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Range dependent radiation leakage from thin scattering columns due to multiple small angle scatter

Yury A. Kravtsov[1], Leon A. Apresyan[2], Ara A. Asatryan[3], Janusz Chrzanowski[4], Józef Kirkiewicz[4]

[1]Space Research Centre, Polish Academy of Sciences, Warszawa, Poland.

[2]General Physics Institute, Centre of Scientific Research, Russian Academy of Sciences, Moscow, Russia, e-mail: lesa@nsc.gpi.ru.

[3]Sydney University of Technology, Sydney, Australia, e-mail: ara.asatryan@uts.edu.au.

[4]Maritime University, Wały Chrobrego 1/2, 70–500 Szczecin, Poland, e-mail: kravtsov@wsm.szczecin.pl.

The phenomenon of range dependent radiation leakage from thin random columns caused by small angle multiple scatter is analysed. On the basis of modified Born approximation (Ishimaru, 1978; Apresyan and Kravtsov, 1996), it is shown that extinction coefficient, describing energy leakage due to multiple scatter on large (as compared to wavelength) inhomogeneities, first increases proportionally to distance $z^{1/2}$ and then saturates on the level which might significantly exceed conventional extinction coefficient, connected with large angle light scatter on small inhomogeneities. The effect might be observed in many physical systems, for example, in a tube filled with water emulsion or suspension, containing small and large particles. The phenomenon may be helpful in distinguishing the contribution of small and large inhomogeneities to total extinction.

Keywords: light scattering, extinction, multiple scattering, radiative transfer.

1. Introduction

Lengthy and thin scattering objects, such as thin scattering columns or thin layers, are not rarity neither in natural nor in laboratory studies. These include: thin layers of fog, turbulent plasma cylinders, thin scattering dielectric rods and fibres, elastic random rods, tubes filled with a dense suspension or emulsion, to name but a few. We perform an analysis for random columns, but note that all phenomena, given below for thin scattering columns, are also characteristic for thin scattering layers.

We speak of a thin scattering column if its diameter D (Fig. 1) is small as compared to extinction length l_{ext}, *i.e.*,

$$D \ll l_{ext}. \tag{1}$$

The characteristic property of the thin random column is that scattered radiation leaves it presumably after the first act of side scatter. Corresponding mathematical approach, treating side scatter as small perturbation, is known as modified Born approximation (MBA), see [1].

According to MBA, in the case of large angle scatter on small (compared to wavelength) particles the energy flow Π_z along the z-axis of a scattering column C (Fig. 1) decays in exponential fashion

$$\Pi_z = \Pi_z^0 \exp(-\alpha z) \tag{2}$$

where $\alpha = (l_{ext})^{-1}$ is an extinction coefficient.

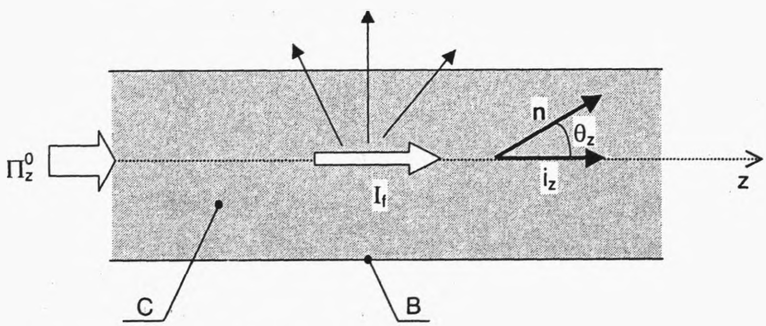

Fig. 1. Light scattering in random column C.

To the best of our knowledge, radiation transfer along the thin random media has not yet been subjected to any further detailed analysis. The main goal of this paper is to describe a new physical phenomenon – range dependent radiation leakage from thin scattering column in the presence of multiple small angle scattering on large inhomogeneities.

