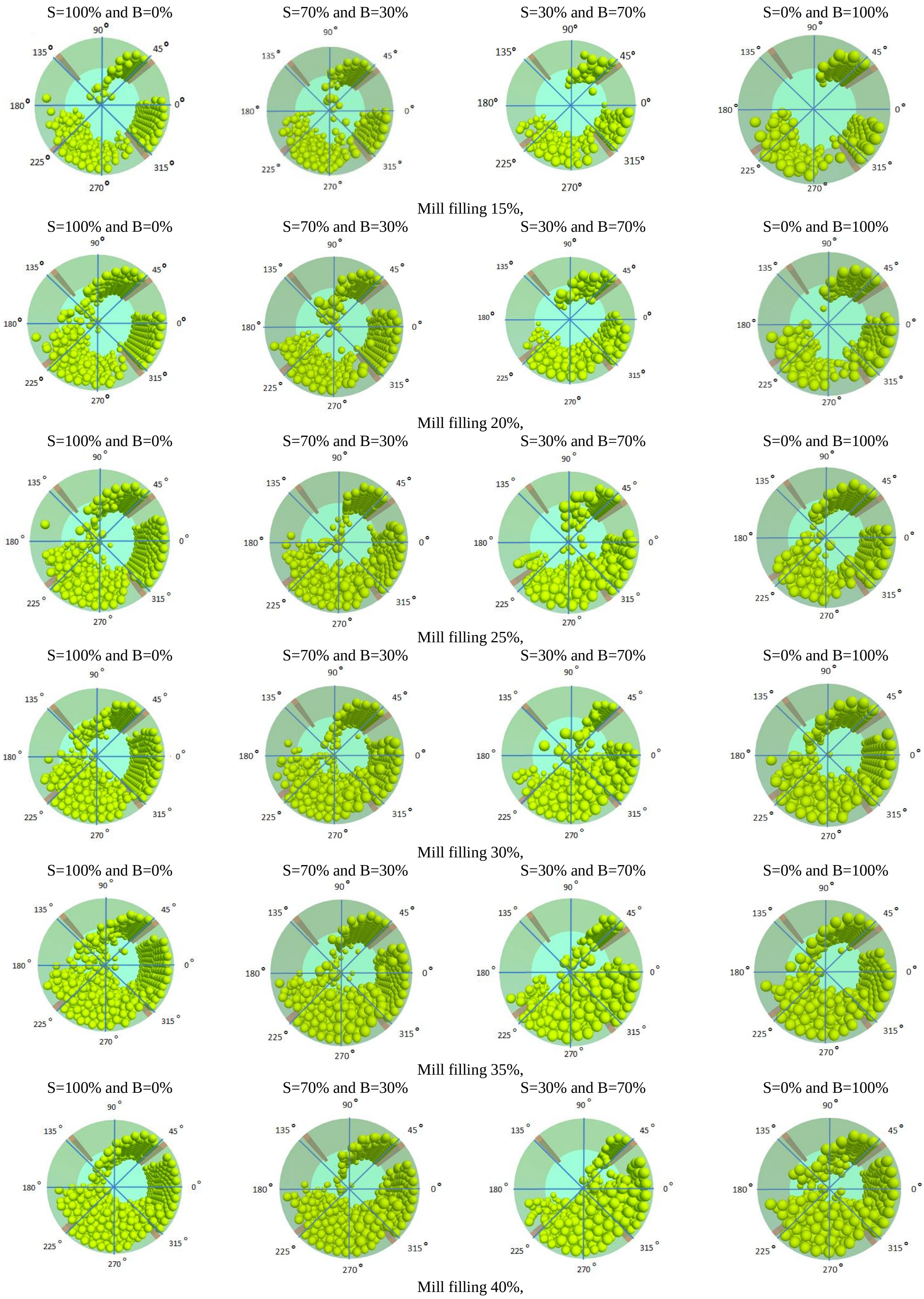

Fig. 7. Changing in charge motion due to changing ball size distribution at different ball milling (S= small balls fraction and B= big balls fraction)

Fig. 7 shows the effect of ball size distribution on the charge motion regime. The results show that, when the mill charge contains mono-sized balls, for all mill filling, the major regime inside the mill is cataracting. Under these conditions, there is only one layer of balls inside the region of 315-45 °, and it can be concluded that impact breakage was the main breakage mechanism and occurred in the toe region. This is especially true when the mill charge contains only the small balls.

For all mill fillings, with increasing the fraction of big balls from 0 to 70%, the toe angle and the layer of balls in the region of 315-45 ° increased. This led the flow of balls into the cascading regime and breakage mechanism to attrition. However, for the low ball mill filling of 15-25%, changes in the ball size distribution had a little effect on the charge motion regime and breakage mechanism. Under these conditions, the ball flow regime transited to the cataracting and impact breakage was the main breakage mechanism.

It is well known that big balls are needed for the effective breakage of big particles, whereas smaller balls are more effective for the breakage of small particles. Therefore, it is a common practice in industry to use a mixture of balls rather than balls of a single size to ensure the efficient grinding of materials of various sizes in the mill. However, the optimum mixture of balls depends on the feed size as well as the product size. Further, the ball size distribution in the mill is not a simple parameter that can be controlled directly, since it depends on the make-up ball charge and wear rate.

6. Conclusion

The effect of ball size distribution increased with increasing the mill filling, and for the mill filling of 35%, the ball size distribution had the maximum effect on the power draw. When the mill charge contained only mono-sized balls, the ball flow regime transited to the cataracting and impact breakage was the main breakage mechanism and occurred in the toe region. This is especially true when the mill charge contains small balls.

Increasing the fraction of big balls from 0 to 70% led the motion of balls to the cascading regime and breakage mechanism to attrition. For the low ball mill filling of 15-25%, changes in the ball size distribution had the little effect on the charge motion regime and breakage mechanism.

The rotational speed was one of the important parameters in tumbling ball mills because it affected the dynamic motion of the balls within the mill. All experiments and simulations in this research were done at 70% of critical speed and, therefore, all presented results based on the critical speed. Further investigations need to be done to find the interaction between the mill speed, ball size distribution and ball charge motion.

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