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# Nanolaser on heavy hole transition in semiconductor nanocrystals

SERGEY I. POKUTNYI*, WŁODZIMIERZ SALEJDA, LUCJAN JACAK, JAN MISIEWICZ, MICHAŁ TYC

Institute of Physics, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

We discuss energy spectrum of electron-hole pairs in a quasi-zero-dimensional system consisting of spherical semiconductor nanocrystals placed in transparent dielectric matrices. Absorption and emission of light in such systems is investigated. We study theoretically the prospect of using hole transitions between equidistant series of quantum levels observed in nanocrystals in design of a nanolaser.

## 1. Introduction

The optical properties of quasi-zero-dimensional structures consisting of spherical semiconductor nanocrystals (SNs) (quantum dots) of radius $a \sim 1-10$ nm, grown in transparent dielectric matrices, are currently being intensively studied [1]. Such heterophase systems are promising materials for creating new nonlinear opto-electronic devices, in particular, elements for controlling optical signals in lasers or optical computers [2]. In this context we point out that the SN size $a$ should be in the range of a few nanometers, in order to provide for the electron and hole energy levels splitting of the order of a few $k_B T_0$ at the room temperature $T_0$.

Since the energy gap of a semiconductor is much smaller than in the dielectric host, the motion of carriers in a semiconductor nanocrystal is restricted by its volume, and optical properties of the systems under study are determined by the energy spectrum of spatially confined electron–hole pairs (EHPs, excitons). Quantum confinement effects have been found in SNs by optical spectroscopic studies of electrons [3] and excitons [4], [5].

The energy spectrum of an exciton in a small SN was obtained in [6]–[14] as a function of the SN radius $a$ in adiabatic approximation under the assumption that $m_e \ll m_h$, where $m_e$ and $m_h$ are the effective masses of an electron and a hole in a SN. In the excitonic Hamiltonian, the electron–hole Coulomb interaction, as well as the polarization interaction of the electron and hole with the SN surface, have been

*Illichivsk Educational Research Centre, Odessa State University, 17a Danchenka str., Illichivsk, Odessa Reg., 68 001 Ukraine.

taken into account. The first order perturbation theory on the electron wave function of a spherical potential well of infinite depth has been used. In papers [15], [16], the finite height of the potential barrier $V_0$ at the boundary between the SN and the dielectric matrix, and the associated penetration of the ambient host by the electron, have been considered. The electron wave functions were obtained in [9], [10] for an electron travelling both in the SN and in the ambient dielectric host.

In the experiments reported in [3], an interesting energy structure was found which consisted of an equidistant series of quantum levels, caused by quantization of the heavy hole energy spectrum in the adiabatic potential of the electron. The purpose of the present paper is to discuss the possibility of applying transitions between these levels in nanolasers.

The paper is organized as follows. Section 2 is concerned with Hamiltonian of electron–hole pairs in SNs. In Section 3, we discuss the energy spectrum of EHPs. Section 4 is devoted to comparison of the theory with experimental data obtained for CdS nanocrystals. In Section 5, we consider absorption and emission of light in SNs. The concept of using CdS nanocrystals as active lasing medium for constructing nanolasers is presented in Section 6.

## 2. Hamiltonian of electron–hole pair in a nanocrystal

A simple model of a quasi-zero-dimensional structure in the form of a neutral spherical SN of radius $a$ and permittivity $\varepsilon_2$, embedded in a medium with permittivity $\varepsilon_1$, was discussed in [6]–[10]. An electron "e" and a hole "h" with effective masses $m_e$ and $m_h$ were assumed to travel within the SN; we use $r_e$ and $r_h$ to denote the distances of an electron and a hole from the center of the SN, respectively. We assume that the permittivities satisfy the relation $\varepsilon_2 \gg \varepsilon_1$, and that the conduction and valence bands are parabolic. In this model, in the framework of the effective mass approximation, the EHP Hamiltonian takes the form [6], [7]

