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Effect of distribution of electromagnetic field inside optical fibres on their luminous flux

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A method for calculation of luminous flux emitted by the front surface of a fibre was proposed. A step-index fibre was considered. Calculations were based upon known electromagnetic field distribution inside the core represented by analytical equations. Particular modes were shown as the superposition of linear polarized plane waves. Light refraction on the fibre front surface was also taken into consideration. It was found that spatial distribution of the luminous flux of a particular fibre depended on its normalized frequency.

Keywords: optical fibres, radiative transfer, photometry, luminous flux.

1. Introduction

Knowledge of spatial distribution of radiant power emitted by the front surface of optical fibres is not necessary in telecommunication applications of optical fibers. However, the increasing use of optical fibers in light engineering makes it necessary to know their luminous flux. This work proposes a method of reckoning fibre's luminous flux, based on distribution of electromagnetic field inside the fibre core. We assumed the following:

1. A step-index optical fibre is considered. The core radius is denoted by a, the refractive index of core is n_1, and the refractive index cladding n_2.

2. The fibre is long enough so that transient effects inside the core can be omitted, and at the same time short enough to ignore absorption of luminous flux. The front surface of the core is plane.

3. Monochromatic radiation at angular frequency ω is guided inside the core.

4. The point light source with constant luminous intensity distribution is placed at the core axis.

5. Radiation propagating in the fibre cladding was ignored.

2. Electromagnetic field distribution inside the fibre core

Distributions of electric and magnetic fields inside the fibre core can be calculated using Maxwell's equations in cylindrical coordinates (r, φ, z). The r, φ and z coordinate of electric and magnetic fields must satisfy the following formulas [1]–[3]:

$$E_z = A_E J_m(ur)\exp[j(m\varphi - \beta z)], \tag{1}$$

$$H_z = A_H J_m(ur)\exp[j(m\varphi - \beta z)], \tag{2}$$

$$E_r = -\frac{1}{u^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\omega\mu_0}{r}\frac{\partial H_z}{\partial \varphi}\right), \tag{3}$$

$$E_\varphi = \frac{1}{u^2}\left(-\frac{\beta}{r}\frac{\partial E_z}{\partial r} + \omega\mu_0\frac{\partial H_z}{\partial \varphi}\right), \tag{4}$$

$$H_r = -\frac{1}{u^2}\left(-\frac{\omega\varepsilon_1}{r}\frac{\partial E_z}{\partial \varphi} + \beta\frac{\partial H_z}{\partial r}\right), \tag{5}$$

$$H_\varphi = -\frac{1}{u^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial r} + \omega\varepsilon_1\frac{\partial H_z}{\partial \varphi}\right) \tag{6}$$

where: m – integer number, A_E, A_H – integration constants, β – propagation constant of the mode in the z-direction, u – solution of Hondros–Debye's equation [1], [2], $J_m(ur)$ – m-th order Bessel's function of the first kind, μ_0 – permeability of free space, ε_1 – permittivity of the fiber core.

Known distributions of electric and magnetic fields do not enable direct calculation of luminous flux, emitted by the front surface of the fibre.

3. Optical fibre modes as superposition of linearly polarizedplane waves

Propagation constants β of particular modes are different. Their values are less than the propagation constant k_1 of electromagnetic waves moving inside a dielectric with a refractive index n_1. This means that the actual waves move at an angle θ in relation to the core axis, and

$$\cos\theta = \frac{\beta}{k_1}. \tag{7}$$

Electromagnetic field, corresponding to the mode guided inside the fibre core, will be replaced by superposition of linearly polarized plane waves. The following conditions must be satisfied at every point of the core:

– resultant values of electric and magnetic fields of component waves must be equal to values given by formulas (1)–(6);

– total power of component waves must be equal to the power of the mode.

