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Laser beam propagation in gain media of diode pumped lasers*

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Two models of gain and thermal guiding effects were derived. In the first one, the complex ABCD matrix for a crystal under gain and thermal guiding was applied to describe the operation of microchip near threshold. In the second one, a simple iterative procedure was proposed to calculate effective fundamental mode parameters of a cavity under thermal and gain guiding for given bare cavity ABCD matrix and pumping parameters, including gain saturation, passive cavity losses and reabsorption ones. The influence of gain guiding effects causes changes of waist width in the range up to 50% compared to expectations derived from thermal guiding theory. Application of such a method for resonators of passively Q-switched lasers was proposed. Results of calculations for microchips were verified with experiment.

Keywords: diode pumped laser, gain guiding, thermal lensing.

1. Introduction

The determination of fundamental mode parameters of the laser cavity under real pumping conditions is one of the oldest and still vital problems in laser physics (see, *e.g.*, [1]–[5]). Due to the proper shaping of pump beam the radially variable gain as well as heat source densities occur in majority of diode pumped lasers [4]–[8]. When analysing such a type of lasers in the framework of space dependent rate equation model [5], [6], [8], [9] it is assumed that laser mode parameters are a priori known. On the other hand, it is well known that the simplest microchip lasers [9] have the confined fundamental mode dependent via thermal guiding effect on the pump beam width and absorbed pump power. However, the thermal guiding effect responsible for mode structure in microchips operating near thresholds [9], [10] does not seem to be a satisfactory mechanism to explain properties of longitudinally pumped rods with high gain. Thus, complementary models taking into account gain guiding, gain related effects, and cavity detuning losses were developed [11]–[18]. It was shown that solely the gain guiding effect can result in generation of non-Gaussian modes of the lowest

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losses [13]–[16]. Moreover, gain saturation and cavity detuning effects can result in changes of the mode structure, waist parameters, threshold powers, *etc.*

The aim of this paper is to analyse the influence of both gain and thermal guiding effects on the properties of several types of longitudinally pumped lasers. We have limited the scope of analysis to resonators operating within stability range for which the complex curvature parameter q exists. A simple linear model valid for cw operation near threshold is presented in Sec. 2. In Section 3, an iterative model taking into account the gain saturation and reabsorption losses has been developed and applied to the analysis of a few types of cw lasers. In the last section conclusions are drawn.

2. Linear model of gain and thermal guiding effects in longitudinally pumped lasers

The simplest method of description of gain and thermal guiding (GTG) effects is the linear ABCD model based on the concept of complex waveguide (see, *e.g.* [1], [11], [19]). Let us assume that we have a cavity formed by complex waveguide with radially variable real and imaginary parts of refractive index and optionally free space and output mirror. The gradients of real part of refractive index are caused by thermal guiding and the gain guiding results in occurrence of transversely variable imaginary part of refractive index. The mean field approximation, the effective index method and near threshold assumption enable us to derive the linearized complex waveguide ABCD matrix M_{GTG} of active element of length l longitudinally pumped by the pump beam of radius w_{p} and instantaneous pump power P as follows:

$$M_{\mathrm{GTG}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\gamma_{\mathrm{GTG}} l) & (n_0\gamma_{\mathrm{GTG}})^{-1}\sin(\gamma_{\mathrm{GTG}} l) \\ -(n_0\gamma_{\mathrm{GTG}})\sin(\gamma_{\mathrm{GTG}} l) & \cos(\gamma_{\mathrm{GTG}} l) \end{bmatrix} \tag{1}$$

with complex GTG waveguide parameter γ_{GTG} being given by

$$(n_0\gamma_{\mathrm{GTG}})^2 = \frac{\beta_{\mathrm{TG}}}{A_p l}\left(1 + i\frac{A_{\mathrm{GTG}}}{d_f(A_p + A_m)}\right)Pd_f \tag{2}$$