$$H = -\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 + E_g + V_{eh}(\vec{r}_e,\vec{r}_h) + V_{hh'}(r_h,a) + V_{ee'}(r_e,a) + V_{eh'}(\vec{r}_e,\vec{r}_h,a) + V_{he'}(\vec{r}_e,\vec{r}_h,a) \tag{1}$$

where the first two terms represent the kinetic energy of the electron and the hole, $V_{eh}(r_e,r_h)$ is the energy of the Coulomb interaction between the hole and the electron

$$V_{eh}(\vec{r}_e,\vec{r}_h) = -\frac{e^2}{\varepsilon_2 a}\frac{a}{(r_e^2 - 2r_e r_h \cos\Theta + r_h^2)^{1/2}} \tag{2}$$

where $\Theta = \sphericalangle(\vec{r}_e,\vec{r}_h)$, $V_{ee'}(r_e,a)$, and $V_{hh'}(r_h,a)$ are the energies of interaction of the electron and the hole with their own images, $V_{eh'}(\vec{r}_e,\vec{r}_h,a)$ and $V_{he'}(\vec{r}_e,\vec{r}_h,a)$ are the energies of interactions with "alien" images, and $E_g$ is the bandgap in the bulk semiconductor with permittivity $\varepsilon_2$.

For arbitrary values $\varepsilon_1$ and $\varepsilon_2$, the terms in Eq. (1) describing the energy of the polarization interaction can be written in an analytic form [11], which is particularly simple for the case $\varepsilon_2 \gg \varepsilon_1$ [6], [7]:

$$V_{hh'}(r_h,a) = \frac{e^2}{2\varepsilon_2 a}\left(\frac{a^2}{a^2-r_h^2}+\frac{\varepsilon_2}{\varepsilon_1}\right), \tag{3}$$

$$V_{ee'}(r_e,a) = \frac{e^2}{2\varepsilon_2 a}\left(\frac{a^2}{a^2-r_e^2}+\frac{\varepsilon_2}{\varepsilon_1}\right), \tag{4}$$

$$V_{eh'}(\vec{r},\vec{r},a) = V_{he'}\ldots = -\frac{e^2}{2\varepsilon_2 a}\frac{a}{[(r_e r_h/a)^2-2r_e r_h\cos\Theta+a^2]^{1/2}}. \tag{5}$$

## 3. EHP spectrum in nanocrystal

The spectrum of EHP in a small semiconductor microcrystal was investigated in [6] and [7] in the case where its size was restricted by the condition

$$a_0 \ll a_h \ll a \leqslant a_e \simeq a_{ex} \tag{6}$$

where $a_h = \varepsilon_2\hbar^2/(m_h e^2)$, $a_e = \varepsilon_2\hbar^2/(m_e e^2)$ and $a_{ex} = \varepsilon_2\hbar^2/(\mu e^2)$ are the Bohr radii of the hole, electron and exciton in the semiconductor with permittivity $\varepsilon_2$, respectively; $\mu = m_e m_h/(m_e+m_h)$ is the reduced effective mass of EHP, and $a_0$ is a characteristic dimension of the interatomic distance order. When condition (6) is satisfied, the polarization interaction is a significant part in the potential energy of the Hamiltonian (1). The above inequalities also allow us to examine the motion of an electron and a hole in the effective mass approxiamtion. The validity of (6) further enables us to consider the motion of a heavy hole ($m_h \gg m_e$) in the electron potential averaged over the motion of the electron (adiabatic approximation).