The coordinates E_r and H_φ will be replaced by superposition of four plane linearly polarized waves, moving at an angle of θ with respect to the fibre axis. Two of them move along the axial section plane of the fibre (Fig. 1). Their electric field vectors are

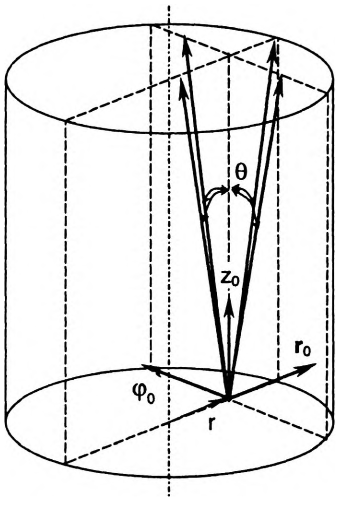

Fig. 1. Linearly polarized flat waves creating electromagnetic field inside the fiber's core.

denoted by $\mathbf{E}_{or}$, and magnetic field vectors by $\mathbf{H}_{o\varphi}$. Vectors $\mathbf{H}_{o\varphi}$ are at angle δ with respect to versor φ_0. Two of the remaining waves move along the plane, perpendicular to the axial section plane of the fiber (Fig. 1). Their electric and magnetic field vectors are $\mathbf{E}_{pr}$ and $\mathbf{H}_{p\varphi}$, respectively. The angle between vector $\mathbf{E}_{pr}$ and versor $\mathbf{r}_0$ equals δ.

The values of E_{or} and E_{pr} can be obtained from equations:

$$E_r = 2E_{or}\cos\theta\cos\delta + 2E_{pr}\cos\delta, \tag{8}$$

$$H_\varphi = 2H_{o\varphi}\cos\delta + 2H_{p\varphi}\cos\theta\cos\delta. \tag{9}$$

Since:

$$H_{o\varphi} = \frac{E_{or}}{Z_1}, \tag{10}$$

$$H_{p\varphi} = \frac{E_{pr}}{Z_1}, \tag{11}$$

$$Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}, \tag{12}$$

the solutions are:

$$E_{or} = \frac{k_1}{2u^2\cos\delta}(k_1 Z_1 H_\varphi - \beta E_r), \tag{13}$$

$$E_{or} = \frac{k_1}{2u^2\cos\delta}(k_1 E_r - \beta Z_1 H_\varphi). \tag{14}$$

Similarly, the components E_φ and H_r will also be replaced by superposition of four plane linearly polarized waves, moving at an angle of θ to the core axis . Two of them move along the axial section plate. Their electric and magnetic vectors are $\mathbf{E}_{o\varphi}$ and $\mathbf{H}_{or}$. The angle between vector $\mathbf{E}_{o\varphi}$ and versor φ_0 equals δ. The remaining waves move perpendicularly to the axial section plane, and their electric and magnetic vectors are $\mathbf{E}_{p\varphi}$ and $\mathbf{H}_{pr}$. The angle between vector $\mathbf{H}_{pr}$ and versor $\mathbf{r}_0$ equals δ. The values $E_{o\varphi}$ and $E_{p\varphi}$ can be calculated according to the following equations:

$$E_\varphi = 2E_{p\varphi}\cos\theta\cos\delta + 2E_{o\varphi}\cos\delta, \tag{15}$$

$$H_r = 2H_{pr}\cos\delta + 2H_{or}\cos\theta\cos\delta, \tag{16}$$

$$H_{pr} = \frac{E_{o\varphi}}{Z_1}, \tag{17}$$

$$H_{or} = \frac{E_{p\varphi}}{Z_1}. \tag{18}$$

The solutions are as follows:

$$E_{p\varphi} = \frac{k_1}{2u^2\cos\delta}(k_1 Z_1 H_r - \beta E_\varphi), \tag{19}$$

$$E_{o\varphi} = \frac{k_1}{2u^2\cos\delta}(k_1 E_\varphi - \beta Z_1 H_r). \tag{20}$$