where n_0 denotes refractive index, $A_p = \pi w_p^2/2$, $A_m = \pi w_{00}^2/2$ denote the averaged pump and laser mode areas, respectively, w_p, w_{00} denote the pump and mode radii, respectively, d_f denotes optional duty factor of pumping regime. Material constants called thermal lensing coefficient β_{TG} and GTG area A_{GTG} are given as follows:

$$\beta_{\mathrm{TG}} = \eta_h\eta_p\frac{n_0\kappa}{K_c}, \qquad A_{\mathrm{GTG}} = \frac{\eta_s\eta_e}{\eta_h}\frac{\lambda K_c}{\kappa I_{\mathrm{sat}}} \tag{3}$$

where η_s, η_p, η_e, η_h denote Stokes, absorption, excitation and heat conversion efficiencies, respectively, $\kappa = \mathrm{d}n/\mathrm{d}T$ denotes the temperature dispersion of refractive

T a b l e. Thermal and gain guiding parameters of a few Nd doped gain media [7] (for $\eta_s = \lambda_p/\lambda_{gen}$, η_P, $\eta_e = 1$, $\eta_h = 1 - \eta_s$).

Gain medium	τ [μs]	σ [10^{-19}cm^2]	K_c [W/cmK]	κ [10^{-6} K^{-1}]	I_{sat} [W/mm^2]	β_{TG} [mm/W]	W_{GTG} [mm]
Nd:YAG	230	3.3	0.14	7.5	24.8	0.235	0.4
Nd:YVO_4	100	25	0.051	8.5	12.1	0.87	0.413
Nd:YLF	480	1.8	0.06	–2	22.1	–0.116	0.539
Nd:YAP	180	3.7	0.11	9.7	27.8	0.438	0.289
Nd:LSB	90	1.3	0.028	4.4	158.4	0.729	0.091
Nd glass Q246	290	0.41	0.0012	8.6	197.3	0.827	0.07

index, K_c – thermal conductivity, $I_{sat} = h\nu/\sigma\tau$ – saturation intensity (h denotes Planck's constant, σ – emission cross-section, τ – lifetime of excited level, ν – laser frequency). Let us introduce the GTG radius similarly as pump or mode radius: $W_{GTG} = (2A_{GTG}/\pi)^{1/2}$. The values of W_{GTG}, β_{TG} for a few laser crystals are collected in the Table.

The W_{GTG} radius defines the limit of domination of thermal or gain guiding effects. If w_p, $w_{00} >> W_{GTG}$, the thermal guiding (TG) effect dominates over gain guiding, which is a typical case for high power (> 100 W) diode pumped lasers. However, for more confined pumping beams if w_p, $w_{00} \leq W_{GTG}$, the gain guiding effect competes with TG one. As shown in the Table the typical microchip lasers made of Nd:YAG or Nd:YVO_4 should work in the GTG regime for pump widths of 0.1 mm. Our linear GTG model is valid near threshold where gain saturation effects and cavity detuning can be neglected. Moreover, we assume a priori known heat conversion efficiency which in real conditions depends on the pump and laser mode intensities. Knowing the matrix M_{GTG} we can calculate the eigenvalue of complex curvature parameter of any cavity consisting of active medium and the remaining elements. The simple thermal guiding model TG (a case of $\mathrm{Im}(M_{GTG}) = 0$) can be included in "bare" cavity ABCD models and does not result in qualitatively new effects. The main consequences of combined GTG effect are the following:

– discordance of the fundamental mode parameters (Rayleigh range, waist location, wavefront radii) of bare cavity and cavity with gain guiding included;

– occurrence of a new type of diffraction losses as a result of discordance of wavefront and mirror radii.