Radiation transfer equation (RTE) for the case of narrow scattering column is presented in Sec. 2. A corresponding method for solution of RTE is considered in Sec. 3. We suggest here an improved version of MBA, which takes into account not only conventional large angle scattering on small particles, but also multiple small angle scatter on large inhomogeneities. The phenomenon of range dependent radiation leakage from a thin random column is analysed in Sec. 4. Section 5 discusses various aspects of the problem: the possibility of experimental observation of growing range dependent energy leakage, reduction of leakage due to light partial reflection from boundaries, opportunity to observe a phenomenon in thin layers.

2. Radiation transfer equation in the case of thin scattering column

Radiation transfer equation, describing radiation propagation and scatter along the thin scattering column C, has the form

$$\begin{aligned} &\frac{\mathrm{d}I}{\mathrm{d}s} + \alpha I = \hat{\sigma} I, \qquad \mathbf{r} \in \mathrm{C}, \\ &\frac{\mathrm{d}I}{\mathrm{d}s} = 0, \qquad \mathbf{r} \notin \mathrm{C}. \end{aligned} \tag{3}$$

Specific intensity, or radiance $I(\mathbf{n}, \mathbf{r})$ characterizes intensity of wave field at a point $\mathbf{r}$ per unit solid angle in direction given by a unit vector $\mathbf{n}$, $\mathrm{d}s$ being elementary displacement in this direction [1]–[8]. In other words, $I(\mathbf{n}, \mathbf{r})$ is an angular spectrum of the wave field at the point of observation $\mathbf{r}$. Energy density w and Pointing vector $\mathbf{S}$ are connected with radiance $I(\mathbf{n}, \mathbf{r})$ by relations:

$$w(\mathbf{r}) = \oint I(\mathbf{n}, \mathbf{r}) \mathrm{d}\Omega_n, \qquad \mathbf{S}(\mathbf{r}) = c \int \mathbf{n} I(\mathbf{n}, \mathbf{r}) \mathrm{d}\Omega_n \tag{4}$$

where $\mathrm{d}\Omega_n$ is an elementary solid angle in space $\mathbf{n}$, and c is the speed of light.

The operator of scattering $\hat{\sigma}$ in Eq. (3) is defined by the formula

$$\hat{\sigma} I = \int \sigma(\mathbf{n}' \to \mathbf{n}) I(\mathbf{n}') \mathrm{d}\Omega_{n'}. \tag{5}$$

The kernel $\sigma(\mathbf{n}' \to \mathbf{n})$ of this operator is differential scattering cross-section per unit volume. This cross-section describes conversion of radiance $I(\mathbf{n}, \mathbf{r})$ from direction $\mathbf{n}'$ into direction $\mathbf{n}$. The term $\hat{\sigma} I$ characterizes radiance increment in direction $\mathbf{n}$ due to energy income from all possible directions $\mathbf{n}'$.

The extinction coefficient α in Eq. (3) describes the rate of radiance $I(\mathbf{n}, \mathbf{r})$ reducing due to joint action of absorption and scattering

$$\alpha = \alpha_{\mathrm{sc}} + \alpha_{\mathrm{abs}}. \tag{6}$$

The scattering term α_{sc} represents total (integrated over the unit sphere) scattering cross-section per unit volume

$$\alpha_{\mathrm{sc}} = \oint \sigma(\mathbf{n}) \mathrm{d}\Omega_n. \tag{7}$$

All the values α, σ, α_{sc} and α_{abs} are considered to be zero outside the column C.

Radiance $I(\mathbf{n}, \mathbf{r})$ should meet initial condition $I^0 = I(z = 0)$ in the initial plane $z = 0$ and boundary condition at cylinder boundary B. The last, in fact, is given by Fresnel reflection formula [2]. Radiance entering column C from outside is considered to be zero. The influence of reflection from boundary B is negligible in the following three cases:

– when refractive indexes of the media inside and outside column C are equal to each other,

– when boundary B presents a smooth transitional layer of thickness, exceeding a wavelength,

– when boundary B is perfectly absorbing, for instance if it is painted from the inside by well absorbing black pigment.

