Optica Applicata, Vol. XXXII, No. 4, 2002

Scattering-induced spectral changes as a singular optical effect

VYACHESLAV K. POLYANSKII, OLEG V. ANGELSKY, PETER V. POLYANSKII

Department of Correlation Optics, University of Chernivtsi, Kotsyubinsky Str. 2, 58012 Chernivtsi, Ukraine.

Scattering-induced spectral changes observed in the forward-scattered component of polychromatic radiation transmitted trough a colourless transparent plate with a surface roughness comparable with the wave length of some spectral component of the probing beam are represented as peculiar effect of singular optics. Spectral modifier governing this process and providing blue -shift or red-shift of spectrum is specified. Scattering-induced spectral changes are both shown numerically and demonstrated.

1. Introduction

Peculiar behaviour of light fields in the neighbourhood of points or lines where a field amplitude has zero value and the phase of the wave is singular is the subject of so-called singular optics [1], [2]. Since the publication of [3], investigations in singular optics concern mainly of a fine field's structure in the vicinity of zeroes of an amplitude of monochromatic and homogeneously polarized ("scalar") optical fields. There are still much fewer papers that deal with "vector" singularities in spatially inhomogeneous polarization monochromatic fields [1], [4], [5], and only lately singular optics of polychromatic radiation has become the subject of interest. It turns out that the Wolf's spectral effect (the effect of correlation-induced or diffraction -induced, or scattering-induced spectral changes [6]–[8]) may be adequately interpreted within the framework of the singular optical concept.

Here we discuss the peculiar case of scattering-induced spectral changes in polychromatic radiation, reported for the first time without attracting the singular optical concept in [9] (further theoretical and experimental results may be found in [10]–[12]). This effect differs from those considered in [7] in that it is observed in the specular component of scattered radiation. We specify the peculiar spectral modifier (using the terminology of [8]) and define the field's complex parameter changing its phase to π at amplitude zero-crossing, which predetermines spectral changes in the forward-scattered component of radiation. Our consideration, to a large extend, is based on [8].

2. Construction of a spectral modifier

First, consider a monochromatic plane wave propagating in z-direction of rectangular Cartesian coordinates,

$$U^{(0)}(\omega) = A_0 e^{i(\omega t - kz)} \tag{1}$$

where ω is frequency, $k = 2\pi/\lambda = \omega/c$ is the associated wave number, λ is the wavelength, c is the speed of light *in vacuo*, which momentarily impinges onto colourless dielectric plate with the relative refraction index n lying on xy-plane, one surface of which is flat and another one is rough. We consider the case of static light scattering [7], and believe that surface roughness is not so large as to destroy the forward-scattered component of the transmitted radiation. It is this component that will be the main subject of our consideration.

The phase delay in the transmitted wave caused by a plate is determined as

$$\Phi(x, y) = e^{ik(n-1)h(x, y)} \tag{2}$$

where $h(x, y)$ is the local deviation of the surface profile from a mean surface line. The boundary field just behind the plate is described by the expression

$$U^{(b)}(x, y;\omega) = A_0 e^{i[\omega t - k(n-1)h(x, y)]}. \tag{3}$$

Partial wave rays transmitted trough the plate at various coordinates (x, y) and involved in the formation of the specular component of scattered field $U^{(s)}(\infty, \omega)$, which is observed at infinity (in the far field in respect of the isolated inhomogeneity of a surface), are added with the phase relations specified by Eq. (2)

$$U^{(s)}(\infty, \omega) = A_0 e^{i\omega t}\int F(h) e^{-ik(n-1)h} \mathrm{d}h \tag{4}$$

where $F(h)\mathrm{d}h$ is the density of a height distribution function of a rough surface. Then, the spectral intensity of the forward-diffracted component of transmitted field $S^{(\infty)}(\omega) = |U^{(s)}(\infty, \omega)|^2$, will be

$$S^{(\infty)}(\omega) = S^{(i)}(\omega) \cdot M^{(\infty)}(\omega) \tag{5}$$

where

$$M^{(\infty)}(\omega) = \left|\int F(h) e^{i[\omega t - k(n-1)h]} \mathrm{d}h\right|^2, \tag{6}$$

and

$$S^{(i)}(\omega) = \left|U^{(0)}(\omega)\right|^2 \equiv \left|A_0(\omega)\right|^2 \tag{7}$$

is the spectral intensity of the incident wave.

