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Analysis of the spatial distribution of luminous intensity at the fibre output

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A method based on weakly guiding approximation which allows calculation of luminous intensity in step-index fibre was proposed. The calculation shows that spatial distribution of luminous intensity is rotationally symmetrical and has a discrete character. The quantity and values of emission angles depend on the normalized frequency of a fibre. Values of luminous intensity depend on the position and radiant power distribution of the light source.

Keywords: optical fibres, photometry, radiative transfer, luminous intensity.

1. Introduction

Application of optical fibres in light engineering is observed to increase, which makes it necessary to calculate of properties of radiation emitted by these fibres. This work presents a method for determining luminous intensity of a fibre, based on distribution of electromagnetic field inside the fibre core.

We consider a step-index fibre with radius a and refractive indices of the core and the cladding n_1 and n_2, respectively. The front surface of the core is plane and perpendicular to the fibre symmetry axes. A monochromatic light source is placed on the core axis. Its wavelength equals quasi-λ and the bandwidth is $\Delta\lambda$. Spatial distribution radiant power of the source is steady, so radiant power guided by every mode is identical. Radiation guided in the cladding is ignored. Since angles of light beam guided inside the core are only slightly different for modes creating the linear polarized modes [1], [2], approximation for weakly guided modes can be used in the following calculations.

2. Luminous flux of the core front surface

Values of particular coordinates of electric and magnetic fields can be calculated according to solutions of Maxwell's equations in cylindrical coordinates (r, φ, z) [1]–[3]. The components E_z and H_z are small in comparison with the others, therefore they can be omitted.

The remaining electromagnetic field coordinates are given as follow:

$$E_r = -j\,A_{mp}\,\frac{k_1+\beta_{mp}}{2u_{mp}}\,J_m(u_{mp}r)\exp\{j[(m\pm 1)\varphi-\beta_{mp}z]\}, \tag{1}$$

$$E_\varphi = -j\,A_{mp}\,\frac{k_1+\beta_{mp}}{2u_{mp}}\,J_m(u_{mp}r)\exp\{j[(m\pm 1)\varphi-\beta_{mp}z]\}, \tag{2}$$

$$H_r = j\,\frac{A_{mp}}{Z_1}\,\frac{k_1+\beta_{mp}}{2u_{mp}}\,J_m(u_{mp}r)\exp\{j[(m\pm 1)\varphi-\beta_{mp}z]\}, \tag{3}$$

$$H_\varphi = -j\,\frac{A_{mp}}{Z_1}\,\frac{k_1+\beta_{mp}}{2u_{mp}}\,J_m(u_{mp}r)\exp\{j[(m\pm 1)\varphi-\beta_{mp}z]\}, \tag{4}$$

where A_{mp} denotes an integration constant for LP_{mp} mode (m, p are natural numbers). The propagation constant in the core is k_1, and β_{mp} is LP_{mp} mode propagation constant in the z-direction. The $J_m(u_{mp}r)$ denotes the m-th order Bessel's function of the first kind. The remaining quantities in Eqs. (1)–(4) can be calculated according to the following:

$$Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}, \tag{5}$$

$$\beta_{mp}^2 = k_1^2 - u_{mp}^2 \tag{6}$$

where μ_0 is permeability of free space, ε_1 – permittivity of the fibre core, whereas u_{mp} and w_{mp} are the solutions of the system of equations:

$$\begin{aligned} \frac{u_{mp}\,a\,J_{m-1}(u_{mp}a)}{J_m(u_{mp}a)} &= -\frac{w_{mp}\,a\,K_{m-1}(w_{mp}a)}{K_m(w_{mp}a)}, \\ u_{mp}^2 + w_{mp}^2 &= \frac{4\pi^2 a^2(n_1^2-n_2^2)}{\lambda^2}, \end{aligned} \tag{7}$$

$K_m(w_{mp}a)$ denotes m-th order modified Bessel's function of the first kind (a – the core radius).

Distribution of the electromagnetic field, described by Eqs. (1)–(4), will be replaced by eight linear polarized plane waves [4]. Neither the field values nor radiant power at any point of the core can change. Waves are moving at an angle of θ_{mp} with respect to the core axis.

For the LP_{mp} mode, the value of angle θ_{mp} equals

$$\cos\theta_{mp} = \frac{\beta_{mp}}{k_1}. \tag{8}$$

Electric field vectors of waves guided along the axial section plane can be obtained from:

$$E_{or} = \frac{k_1}{2u_{mp}^2\cos\delta}\,(k_1 Z_1 H_\varphi - \beta_{mp}E_r), \tag{9}$$

$$E_{o\varphi} = \frac{k_1}{2u_{mp}^2\cos\delta}\,(k_1 E_\varphi - \beta_{mp}Z_1 H_r), \tag{10}$$

while in the case of waves, guided along the plane perpendicular to the one mentioned above, they are calculated from the following formulas:

$$E_{pr} = \frac{k_1}{2u_{mp}^2\cos\delta}\,(k_1 E_r - \beta_{mp}Z_1 H_\varphi), \tag{11}$$

$$E_{p\varphi} = \frac{k_1}{2u_{mp}^2\cos\delta}\,(k_1 Z_1 H_r - \beta_{mp}E_\varphi) \tag{12}$$

where δ is an angle between the electric field vector and the front surface of the core for the waves signed with t index or an angle between the magnetic field vector and the front surface of the core for waves signed with s index.