In the adiabatic approximation we assume that the electron kinetic energy is the largest term and we take the last four terms in Eq. (1) together with the nonadiabatic operator as a perturbation. Then, taking into account only the first order perturbation theory, one can easily obtain the spectrum of an EHP, $E^{n_h,l_h,m_h}_{n_e,l_e,m_e}(a)$, in the state $(n_e,l_e,m_e;n_h,l_h,m_h)$, (here $n_e,l_e,m_e$ and $n_h,l_h,m_h$ are the main, orbital and magnetic, quantum numbers of an electron and a hole) [7], [9]

$$E^{n_h,l_h,m_h}_{n_e,l_e,m_e}(a) = \frac{\hbar^2}{2m_e a^2}u^2_{n_e,l_e} + \bar{V}_{ee'}(a) + \bar{\chi}^{n_h,l_h,m_h}_{n_e,l_e,m_e}(a) + E_g \tag{7}$$

where the first term is the kinetic energy of an electron in an infinite spherical well, and $\bar{V}_{ee'}(a)$ is the average value of the electron interaction with the self-image with the infinitely deep spherical well functions

$$\Psi_{n_e,l_e,m_e} = \sqrt{\frac{2}{a^2 r_e}}\,Y_{l_e,m_e}(\Theta,u)\frac{J_{l_e+1/2}(u_{n_e,l_e}r_e/a)}{J_{l_e+3/2}(u_{n_e,l_e})} \tag{8}$$

where $Y_{l_e,m_e}$ are normalized spherical functions, $u_{n_e,l_e}$ are the roots of the Bessel

function $J_{l_e+1/2}(u_{n_e,l_e}) = 0$. The quantity $\lambda^{n_h,l_h,m_h}_{n_e,l_e,m_e}(a)$ is an eigenvalue of the heavy hole Hamiltonian

$$H_h = -\frac{\hbar^2}{2m_h}\nabla_h^2 + V_{hh'}(r_h,a) + \bar{V}_{n_e,l_e,m_e}(r_h,a) \tag{9}$$

where

$$\bar{V}_{n_e,l_e,m_e}(r_h,a) = \bar{V}_{eh}(r_h,a) + \bar{V}_{eh'}(r_h,a) + \bar{V}_{he'}(r_h,a) \tag{10}$$

is the average value of the Coulomb interaction and of the electron and hole interaction with "alien" images on "free" electron states (8).

Quantitative results for the EHP spectrum (7) are obtained here only for the simple case $l_e = 0$. Using expressions (3)–(5), (8) and (10), one can obtain:

$$\bar{V}^{n_e,0,0}_{eh}(x,s) = s^{-1}\left[\frac{\sin(2\pi n_e x)}{\pi n_e x} - 2\mathrm{Ci}(2\pi n_e x) + 2\mathrm{Ci}(2\pi n_e) + 2\ln x - 2\right], \tag{11}$$

$$\bar{V}^{n_e,0,0}_{eh'}(x,s) + \bar{V}^{n_e,0,0}_{he'}(x,s) = -2s^{-1}, \tag{12}$$

$$\bar{V}^{n_e,0,0}_{ee'}(x,s) = s^{-1}\left(Z_{n_e,0} + \frac{\varepsilon_2}{\varepsilon_1}\right), \quad \bar{V}_{hh'}(x,s) = s^{-1}\left(\frac{1}{1-x^2} + \frac{\varepsilon_2}{\varepsilon_1}\right) \tag{13}$$

where

$$Z_{n_e,0} = 2\int_0^1 \mathrm{d}x_e \frac{\sin^2(\pi n_e x_e)}{1-x_e^2},$$

and $\mathrm{Ci}(y)$ is the integral cosine. Here and below the energy is measured in units $R_h = \hbar^2/(2m_h a_h^2)$ and dimensionless lengths $x = r_h/a$ and $s = a/a_h$ are used.

Note that in the approximation considered the interaction of an electron with the images (its own and "alien") (12), (13) and the interaction of a hole with an "alien" image (12) yield a constant addition $\sim s^{-1}$ the hole energy.