In the case of microchip laser (*i.e.*, cavity formed by active medium with mirrors deposited at the plane parallel facets) working in linear GTG regime the following formulae on waist area and radius $A_{0,\,mc}$, $W_{0,\,mc}$, output beam area and radius $A_{OC,\,mc}$, $W_{OC,\,mc}$ and diffraction losses $L_{diff,\,mc}$ were derived

$$A_{0,\,mc} = \frac{\pi W_{0,\,mc}^2}{2} = \sqrt{\left(\frac{\lambda}{2}\right)^2 \frac{lA_p}{\beta_{TG} P d_f} \frac{\sqrt{1+X^2}+1}{2(1+X^2)}}, \tag{4}$$

$$A_{\mathrm{OC,\,mc}} = \frac{\pi W_{\mathrm{OC,\,mc}}^2}{2} = \sqrt{\left(\frac{\lambda}{2}\right)^2 \frac{lA_p}{\beta_{\mathrm{TG}} P d_f} \frac{2}{\sqrt{1+X^2}+1}}, \tag{5}$$

$$L_{\mathrm{diff,\,mc}} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+X^2}}\right) \tag{6}$$

where $X = A_{\mathrm{GTG}}/(A_p+A_m)d_f$ denotes undimensional GTG parameter.

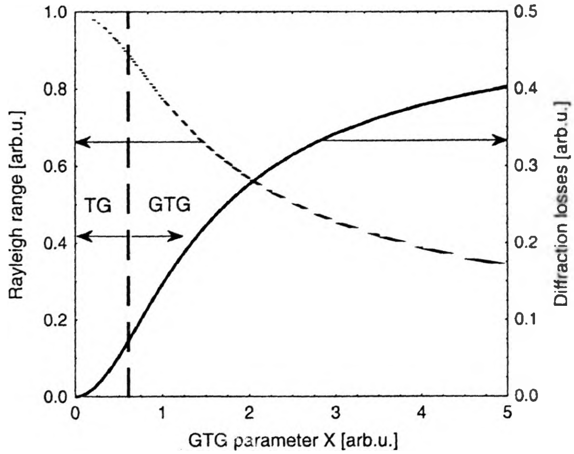

Fig. 1. Rayleigh range, diffraction losses *vs.* GTG parameter X.

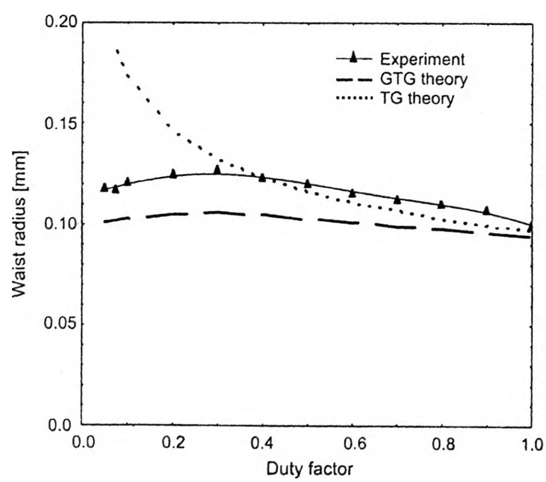

Fig. 2. Comparison of GTG and TG models with experiment.

As shown in Fig. 1, the Rayleigh range decreases with an increase of parameter X. The increase of diffraction losses (see Fig. 1) means that the Gaussian beam being the TEM_{00} mode of microchip under thermal guiding ($X = 0$) ceases the lowest losses property. Such a result confirms quantitatively the non-Gaussian conic-like behaviour of emission of gain guided microchips [14]–[16]. We have verified the GTG model experimentally (see Fig. 2). For unchanged instantaneous pump power P, we did not achieve any significant change of waist diameter with the change of duty factor (*i.e.*, average pump power $P_{avg} = d_f P$), whereas conventional TG model indicates a significantly different dependence. However, for cw pumping ($d_f = 1$) there are no significant differences in measurable parameters of microchip generation. Thus, the TG model gives a satisfactory qualitative accordance with experiment for the majority of cases.