Putting Eqs. (1)–(4) into (9)–(12) we can obtain the following relations:

$$E_{or} = E_{o\varphi} = E_{pr} = E_{p\varphi} = A_{mp}\frac{k_1}{4u_{mp}\cos\delta}J_m(u_{mp}r). \tag{13}$$

The radiant power density guided by all waves equals:

$$S = A_{mp}^2\frac{k_1^2}{4u_{mp}^2\cos^2\delta}J_m^2(u_{mp}r), \tag{14}$$

and its component in the z-axis can be calculated according to the following formula:

$$S_z = S\frac{\beta_{mp}}{k_1}. \tag{15}$$

Otherwise, the z-coordinate of radiant power density equals the z-component of Poynting's vector, therefore

$$S_z = \mathrm{Re}\{E_r H_\varphi^* - E_\varphi H_r^*\}. \tag{16}$$

Comparing Eqs. (15), (16) and taking formulas (1)–(4) into account, the following results can be obtained:

$$\cos^2\delta = \frac{2k_1\beta_{mp}}{(k_1+\beta_{mp})^2}. \tag{17}$$

The whole power, guided by the LP_{mp} mode through the core, equals

$$P_{mp} = 2\pi\int_0^a Sr\mathrm{d}r. \tag{18}$$

Substituting formulas (14) and (17) into (18) we obtain after integrating

$$P_{mp} = A_{mp}^2 \frac{\pi a^2 k_1^2 (k_1+\beta_{mp})^2}{8\beta_{mp} u_{mp}^2 Z_1}\left[J_m^2(u_{mp}a) - J_{m-1}(u_{mp}a)J_{m+1}(u_{mp}a)\right]. \tag{19}$$

As the light source is placed at the core axis, and its radiant power has steady distribution, the power in every linear polarized mode is the same. Assuming that $P_{mp} = 1$, the integrating constant A_{mp} can be defined as

$$A_{mp}^2 = \frac{8\beta_{mp} u_{mp}^2 Z_1}{\pi a^2 k_1^2 (k_1+\beta_{mp})^2\left[J_m^2(u_{mp}a) - J_{m-1}(u_{mp}a)J_{m+1}(u_{mp}a)\right]}. \tag{20}$$

The linear polarized plane waves arriving at the front plane of the core are refracted according to Snell's law and leave it at an angle of γ_{mp}

$$n_1 \sin\theta_{mp} = \sin\gamma_{mp}. \tag{21}$$

To calculate the radiant power density, emitted by the front surface of the fibre, every wave, the electric field vectors of which are E_{or}, E_{pr}, $E_{o\varphi}$ and $E_{p\varphi}$, will be divided into two waves. The electric field vector of one of them (marked with index t) is tangent to the front surface. The second wave, denoted by index s, has its magnetic field vector tangent to the front surface. These vectors are:

$$E_{ors} = E_{or}\cos\delta, \tag{22}$$

$$E_{ort} = E_{or}\sin\delta, \tag{23}$$

$$E_{prs} = E_{pr}\sin\delta, \tag{24}$$

$$E_{prt} = E_{pr}\cos\delta, \tag{25}$$

$$E_{o\varphi s} = E_{o\varphi}\sin\delta, \tag{26}$$

$$E_{o\varphi t} = E_{o\varphi}\cos\delta, \tag{27}$$

$$E_{p\varphi s} = E_{p\varphi}\cos\delta, \tag{28}$$

$$E_{p\varphi t} = E_{p\varphi}\sin\delta. \tag{29}$$

According to Fresnel's formulas the waves signed with index t, when crossing the border surface, have radiant power density given by

$$q_{et} = \frac{C_{tmp}}{2Z_0}E_t^2. \tag{30}$$

In the same case, the radiant power density of waves signed with s is given by

$$q_{es} = \frac{C_{smp}}{2Z_0}E_s^2. \tag{31}$$

Coefficients C_{smp} and C_{tmp} can be obtained from the following formulas:

$$C_{tmp} = \frac{4\beta_{mp}^2 n_1^2}{n_1^2\sqrt{k_0^2 - u_{mp}^2} + \beta_{mp}^2}, \tag{32}$$

$$C_{smp} = \frac{4\beta_{mp}^2}{\sqrt{k_0^2 - u_{mp}^2} + \beta_{mp}^2} \tag{33}$$

where k_0 denotes the propagation constant in free space, while Z_0 is defined by the formula

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{34}$$

where ε_0 is permittivity of free space.