In all these cases one can consider boundary B as perfectly absorbing surface. Noticeable reflection from boundary B can be the reason for waveguide effects. In order to simplify further analysis we neglect reflection from B and thereby exclude waveguide effects from consideration. However, in Sec. 5 we shall note that partial reflection from boundaries might reduce range dependent energy leakage.

3. Modified Born approximation in the presence of multiple forward small angle scatter

Modified Born approximation makes use of side scatter smallness from thin scattering media [1], [2]. Unlike traditional Born approximation [1], [9], which is valid only for small (compared to l_{ext}) scattering areas, MBA is not restricted by a requirement $z \ll l_{ext}$ and admits exponentional decay of the primary wave field at distances $z \geq l_{ext}$. That version of MBA, which is outlined below, takes into account the effects of multiple forward scatter on large scale inhomogeneities and thereby makes it possible to describe the range dependent radiation leakage phenomenon.

Whenever inequality (1) holds, it is reasonable to present radiance I as a sum

$$I = I_f + I_s \tag{8}$$

where the first leading term I_f describes multiple forward scatter inside column C, while the second term I_s, responsible for side scattering, is considered as to be a small perturbation.

The term I_f is supposed to obey RTE in small angle approximation:

$$\begin{aligned} \frac{\mathrm{d}I_f}{\mathrm{d}s} + \alpha I_f &= \hat{\sigma}_f I_f, \qquad \mathbf{r} \in C, \\ \frac{\mathrm{d}I_f}{\mathrm{d}s} &= 0, \qquad \mathbf{r} \notin C \end{aligned} \tag{9}$$

where

$$\hat{\sigma}_f I_f = \int \sigma_f(\mathbf{n}' \to \mathbf{n}) I_f(\mathbf{n}') \mathrm{d}\Omega_n, \tag{10}$$

and small angle component σ_f is defined as

$$\sigma_f(\mathbf{n}' \to \mathbf{n}) = G_f(\mathbf{n}' \to \mathbf{n})\sigma(\mathbf{n}' \to \mathbf{n}). \tag{11}$$

"Filtering" function G_f, which rejects small angle component of σ, can be chosen in many ways. The simplest one is to choose a stepwise function

$$G_f(\mathbf{n}' \to \mathbf{n}) \equiv G_f(\theta) = \begin{cases} 1, & \theta \le \theta_b, \\ 0, & \theta > \theta_b \end{cases} \tag{12}$$

where θ is a scattering angle, that is, angle between unit vectors **n** and **n'** ($\mathbf{nn}' = \cos\theta$), and θ_b is boundary angle. A reasonable choice of the boundary angle θ_b will be discussed at the end of this section.

It is natural to treat a difference between components σ and σ_f as side scattering component σ_s:

$$\sigma_s(\theta) = \sigma(\theta) - \sigma_f(\theta) = [1 - G_f(\theta)]\sigma(\theta) \equiv G_s(\theta)\sigma(\theta). \tag{13}$$

Stepwise approximation (12) for function G_f leads to the following stepwise approximation for G_s:

$$G_s(\theta) = 1 - G_f(\theta) = \begin{cases} 0, & \theta \le \theta_b, \\ 1, & \theta > \theta_b. \end{cases} \tag{14}$$

Unlike component σ_f, side scattering cross-section component σ_s is zero inside small angle cone $\theta \le \theta_b$ and coincides with σ outside this cone. Note that along with stepwise approximation (12) one can also use continuous models for the filtering functions G_f, say Gaussian model $G_f(\theta) = \exp(-\theta^2/\theta_b^2)$ or somewhat more complicated model $G_f(\theta) = \exp[-\tan^2(\theta/2)/\tan^2(\theta_b/2)]$ which guarantees the zero values for function G_f at $\theta = 180°$.

The procedure for separating the small angle components considered above is quite similar to hybrid approach in the theory of wave propagation and scattering in random media, containing both large scale and small scale inhomogeneities [10]. Insignificant difference is that hybrid approach deals with spatial spectrum of the medium respective index fluctuations, while RTE operates with angular behaviour of the scattering cross -section. Like hybrid approach the resulting solution $I = I_f + I_s$ should only slightly depend on boundary value θ_b.