3. Iterative model of GTG cavity with gain saturation effect – cw pumping

The main limitation of linear GTG model is the near threshold approximation which results in neglecting the gain saturation effects. The ABCD model of gain guiding for high intracavity intensities assuming the transversally Gaussian distribution of gain profile was proposed lately by DENCHEV *et al.* [19], which applied to the analysis of unstable resonators with saturable gain profile. We decided to use for our purposes a different and more general approach proposed lately by GRACE *et al.* [20]. The main concept consists in application of analytical solution of saturable gain (or absorption) equation for homogenously broadened medium as follows

$$\frac{1}{I_r}\frac{\mathrm{d}I_r}{\mathrm{d}z} = \frac{g}{1+I_r} \Rightarrow I_r \exp(I_r) = G(z) = \exp(gz) \tag{7}$$

where g denotes small signal gain coefficient, $I_r = I/I_{sat}$ – the relative intensity. The general integral of such a type of the first order ordinary differential equation is known as Lambert W function (see, *e.g.* [21]–[23]) and is defined as

$$W(z)\exp[W(z)] = z. \tag{8}$$

Considering the results of [21] we can express the relative intensity $I_{r,1}$ after passing through an active medium of length l as the explicit function of incident intensity $I_{r,0}$ and given small signal logarithmic gain gl as follows

$$I_{r,1}\exp(I_{r,1}) = I_{r,0}\exp(gl + I_{r,0}). \tag{9}$$

Applying Eq. (8) we have the output intensity given by

$$I_{r,1} = W[I_{r,0}\exp(I_{r,0})G(l)]. \tag{10}$$

For real positive arguments of Lambert W function the well defined simple analytical approximation formulae were derived [21], [22].

Knowing the output intensity at any point for a given incident one and small signal gain we can calculate the output profile for any incident beam and gain profile. Further, Grace's concept consists in approximation of the output beam by the Gaussian one and in calculation of the parameters of virtual "gain diaphragm" ABCD matrix M_{SG} as follows:

$$M_{SG} = \begin{bmatrix} 1 & 0 \\ C_{SG} & 1 \end{bmatrix}, \quad C_{SG} = \frac{i\lambda}{\pi w_{SG}^2} \tag{11}$$

where λ denotes the wavelength, w_{SG} denotes the effective radius of gain diaphragm defined as

$$w_{SG}^{-2} = w_{out,\,SG}^{-2} - w_{inp}^{-2}, \tag{12}$$

w_{inp} denotes the radius of incident Gaussian beam and $w_{out,\,SG}$ denotes the radius of output approximated Gaussian beam. The value of $w_{out,\,SG}$ can be determined according to Siegman's definition [24] using second moments of output intensity profile as follows:

$$w_{out,\,SG}^2 = 2\frac{\int_0^\infty x^3 I_{r,\,out}(x)\mathrm{d}x}{\int_0^\infty x I_{r,\,out}(x)\mathrm{d}x}. \tag{13}$$

To analyze the complex cavities under GTG effect we have developed the iterative procedure working in the following way (see Fig. 3).

We start with calculation of the bare cavity matrix M_{BC} including the effective thermal lensing power of a rod to determine the incident Gaussian beam profile at the

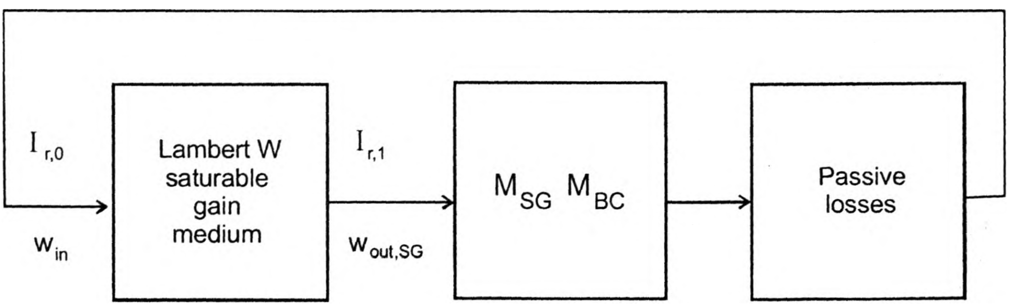

Fig. 3. Scheme of iterative procedure of GTG model for cw laser operating in free running regime.