The coordinate E_z is replaced by superposition of four similar waves, moving as above. Their electric and magnetic field vectors are $\mathbf{E}_{sz}$ and $\mathbf{H}_{tz}$

$$E_{sz} = \frac{k_1}{4u}E_z. \tag{21}$$

The coordinate H_z is described in a similar way, but the electric and magnetic field vectors are denoted $\mathbf{E}_{tz}$ and $\mathbf{H}_{sz}$

$$E_{tz} = \frac{k_1}{4u}Z_1 H_z. \tag{22}$$

The density of radiant power S, taken by all the plane waves, at any point of the core equals

$$S = \frac{E_{or}^2 + E_{pr}^2 + E_{o\varphi}^2 + E_{p\varphi}^2 + 2E_{sz}^2 + 2E_{tz}^2}{Z_1}. \tag{23}$$

The density of radiant power, guided by all the waves along the z-axis equals

$$S_z = S\cos\theta. \tag{24}$$

At the other side, density of radiant power S_z is the z coordinate of Poynting vector

$$S_z = \mathrm{Re}(E_r H_\varphi^* - E_\varphi H_r^*). \tag{25}$$

Comparing Eqs. (24) and (25) makes it possible to obtain the value of the angle δ.

$$\cos\delta = \sqrt{\frac{(E_{or0}^2 + E_{pr0}^2 + E_{o\varphi 0}^2 + E_{p\varphi 0}^2)\cos\theta}{Z_1 S_z - 2(E_{sz}^2 + E_{tz}^2)\cos\theta}} \tag{26}$$

where E_{or0}, E_{pr0}, $E_{o\varphi 0}$ and $E_{p\varphi 0}$ denote the values of E_{or}, E_{pr}, $E_{o\varphi}$ and $E_{p\varphi}$ for $\delta = 0$.

4. Luminous flux emitted by the front surface of the fibre

Plane linearly polarized waves, arriving at the front surface of the fibre, are refracted according to Snell's law. They move at an angle γ with respect to the end surface of the fibre core, whereas

$$n_1 \sin\theta = \sin\gamma. \tag{27}$$

In order to calculate radiant power density, emitted by the front surface of the fibre, every wave, the electric field vectors of which are $\mathbf{E}_{or}$, $\mathbf{E}_{pr}$, $\mathbf{E}_{o\varphi}$ and $\mathbf{E}_{p\varphi}$, will be divided into two waves. The electric field vector of one of them (marked with index t) is tangent to the front surface. The second wave, marked with index s, has its magnetic field vector tangent to the front surface. These vector components equal:

$$E_{ors} = E_{or}\cos\delta, \tag{28}$$

$$E_{ort} = E_{or}\sin\delta, \tag{29}$$

$$E_{prs} = E_{pr}\sin\delta, \tag{30}$$

$$E_{prt} = E_{pt}\cos\delta, \tag{31}$$

$$E_{o\varphi s} = E_{o\varphi}\sin\delta, \tag{32}$$

$$E_{o\varphi t} = E_{o\varphi}\cos\delta, \tag{33}$$

$$E_{p\varphi s} = E_{p\varphi}\cos\delta, \tag{34}$$

$$E_{p\varphi t} = E_{p\varphi}\sin\delta. \tag{35}$$

Waves marked with the index t, while crossing the border surface, have equal radiant power density q_{et}, according to Fresnel's formulas,

$$q_{et} = \frac{C_t}{2Z_0} E_t^2. \tag{36}$$

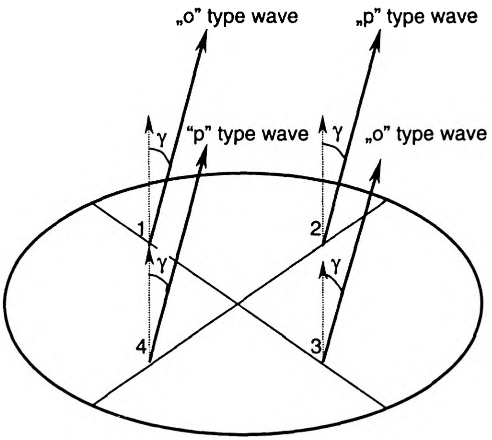

Fig. 2. Points of the fiber's front surface emitting the radiant power into direction g.