Subtracting small angle approximation (9) from the total radiation transfer equation (3), one obtains the equation for side scattered component I_s

$$\begin{cases} \dfrac{\mathrm{d}I_s}{\mathrm{d}s} + \alpha I_s = \hat{\sigma} I_s + \hat{\sigma}_s I_f, & \mathbf{r} \in C, \\ \dfrac{\mathrm{d}I_s}{\mathrm{d}s} = 0, & \mathbf{r} \notin C. \end{cases} \tag{15}$$

A solution to this equation can be presented in the form of series in multiplicity of side scatter

$$I_s = \sum_{m=1}^{\infty} I_s^{(m)}. \tag{16}$$

A singly side scattered term is assumed to satisfy the equation

$$\begin{cases} \dfrac{dI_s^{(1)}}{ds} + \alpha I_s^{(1)} = \hat{\sigma}_s I_f, & \mathbf{r} \in C, \\ \dfrac{dI_s^{(1)}}{ds} = 0, & \mathbf{r} \notin C, \end{cases} \tag{17}$$

whereas multiply scattered terms $I_s^{(1)}$, $m \geq 2$, can be found from a recurrent system of equations

$$\begin{cases} \dfrac{dI_s^{(m)}}{ds} + \alpha I_s^{(m)} = \hat{\sigma}\, I_s^{(m-1)}, & \mathbf{r} \in C, \\ \dfrac{dI_s^{(m)}}{ds} = 0, & \mathbf{r} \notin C. \end{cases} \tag{18}$$

Unlike Eq. (17), which contains the side cross-section σ_s in the right-hand part and describes only side scattering, Eq. (18) includes the total cross-section $\sigma = \sigma_s + \sigma_f$ and therefore the terms $I_s^{(m)}$ with $m \geq 2$ partially contribute to forward scattering along with component I_f.

In what follows we restrict our analysis by only a single side scatter which corresponds to the first approximation of MBA method. In this approximation, the solution (8) takes the form

$$I \approx I_f + I_s^{(1)}. \tag{19}$$

A formal solution of Eq. (17) outside the scattering column is

$$I_s^{(1)} = \exp(-\alpha l_n) \int_0^{l_n} \hat{\sigma}_s\, I_f \exp(\alpha s')\, ds'. \tag{20}$$

Here, l_n is a ray path length inside the scattering column (Fig. 2), depending on unit vector $\mathbf{n}$ direction.

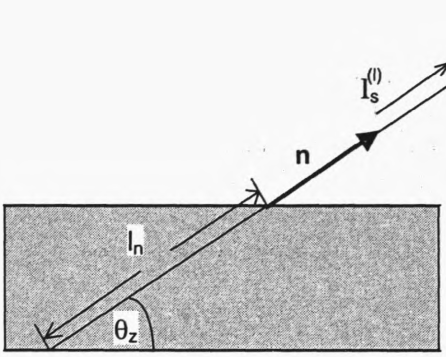

Fig. 2. Path length l_n of the singly scattered radiationnce $I_s^{(l)}$ inside scattering column C.

Taking into account that forward scattered component I_f is concentrated inside narrow cone directed along the z-axis, the term $\hat{\sigma}_s I_f$ can be estimated as $\sigma_s(\theta)\int I_f(\mathbf{n})d\Omega_n$. Then Eq. (20) gives

$$I_s^{(l)}(\theta) \approx \frac{1-\exp(-\alpha l_n)}{\alpha}\sigma_s(\theta)\int I_f(\mathbf{n})d\Omega_n. \tag{21}$$

In the frame of single side scatter approximation the value αl_n should be small enough, $\alpha l_n \ll 1$, otherwise one should account for the highest term in Eq. (16). As a result, Eq. (21) takes a simplified form

$$I_s^{(l)}(\theta) \approx l_n \sigma_s(\theta)\int I_f(\mathbf{n})d\Omega_n. \tag{22}$$