entrance of a gain medium. We take small value of incident intensity compared to saturation one. In each step of iterative procedure we calculate the output intensity profile after passing through the gain medium according to formulae (9)–(11) and determine the effective gain matrix M_{SG}. Further, we pass the beam through the cavity applying the product of both matrices M_{SG} M_{BC}. In each step we introduce the logarithmic passive losses of cavity δ_{pas} multiplying the intensity profile at output mirror by the factor $\exp(-\delta_{pas})$. Because of saturated gain profile and passive losses the peak intensity converges with the number of roundtrips to finite value, for which the procedure stops. An example of the results of calculation of GTG effect for cw

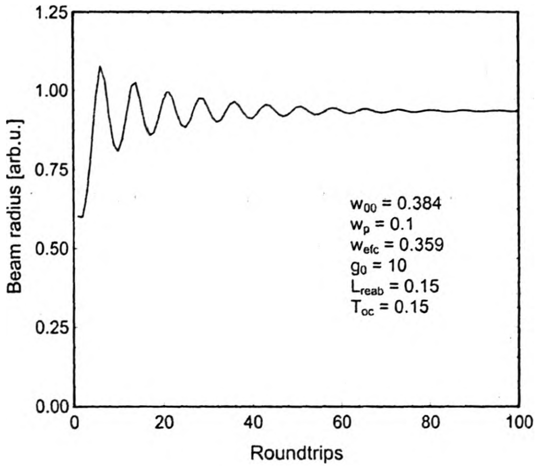

Fig. 4. Instantaneous fundamental mode radius *vs*. roundtrip number (w_{00} – fundamental mode radius of "bare" cavity, w_{efc} – "effective" Gaussian beam radius, L_{reab} – reabsorption losses, T_{OC} – transmission of output coupler).

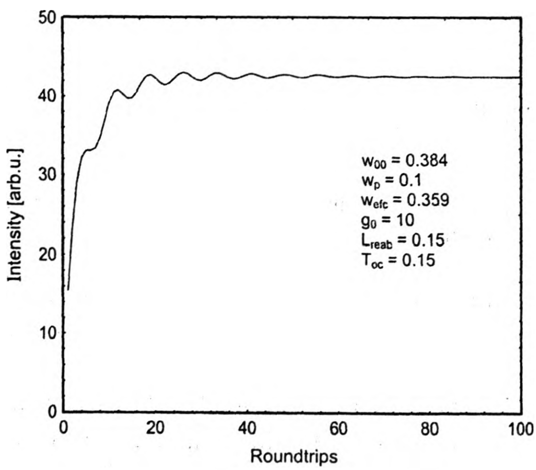

Fig. 5. Intracavity intensity *vs*. roundtrip number.

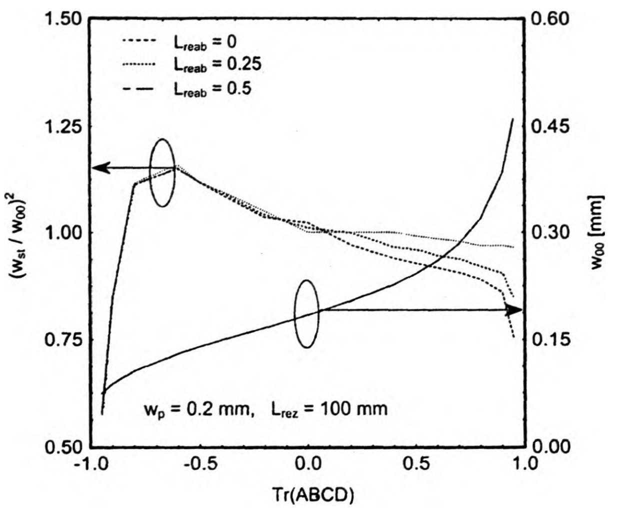

Fig. 6. Dependences of bare cavity waist radius w_{00}, relative stationary beam radius on trace of round trip matrix ABCD; constant pump width w_p = 0.2 mm.