In the same case, radiant power density of waves marked with s is given by

$$q_{es} = \frac{C_s}{2Z_0} E_s^2. \tag{37}$$

Coefficients C_s and C_t can be obtained from the following formulas:

$$C_t = \frac{4\beta^2 n_1^2}{n_1^2 \sqrt{k_0^2 - u^2 + \beta^2}}, \tag{38}$$

$$C_s = \frac{4\beta^2}{\sqrt{k_0^2 - u^2 + \beta^2}} \tag{39}$$

where k_0 is the propagation constant in the air, and Z_0 is given by

T a b l e Dependence of fibre's luminous flux on emission angle γ.

Mode	β [μm^{-1}]	θ [°]	γ [°]	Φ_γ
HE_{11}	16.7243	3.47	5.14	0.9647
HE_{12}	16.6000	7.80	11.59	0.9732
TE_{01}	16.6781	5.49	8.15	0.9679
TM_{01}	16.6777	5.51	8.17	0.9679
HE_{21}	16.6778	5.51	8.17	0.9679
HE_{31}	16.6181	7.33	10.89	0.9721
EH_{11}	16.6185	7.32	10.87	0.9719
HE_{41}	16.5483	9.01	13.40	0.9772
EH_{21}	16.5492	8.99	13.38	0.9772

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{40}$$

where ε_0 is permittivity of free space.

Radiant power density coefficient $q_{e\gamma}$, emitted in any direction ψ, is the sum of radiation emitted by points 1, 2, 3 and 4 (Fig. 2). In order to obtain coefficient $q_{e\gamma}$, it is necessary to add waves emitted by points 1 and 3 into the axis section plane of the core, and the energy emitted by points 2 and 4 into the perpendicular plane

$$\begin{aligned} q_{e\gamma} = {} & \frac{1}{Z_0}[C_s(E_{ors}^2 + E_{prs}^2 + E_{o\varphi s}^2 + E_{p\varphi s}^2 + 2E_{sz}^2) \\ & + C_t(E_{ort}^2 + E_{prt}^2 + E_{o\varphi t}^2 + E_{p\varphi t}^2 + 2E_{tz}^2)]. \end{aligned} \tag{41}$$

The whole radiant power, emitted by the front surface of the core is therefore expressed by

$$P_{e\gamma} = 2\pi\int_0^a q_{e\gamma}\, r\mathrm{d}r, \tag{42}$$

and luminous flux

$$\Phi_\gamma = K_m V(\lambda) P_{e\gamma} \tag{43}$$

where $K_m = 680$ lm/W and $V(\lambda)$ – relative sensitivity of human eye at wavelength λ.

The Table shows the results of luminous flux calculation for the fibre with the following parameters: $n_1 = 1.48$, $n_2 = 1.46$, $a = 2$ μm, $\lambda = 555$ nm.

5. Conclusions

The model of linearly polarized plane waves enables calculation of luminous flux emitted by the front surface of a fibre. The distribution of a light beam leaving the fibre has a rotational symmetry. Luminous flux is emitted at specified angles. Their quantity and values depend on the fibre parameters and the wavelength of light. The monochromatic light is emitted in the form of parallel beams. Values of luminous flux emitted in a defined direction depend on power transmitted into the fibre, therefore, on the position and spatial distribution of light source. Angles of light distribution are slightly different for the modes creating linearly polarized modes, so that the weakly guided modes approximation can be used in calculations.

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