The product σl_n serves here as a parameter, characterizing the smallness of singly side scattered radiance $I_s^{(l)}$ as compared to the leading term I_f. Assuming this parameter to be small enough, one arrives at the condition

$$l_n \sigma_s(\theta) = \mu < 1 \tag{23}$$

which, in fact, is an equation for the boundary angle θ_b. Parameter μ, which has the sense of scattering probability on the path l_n, is reasonable to be chosen between 1/2 and 1/3. Taking into account that $l_n \approx D/\sin\theta$, one can rewrite Eq. (23) in the form

$$D\sigma_b = \mu \sin\theta_b, \qquad \sigma_b \equiv \sigma_s(\theta_b). \tag{24}$$

By virtue of requirement (1) the product $D\sigma_b$ as well as the boundary angle θ_b should be small enough

$$\theta_b \approx \frac{D\sigma_b}{\mu} \ll 1. \tag{25}$$

4. Range dependent radiation leakage

Let us consider energetic balance of radiation in scattering column using both qualitative and quantative arguments. Energy flow in scattering column decreases firstly due to the large angle scatter, and secondly due to radiation exit from the scattering column by virtue of multiple small angle scatter. The latter is responsible for new physical phenomenon which we shall refer to as growing radiation leakage. Both effects, the energy leakage due to large angle scattering and growing leakage due to partial migration of the small angle components I_f out of column C will be described below in a similar fashion.

The energy flow along the z-axis equals

$$\Pi_z = \frac{\pi D^2}{4} S_{fz}, \quad S_{fz} = c \oint n_z I(\mathbf{n}, \mathbf{r}) \mathrm{d}\Omega_n \tag{26}$$

where $\pi D^2/4$ is a column cross-section, S_{fz} is z-component of Poynting vector $\mathbf{S}_f$, and c is a light velocity. Energy leakage through the side surface of the length Δz is given by

$$\Delta\Pi_\perp = \int_B \mathbf{S}\mathrm{d}\mathbf{a} = \int_B \mathrm{d}\mathbf{a} \oint \mathbf{n}\, I(\mathbf{n}, \mathbf{r}) \mathrm{d}\Omega_n = \pi D \Delta z S_\perp \tag{27}$$

where d**a** is an oriented element of a side surface, $\pi D \Delta z$ is a side surface of a cylinder section of the length Δz, $S_\perp = \oint (\mathbf{n} \cdot \mathbf{n}_a) I(\mathbf{n}, \mathbf{r}) \mathrm{d}\Omega_n$ is a transverse component of the Poynting vector, responsible for side leakage of wave energy and $\mathbf{n}_a$ is a unit vector along element d**a**.

Energy flow $\Delta\Pi_\perp$ leaving column C through the side surface reduces the longitudinal flow Π_z at $\Delta\Pi_z = -\Delta\Pi_\perp$. In these conditions relative decrement of energy flow per unit length serves as an effective extinction coefficient

$$\alpha_{\mathrm{eff}} = -\frac{1}{\Pi_z}\frac{\Delta\Pi_z}{\Delta z} = \frac{1}{\Pi_z}\frac{\Delta\Pi_\perp}{\Delta z} = \frac{\pi D S_\perp}{(\pi D^2/4) S_{fz}} = \frac{4 S_\perp}{D S_{fz}}. \tag{28}$$

Energy losses $\Delta\Pi_{\perp s}$ due to side scatter can be calculated from Eqs. (28) and (29) using component $I_s^{(l)}$, Eq. (22), instead of I. After comparatively simple though lengthy calculations one can find "large angle" extinction coefficient $\alpha_{\perp s}$, which happens to be equal to integral side scattering cross-section

$$\alpha_{\perp s} \approx \alpha_s \equiv \oint \sigma_s(\mathbf{n}) \mathrm{d}\Omega_n \tag{29}$$

where $\sigma_s(\mathbf{n})$ is the side scattering differential cross-section (13).

For small angle scattering, when I should be substituted by I_f in Eq. (27), the scalar product $(\mathbf{n}\mathbf{n}_a)$ is of order θ_A, where θ_A is an angular width of forward scattered beam I_f.