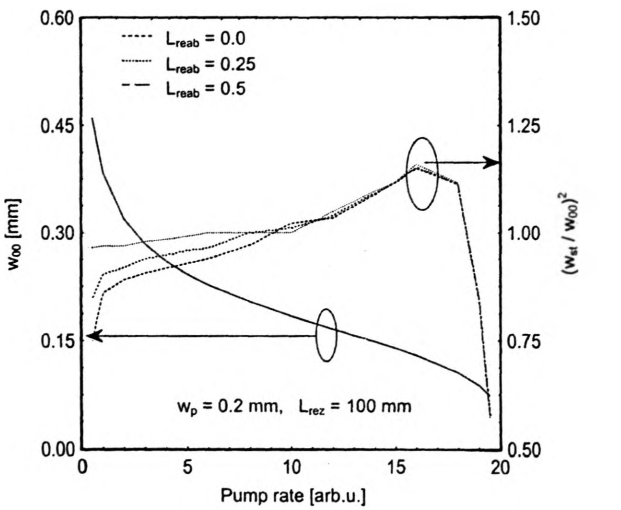

Fig. 7. Dependences of bare cavity waist radius w_{00}, relative stationary beam radius on pumping rate; constant pump width w_p = 0.2 mm (L_{rez} – rezonator length).

pumped cavity is shown in Figs. 4 and 5. Note that such a procedure is similar to Fox Li approach (see, *e.g.* [1]–[5]), however instead of exact calculation of diffraction integrals for each roundtrip we pass the Gaussian beam modified by the gain medium through the cavity applying Kogelnik's ABCD formulae (see, *e.g.* [1]). As a result we have dealt with "effective" Gaussian beam of a GTG cavity, the parameters of which change in each roundtrip converging to stationary value. The solution of GTG cavity depends both on parameters of bare cavity M_{BC} as well as magnitude of gain and its profile. We have chosen for analysis of GTG effects a simple linear cavity with gain medium placed at rear mirror and flat output facet assuming the constant pump radius w_p. Due to thermal guiding the thermal lensing power of rod increases with pump rate, moving the cavity across stability region (see Figs. 6, 7). The magnitude of gain proportional to the pump rate causes via gain guiding deviations from TG guiding model (see dotted, dashed curves in Figs. 6 and 7). The influence of additional reabsorption losses according to our numerical experiments is of the second order, such an effect should be taken into consideration only for $w_{00} > w_p$. For the low pump rate such an effect is much more significant (see Fig. 7), because of the low value of thermal lensing resulting in high value of fundamental mode width and low gain compared to reabsorption losses. Depending on the cavity and pumping parameters the differences between results of TG and saturable GTG models are in the limits of ±50%.

4. Conclusions

Two models of gain and thermal guiding effects were derived. In the first one, a complex ABCD matrix for a crystal under gain and thermal guiding was applied to describe the operation of microchip near threshold. In the second one, a simple iterative procedure was proposed to calculate effective fundamental mode parameters of a cavity under thermal and gain guiding for a given bare cavity ABCD matrix and pumping parameters, including gain saturation, passive cavity losses and reabsorption ones. Application of such a method for resonators of passively Q-switched lasers and cavities with nonlinear crystals for frequency conversion, parametric generation, *etc.* is feasible. In the case of a free running laser, the change of the beam width compared to fundamental mode width is in the range of ±50%. The gain and thermal guiding effects should be taken into consideration in designing the cavities destined for high gain, high power lasers especially with decreased thermal load. The iterative model based on Lambert W function can be applied for low and medium power Q-switched microlasers, lasers with intracavity conversion, *etc.*

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