Let us briefly discuss the sense of the function $M^{(\infty)}(\omega)$. For the sake of simplicity (but without loss of generality of the main conclusions), we use here the simplest height distribution, such as rectangular one $F(h) = H^{-1}$, where H is the maximum deviation of the surface profile from the mean surface line. For such a height distribution $M^{(\infty)}(\omega)$ takes the form [9]

$$M^{(\infty)}(\omega) = \operatorname{sinc}^2\left[\frac{\pi(n-1)}{\lambda}H\right] \equiv \operatorname{sinc}^2\left[\frac{\omega}{2c}(n-1)H\right] \tag{8}$$

where $\operatorname{sinc}x = \sin x/x$. The argument of sinc-function $\pi(n-1)H/\lambda$, may be called the roughness strength. Function $M^{(\infty)}(\omega)$ vanishes firstly, when $(n-1)H/\lambda = 1$. As a consequence, function $S^{(\infty)}(\omega)$ vanishes for the specified ω. Thus, in the case of monochromatic radiation, the function $M^{(\infty)}(\omega)$ determines the relative power of the forward-scattered component of transmitted radiation field of frequency ω.

Now, consider the case where the incident wave is polychromatic. Assume the spectrum of the incident field to be of Gaussian profile centered at frequency ω_0 and with r.m.s. width σ

$$S^{(i)}(\omega) = S_0\exp\left[-\frac{(\omega-\omega_0)^2}{2\sigma^2}\right], \tag{9}$$

S_0 is a constant. It is known [13] that the forward scattering is always a coherent one. It means that partial wave rays for each monochromatic component of polychromatic radiation involved in the formation of the forward-scattered component are added interferentinally, just as in Eq. (4). Under assumption made for Eq. (8), one just obtains instead of Eq. (5)

$$S^{(\infty)}(\omega) = \exp\left[-\frac{(\omega-\omega_0)^2}{2\sigma^2}\right]\cdot \operatorname{sinc}^2\left[\frac{\omega}{2c}(n-1)H\right]. \tag{10}$$

The second multiplier of Eq. (10) is a peculiar spectral modifier governing the scattering-induced spectral changes in the forward-scattered component of transmitted radiation. It shows that the spectrum $S^{(i)}(\omega)$ of the incident polychromatic wave is modified in its forward-scattered component. Let us discuss a physical meaning of this conclusion, give numerical examples and some experimental demonstrations.

Polychromatic light results from additive mixing of numerous monochromatic components of various frequency ω. If any monochromatic components are attenuated (subtracted), then the spectrum of the resulting radiation is changed. Obviously, spectral changes are most pronounced when some spectral component of polychromatic radiation vanishes. In turn, the vanishing of the spectral intensity of some monochromatic component of the incident polychromatic field corresponds to singularity of the phase of sinc-function for the associated component. The peculiarity of the result obtained consists in the following. We explore the case where the phase of roughness strength for specified frequencyω is singular, rather than the phase of conventional complex amplitude of a light field. For that purpose, one observes spectral changes within homogeneous forward-scattered component of the transmitted radiation for rough surface specimens taken in turn, rather than spatial or angular redistribution of spectral components of polychromatic field, as in the Wolf's spectral effect [6]–[8].

3. Numerical results and demonstrations

Using Eq. (10) we obtained the following results. Figure 1 shows the normalized spectrum of a polychromatic incident field corresponding approximately to the relative sensitivity of a bright-adapted human eye and the spectral modifiers associated with