In these conditions the corresponding small angle extinction coefficient $\alpha_{\perp f}$ can be estimated as

$$\alpha_{\perp s} \approx \frac{\theta_A}{D}. \tag{30}$$

According to diffusion approximation for RTE, which demonstrates its efficiency, namely for small angle scatter (see [1] and [2]), the angular width of radiation in random half-space is given by

$$\theta_A(z) = (4D_{\mathrm{ang}}\, z)^{1/2} = (4\langle v^2\rangle_{\mathrm{f}}\, \alpha_{\mathrm{f}}\, z)^{1/2} \tag{31}$$

where

$$D_{\mathrm{ang}} = \alpha_{\mathrm{f}}\langle v^2\rangle_{\mathrm{f}} \tag{32}$$

is an angular diffusion coefficient,

$$\langle v^2\rangle_{\mathrm{f}} = \frac{\int v^2 \sigma_{\mathrm{f}}(v)\mathrm{d}^2 v}{\int \sigma_{\mathrm{f}}(v)\mathrm{d}^2 v} \tag{33}$$

is an angular width of small angle cross-section σ_{f}, and $\mathbf{v} \equiv \mathbf{n}_{\perp}$ is transverse (relative to the z-axis) component of a unit vector $\mathbf{n}$. The value

$$\alpha_{\mathrm{f}} = \oint \sigma_{\mathrm{f}}(\mathbf{n})\mathrm{d}\Omega_n \approx \int \sigma_{\mathrm{f}}(v)\mathrm{d}^2 v \tag{34}$$

is an integral cross-section, corresponding to small angle scatter.

One can estimate the quantities $\langle v^2\rangle_{\mathrm{f}}$ and α_{f} assuming that cross-section σ_{f} preserves a constant value within a cone, restricted by a boundary angle θ_{b}. Then

$$\langle v^2\rangle_{\mathrm{f}} \approx \theta_{\mathrm{b}}^2, \qquad \alpha_{\mathrm{f}} = \sigma_{\mathrm{f0}}\pi\theta_{\mathrm{b}}^2 \tag{35}$$

where $\sigma_{\mathrm{f0}} = \sigma_{\mathrm{f}}(\mathbf{i}_z)$ is scattering cross-section exactly along the z-axis (here $\mathbf{i}_z$ is a unit vector along the z-axis). In this approximation

$$\theta_A(z) \approx \theta_{\mathrm{b}}(\alpha_{\mathrm{f}} z)^{1/2}. \tag{36}$$

According to Eq. (36) the angular width θ_A grows proportionally to $z^{1/2}$. It becomes comparable with boundary angle θ_{b} at a distance $z \approx \alpha_{\mathrm{f}}^{-1} \equiv l_{\mathrm{f}}$. One can assume that at this distance the angular width θ_A will saturate, because at $\theta_A > \theta_{\mathrm{b}}$ significant part of radiation will leave the column not experiencing additional acts of scattering. Thus,

$$\theta_A(z) \approx \begin{cases} \theta_{\mathrm{b}}(\alpha_{\mathrm{f}} z)^{1/2}, & z < l_{\mathrm{f}}, \\ \theta_{\mathrm{b}}, & z > l_{\mathrm{f}}. \end{cases} \tag{37}$$

Using relations (25) and (30), one can rewrite range dependent extinction coefficient in the form

$$\alpha_{\perp f}(z) \approx \begin{cases} \alpha_{\infty}(\alpha_{f} z)^{1/2}, & z < l_{f}, \\ \alpha_{\infty}, & z > l_{f} \end{cases} \tag{38}$$

where

$$\alpha_{\infty} = \frac{\theta_{b}}{D} = \frac{\sigma_{b}}{\mu} \tag{39}$$

is an extinction coefficient in saturation area $z > l_f = \alpha^{-1}$. Thus, the effective extinction coefficient α_{eff} in the case of thin scattering column might be is estimated as

$$\alpha_{eff} = \alpha_{abs} + \alpha_{s} + \alpha_{\perp f}(z). \tag{40}$$

For completeness we used here the term α_{abs}, describing an absorption in a column.