The radiant power density, emitted by a given point of the core front surface, is calculated according to the following formula

$$q_{e\gamma} = \frac{1}{Z_0}[C_{smp}(E_{ors}^2 + E_{prs}^2 + E_{o\varphi s}^2 + E_{p\varphi s}^2) + C_{tmp}(E_{ort}^2 + E_{prt}^2 + E_{o\varphi t}^2 + E_{p\varphi t}^2)]. \tag{35}$$

If formulas (13), (17) and (22)–(29) are allowed, we get

$$q_{e\gamma} = A_{mp}^2 \frac{k_1(k_1 + \beta_{mp})^2 (C_{smp} + C_{tmp})}{16\beta_{mp} u_{mp}^2 Z_0} J_m(u_{mp} r). \tag{36}$$

The total radiant power emitted by the fibre front surface at an angle γ is calculated from the formula

$$P_{e\gamma} = 2\pi \int_0^a q_{e\gamma} r \, \mathrm{d}r. \tag{37}$$

After integrating we obtain

$$P_{e\gamma} = \frac{C_{smp} + C_{tmp}}{2n_1}. \tag{38}$$

In the case of quasi-monochromatic light of wavelength λ and bandwidth $\Delta\lambda$, the radiant powers emitted at boundary wavelengths $\lambda \pm 0.5\Delta\lambda$, are almost the same. Since the bandwidth is narrow, the relative sensitivity of human eye $V(\lambda)$ is practically constant in the whole spectral range. So, luminous flux emitted at the angle γ equals

$$\Delta\Phi_\gamma = K_m V(\lambda) P_{e\gamma} \tag{39}$$

where $K_m = 680$ lm/W.

3. Luminous intensity spatial distribution

The luminous flux $\Delta\Phi_\gamma$ is emitted into the solid angle of $\Delta\Omega_{mp}$. It is delimited by angles γ_1 and γ_2 corresponding to the light of wavelengths $\lambda - 0.5\Delta\lambda$ and $\lambda + 0.5\Delta\lambda$, respectively, whereas

$$\Delta\Omega_{mp} = 2\pi(\cos\gamma_1 - \cos\gamma_2). \tag{40}$$

Luminous intensity in the direction of γ is calculated based on its definition

$$I_\gamma = \frac{\Delta\Phi_\gamma}{\Delta\Omega_{mp}}. \tag{41}$$

Table. Parameters of the two fibres analysed.

Fibre	a [μm]	n_1	n_2	λ [nm]	$\Delta\lambda$ [nm]
1	2	1.48	1.46	555	5
2	5	1.48	1.46	555	5

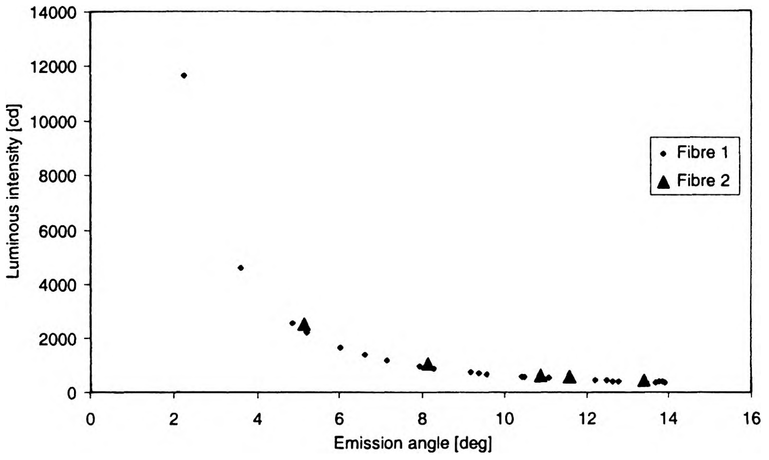

Figure. Characteristic of the luminous intensity of fibre against emission angle γ.

Both the luminous flux and the luminous intensity can be obtained by means of numerical methods. The dependence of luminous intensity on the angle γ is presented in the Figure. It has been obtained for two fibres, the parameters of which are collected in the Table.

4. Conclusions

The spatial distribution of luminous intensity shows a rotational symmetry and has a discrete character. The quantity and values of the emission angles depend on fibre parameters and light wavelength. They are the function of the normalized frequency V of fibre. Light distribution becomes nearly continuous when the value of frequency V increases. The shape of luminous intensity distribution $I_\gamma = f(\gamma)$ depends on radiant power guided by the particular mode, which is related to the position and radiant power distribution of the light source.

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References

[1] EINARSSON G., *Basis of the Optical Fibre Communication*, WKiŁ, Warszawa 1998 (in Polish).
[2] SIUZDAK T., *Introduction to Optical Fibre Communication*, WKiŁ, Warszawa 1997 (in Polish).
[3] MAJEWSKI A., *Optical Fibres – Theory and Designing*, WNT, Warszawa 1991 (in Polish).
[4] NIKOŁAJEW A., DOROSZ J., Opt. Appl. **33** (2003), 205.

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