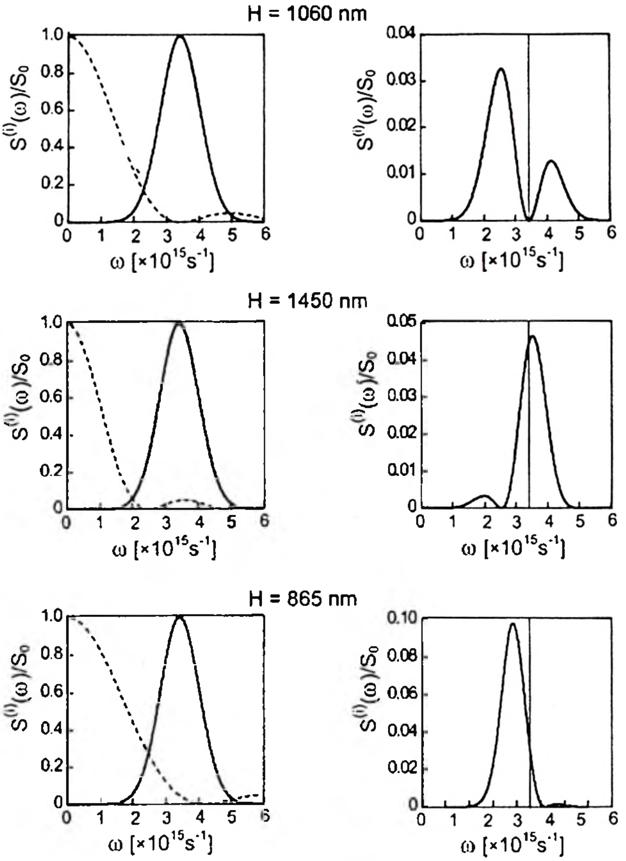
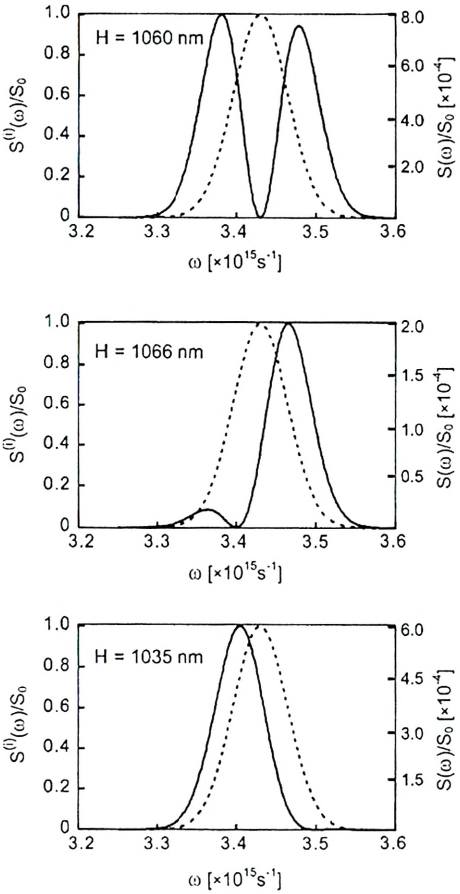
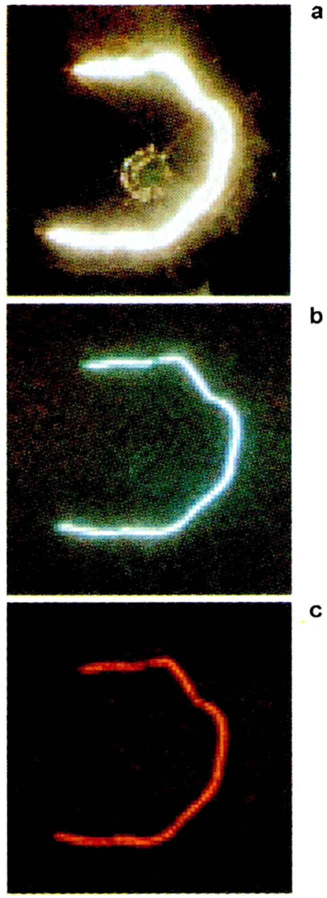


Fig. 1. Normalized spectrum $S^{(i)}(\omega)/S_0 = \exp[-(\omega - \omega_0)/2\sigma^2]$ of the incident field, with $\omega_0 = 3.43\times10^{15}$ s^{-1}, $\sigma = 6\times10^{14}$ s^{-1} ($\sigma/\omega_0 \approx 0.175$), $n = 1.52$ (left, solid curves); spectral modifiers with the first zero-crossings at $\omega = 3.43\times10^{15}$ s^{-1}, $\omega = 2.50\times10^{15}$ s^{-1}, and $\omega = 4.19\times10^{15}$ s^{-1}, the associated magnitudes of H are labelled (left, dashed curves); the corresponding normalized spectra of the forward -scattered component of transmitted radiation (right).