Dependence of the effective extinction coefficient (Eq. (40)) on distance z is presented in Fig. 3. The horizontal dashed line corresponds to constant component $\alpha_s + \alpha_{abs}$, whereas the range dependent part $\alpha_{\perp f}$ describes spreading and saturation of the radiance I_f angular spectrum width θ_A.

According to Eq. (28), energy flow obeys the equation

$$\frac{d\Pi_z}{dz} + \alpha_{eff}\Pi_z = 0,$$

and reduces to

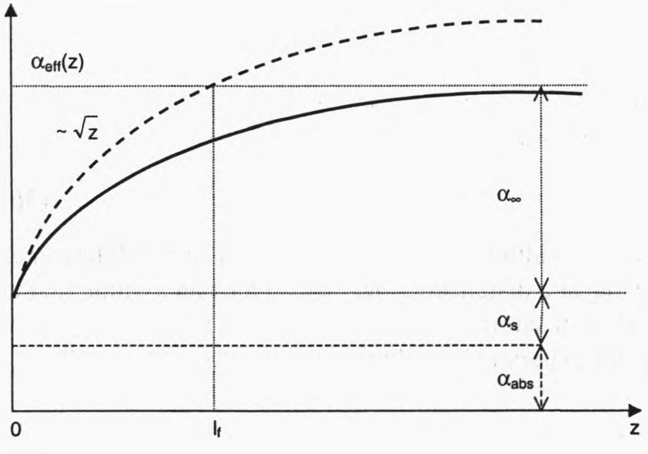

Fig. 3. Dependence of extinction coefficient α_{eff} on distance z in the presence of small and large inhomogeneities: constant term $\alpha_s + \alpha_{abs}$ is due to single side scatter $I_s^{(l)}$ and absorption, whereas range dependent term $\alpha_{\perp f}$ reflects the influence of multiple small angle scatter.

$$\Pi_z(z) = \Pi_z^0 \exp[-E(z)] \tag{41}$$

where the value

$$E(z) = \int \alpha_{\mathrm{eff}}(z)\mathrm{d}z = (\alpha_{\mathrm{abs}} + \alpha_{\mathrm{s}})z + \int_0^z \alpha_{\perp\mathrm{f}}(z)\mathrm{d}z \tag{42}$$

with

$$\int_0^z \alpha_{\perp\mathrm{f}}(z)\mathrm{d}z \approx \begin{cases} \frac{2}{3}\alpha_\infty \alpha_{\mathrm{f}}^{1/2} z^{3/2}, & \text{for} \quad z < l_{\mathrm{f}}, \\ \frac{2\alpha_\infty}{3\alpha_{\mathrm{f}}} + \alpha_\infty(z - l_{\mathrm{f}}), & \text{for} \quad z > l_{\mathrm{f}}, \end{cases}$$

characterizes decrement of the total energy flow along a scattering column.

For media containing only small scale inhomogeneities, the role of small angle forward scatter becomes negligible. In this case, coefficients $\alpha_{\perp\mathrm{f}} \ll \alpha_{\mathrm{s}}$ and $\alpha_{\mathrm{eff}} \approx \alpha_{\mathrm{s}} + \alpha_{\mathrm{abs}}$. However, in the opposite case, when random medium contains presumably large inhomogeneities, side scatter becomes negligible and then the term α_∞ prevails in a saturation area: $\alpha_\infty \gg \alpha_{\mathrm{s}}$. In this case, one can speak of enhanced radiation leakage.