Taking into account formulae (3) and (10)–(12), we write the potential energy in the heavy hole Hamiltonian (9) as follows:

$$\begin{aligned}\bar{U}^{n_e,0,0}(x,s) &= V_{hh'}(x,s) + \bar{V}^{n_e,0,0}(x,s) \\ &= s^{-1}\left[\frac{1}{1-x^2} + \frac{\sin(2\pi n_e x)}{\pi n_e x} - 2\mathrm{Ci}(2\pi n_e x) + 2\mathrm{Ci}(2\pi n_e)\right. \\ &\quad \left. + 2\ln x + \frac{\varepsilon_2}{\varepsilon_1} - 4\right].\end{aligned} \tag{14}$$

The minimum of the potential energy (14)

$$\bar{U}^{n_e,0,0}_{\min}(s) = \bar{U}^{n_e,0,0}(x=0,s) = \frac{P_{n_e,0}}{s}$$

where

$$P_{n_e,0} = 2\mathrm{Ci}(2\pi n_e) - 2\ln(2\pi n_e) - 2\gamma + \frac{\varepsilon_2}{\varepsilon_1} - 1$$

($\gamma = 0.577$ is the Euler constant) is reached at the point $x = 0$. A series expansion of the potential (14) with respect to the parameter $x^2 \ll 1$ with accuracy up to the first two terms gives the hole spectrum $\lambda^{t_h}_{n_e,0,0}(\bar{s})$ in oscillator from [7], [9]

$$\lambda^{t_h}_{n_e,0,0}(\bar{s}) = \frac{P_{n_e,0}}{\bar{s}} + \bar{\omega}(\bar{s},n_e)\left(t_h + \frac{3}{2}\right) \tag{15}$$

where

$$\bar{\omega}(\bar{s},n_e) = 2.232\left(1 + \frac{2}{3}\pi^2 n_e^2\right)^{1/2} \bar{s}^{-3/2}, \tag{16}$$

and $\bar{\omega}(\bar{s},n_e)$ is the frequency of the hole oscillator vibrations, $t_h = 2n_r + l_h = 0, 1, \ldots$ is the hole main quantum number ($n_r$ is the hole radial quantum number).

Taking into account formulae (13), (15) and (16) we obtain the EHP spectrum $[E^{t_h}_{n_e,l_e=0}(\bar{s})]$ in the state $(n_e, l_e = 0; t_h)$ for the SN of radius ($\bar{s} = \bar{a}/a_h$) [8], [10]

$$E^{t_h}_{n_e,l_e=0}(\bar{s}) = E_g + \frac{\pi^2 n_e^2}{\bar{s}^2} K\beta + \bar{s}^{-1}\left(Z_{n_e,0} + P_{n_e,0} + \frac{\varepsilon_2}{\varepsilon_1}\right) + \bar{\omega}(\bar{s},n_e)\left(t_h + \frac{3}{2}\right) \tag{17}$$

where $K = 0.67$ represents the spread of the SN radii [4] and $\beta = m_e/m_h$. Moreover, numerical analysis of X-ray data [5], which took the spread of the SN radii into account, shows that the mean radius evaluated over the Lifshits–Slezov distribution [17] was $\bar{s} = 0.86s$ [5], where $s$ is the SN radius obtained in the small variance approximation.

The main contribution to the EHP spectrum (17) is provided by the second term (electron kinetic energy), which comes from the purely spatial restrictions imposed on the quantization region. The last two terms, which are associated with the Coulomb and polarization interactions between the electron and the hole, are regarded as corrections. The polarization interaction between the hole and the electron, on the one hand, and the surface of the SN, on the other, contributes, like the size quantization of carriers, to the renormalization of the energy gap of the SN given by Eq. (17). The polarization interaction, which is of the same order of magnitude as the exciton binding energy in the SN, in this case is found to be much smaller than the size quantization energy of the charge carriers in the SN.

We also note that the exciton spectrum (17) is valid only for lower states of the exciton $(n_e, 0, t_h)$ in the SN, for which $(E_{n_e,0} - E_g) \ll V_0$, where $V_0$ is the depth of the potential well for an electron in the SN (*e.g.*, in the cadmium sulphide SN, $V_0 = 2.3 - 2.5$ eV when condition (6) is satisfied [18]).