▲

Fig. 2. Normalized spectrum $S^{(i)}(\omega)/S_0 = \exp[-(\omega - \omega_0)/2\sigma^2]$ of the incident field, with $\omega_0 = 3.43\times10^{15}$ s^{-1}, $\sigma = 3.43\times10^{13}$ s^{-1} ($\sigma/\omega_0 \approx 0.01$), $n = 1.52$ (dashed curves); the normalized spectra of the forward-scattered component of transmitted radiation provided by spectral modifiers with the first zero-crossings at $\omega = 3.43\times10^{15}$ s^{-1}, $\omega = 3.40\times10^{15}$ s^{-1}, and $\omega = 3.50\times10^{15}$ s^{-1}, the associated magnitudes of H are labelled (solid curves).

Fig. 3. Spectral changes in the forward-scattered component of polychromatic radiation transmitted through a glass ($n = 1.52$) with rough surface: primary source (a lamp filament) (**a**); blue-shifted (**b**) and red-shifted (**c**) forward-scattered component.

three magnitudes of H. Depending on the magnitude of deviation H, one observes the splitting of the spectrum of incident field into two lines, either blue-shifted or red -shifted spectrum.

Figure 2 shows the normalized spectrum of polychromatic incident field with the same value of frequency ω_0 but with much smaller width σ, and the spectral modifiers associated with three magnitudes of deviation H. Comparison with Fig. 1 shows that the narrowing of the spectrum of incident field leads to the narrowing of the range of surface roughness exhibiting scattering-induced spectral changes observed in the forward-diffracted component of transmitted radiation.

In Figure 3, we demonstrate scattering-induced spectral changes of a polychromatic incident field observed in the forward-diffracted component of transmitted radiation. The conventional lamp filament is used as a primary source of polychromatic (white) light. Blue-shifted and red-shifted forward-scattered components (filament images) have been obtained with a glass ground using corundum with a mean diameter of grain 10 μm, but with varying grinding duration. The colouring of the forward-diffracted component is evident. More detailed experimental results will be reported in a future paper.

4. Conclusions

Summarizing, the singularity of the phase of roughness strength for some monochromatic component of the incident polychromatic field causes pronounced spectral changes in the forward-scattered component of radiation transmitted trough a colourless dielectric plate with a surface roughness comparable with a wavelength associated with the specified spectral component of the incident field. Similarly to the Wolf's effect, both blue- and red-shifts are concurrently realized within the limits of spectral band of the incident field.

The specific form of spectral modifier governing scattering-induced spectral changes is altered for the height distribution function different from the one used in this paper, as well as for the case of the specular component of polychromatic radiation reflected by a rough surface. So, in the case of transmitting phase-only holographic grating the spectral modifier is represented by the zero-order Bessel function of the first kind as a function of phase modulation percentage [12]. But in all cases elaborated by us, the singular optical mechanism of the colouring of the specular component is clearly recognized. Here, the ratio H/λ is the governing parameter in any case.

References

[1] Nye J.F., *Natural Focusing and the Fine Structure of Light*, Institute of Physics, Bristol 1999.
[2] Soskin M.S., Vasnetsov M.V., [In] *Progress in Optics*, [Ed.] E. Wolf, Elsevier, Amsterdam 2001, Vol. 42, p. 219.
[3] Nye J.F., Berry M.V., Proc. R. Soc. Lond. A **336** (1974), 165.
[4] Angelsky O.V., Mokhun I.I., Mokhun A.I., Soskin M.S., Phys. Rev. E. **65** (2002), 3.

[5] FREUND I., MOKHUN A.I., SOSKIN M.S., ANGELSKY O.V., MOKHUN I.I., Opt. Lett. **27** (2002), 545.
[6] WOLF E., Nature **326** (1987), 363.
[7] WOLF E., JAMES D.F.V., Rep. Prog. Phys. **59** (1996), 771.
[8] GRUB G., VISSER T.D., WOLF E., Phys. Rev. Lett. **88** (2002), 1.
[9] POLYANSKII V.K., J. Appl. Spectrosc. **13** (1970), 1039.
[10] POLYANSKII V.K., KOTLYAROVA I.B., Opt. Spectrosk. **30** (1971), 1142 (in Russian).
[11] KNOP K., Opt. Commun. **18** (1976), 298.
[12] STRINADKO L.V., POLYANSKII V.K., STRINADKO M.T., Opt. Spectrosk. **49** (1980), 952.
[13] VAN DE HULST H.C., *Light Scattering by Small Particles*, Wiley, New York 1957.

Received July 7, 2002