Relations (41) and (42) one can be rewriten in the following form:

$$\Pi_z = \Pi_z^0 \exp[(\alpha_{\mathrm{abs}} + \alpha_{\mathrm{s}})z]Q(z) \tag{43}$$

where

$$Q(z) = \exp\left(-\int_0^z \alpha_{\perp\mathrm{f}}(z)\,\mathrm{d}z\right) \approx \begin{cases} \exp\left(-\frac{2}{3}\alpha_\infty \alpha_{\mathrm{f}}^{1/2} z^{3/2}\right) & \text{for} \quad z < l_{\mathrm{f}}, \\ \exp\left(\frac{2\alpha_\infty}{3\alpha_{\mathrm{f}}} - \alpha_\infty(z - l_{\mathrm{f}})\right) & \text{for} \quad z > l_{\mathrm{f}}. \end{cases} \tag{44}$$

The factor $Q(z)$ characterizes the share of energy flow, which is confined by scattering column and can be named "confinement factor". The difference $1 - Q(z)$ corresponds to energy share which released the column forever.

It is worth noting that extinction coefficient α_∞ which characterizes energy leakage from a thin scattering column in a saturation regime is much larger than large scale component α_f of the total extinction coefficient $\alpha = \alpha_{\mathrm{s}} + \alpha_{\mathrm{f}}$, that is

$$\frac{\alpha_\infty}{\alpha_{\mathrm{f}}} \gg 1. \tag{45}$$

Large difference in values between coefficients α_f and α_∞ may be explained by a different role each of these values plays in extinction phenomenon. The extinction coefficient α_f, entering the radiation transfer Eq. (9), describes angular energy redistribution due to small angle scattering events, while the coefficient $\alpha_{\perp f}$ and its saturation limit α_∞ are connected with an energy outcome from a thin scattering column. In a thick scattering medium angular redistribution of radiation is accompanied by a transverse energy migration, but energy flow along the z-axis on average is almost a constant value, since energy outcome from each thin cylinder is compensated by the energy income from neighbouring cylinders. This is not the case for a thin cylinder: the energy outcome cannot be compensated now by energy flow from the outside, and therefore the energy content in a scattering cylinder decreases sufficiently quickly (proportionally to factor $Q(z)$ in Eq. (43)). Thus, the values α_f and α_∞ characterize different aspects of the extinction phenomenon in random media.

5. Discussion

One of the examples of phenomena allowing experimental checking is a light scattering in a tube, blackened from the inside and filled with water emulsion or suspension, containing small and large particles. Blackened walls of a tube play a role of transparent boundary, allowing radiation to leave the column, but preventing radiation income from the outside. Having measured the energy flow $\Pi_z(z)$ along the column, one can determine the value $E(z) = \ln[\Pi_z(z)/\Pi_z^\theta]$, which describes general decreasing of energy flow. It is natural to identify the term E_{lin}, which linearly grows with distance z, with conventional extinction, connected with absorption and side scat-tering on small scale inhomogeneities. At the same time nonlinearly growing component $E_{\text{nonlin}} = E - E_{\text{lin}}$ can be associated with the influence of large inhomogeneities. As a result of the analysis of function $E(z)$ one can distinguish the role of small and large scale inhomogeneities of a random medium.

In the case of partially reflecting boundary Fresnel transition coefficient $T(\theta_A)$ should be introduced into energy flow, leaving the column (Eq. (27)) and thereby into small angle extinction coefficient $\alpha_{\perp f}$ (Eq. (38)). At very small angles θ_A transmission factor $T(\theta_A)$ tends to zero, which can significantly reduce the leakage of radiation from the column.

In the present paper, we have restricted ourselves to analysis in thin scattering cylinder. Meanwhile, the effect of range dependent radiation leakage is to be inherent to all thin scattering systems, containing large inhomogeneities. In particular, growth and saturation of the extinction coefficient should be observed in thin random layers.

6. Conclusions

The effect of range dependent radiation leakage, which has been described above, is characteristic of thin scattering cylinders and layers containing large scale inhomogeneities. In such systems, significant part of scattered energy leaves the random medium nonuniformly, demonstrating firstly growth, and then saturation of energy leakage.

The phenomenon of range dependent leakage might be of interest for different applications, connected with studies of multiple scattering. In particular, the phenomenon might be helpful in separation of small angle and large angle scatter by observing extinction in the blackened tubes, filled with dense suspension or emulsion.

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