## 4. Comparison of theory with experimental data

In papers [3], [19] interband absorption spectra of CdS SNs ($\varepsilon_2 = 9.3$) of sizes varying from 1 to $10^2$ nm dispersed in a transparent dielectric matrix of silicate

glass were investigated. The effective masses of electrons and holes in CdS were, respectively, $m_e/m_0 = 0.205$ and $m_h/m_0 = 5$ (*i.e.*, $m_e \ll m_h$). The dimensional quantization phenomenon found in [4] for the electron energy spectrum was qualitatively described by the formula

$$E_{n_e,l_e}(a) = E_g + \frac{\hbar^2}{2\mu a^2} u^2_{n_e,l_e}. \tag{18}$$

In this case, in the region of SN sizes $a$ comparable with the exciton radius, $a \leqslant a_{ex} \simeq 3$ nm, the experimental points were observed to deviate from the theoretical dependence (18). The spectrum of electron–hole pairs in the interval of SN sizes $a \leqslant a_{ex}$ calculated in [6]–[10] gave a qualitatively correct description of the dependence of the experimental spectrum [3] on the SN radius $a$.

In the experiment reported in [5], an energy structure consisting of an equidistant series of levels was found in the region of transitions to the lower level of dimensional quantization of the electron ($n_e = 1$). The above structure is due to quantization of the hole energy spectrum in the adiabatic potential of the electron.

From the comparison of formula (16) (for $n_e = 1$) with the experimental dependence of the splitting magnitude on the SN size $\bar{a}$ obtained in [5], it follows that for SNs of radii $1.5 \leqslant \bar{a} \leqslant 3.0$ nm, the splitting $\bar{\omega}(\bar{s})$ (Eq. (16)) is in good agreement with the experimental data [5] and differs from the latter only slightly ($\leqslant 4\%$).

## 5. Absorption and emission of light in nanocrystal

The interband absorption of light in SNs was studied theoretically in [20] using the dipole approximation in the framework of the model considered here, and under the assumption that the absorption length $\lambda \gg a$.

An expression for the quantity $K(\bar{s},\omega)$ defined by the hole optical transition from the energy level $t_h = 2n_h$ to the lowest electron level ($n_e = 1$, $l_e = m_e = 0$) was derived in the following form:

$$K(\bar{s},\omega) = A \sum_{n_h} L_{n_h}(\bar{s}) \times \delta\left[\Delta(\omega) - \frac{\pi^2}{\bar{s}^2}\frac{m_h}{m_e} - \frac{1}{\bar{s}}\left(Z_{1,0} + P_{1,0} + \frac{\varepsilon_2}{\varepsilon_1}\right) - \bar{\omega}(\bar{s}, n_e = 1)\left(2n_h + \frac{3}{2}\right)\right] \tag{19}$$

where $\Delta(\omega) = \hbar\omega - E_g$, $\omega$ – incident light frequency, and $A$ is proportional to the square of the absolute value of the dipole moment matrix element calculated with Bloch functions. The quantity $K(\bar{s},\omega)$ (19) connects the energy absorbed by SN in a time unit with the time average of electric field square of incident wave. Moreover, the product of $K(\bar{s},\omega)$ and SN concentration in the dielectric matrix gives electric conductivity of the quasi-zero-dimensional system under consideration for the frequency $\omega$, which is connected with light absorption coefficient in the usual way.

We determine the quantity $K(\bar{s},\omega)$ (19) corresponding to the hole optical transition from the energy level $t_h = 2n_h$ to the lowest electron level ($n_e = 1$, $l_e = m_e = 0$). In this case, the expression for the quantity $L_{n_h}(\bar{s})$, given by the square of the overlap integral of the electron and hole wave functions, takes the form [20]

$$L_{n_h}(\bar{s}) = \frac{2\pi^{5/2}}{(1+2/3\pi^2)^{3/4}} \frac{n_h+1}{2^{2n_h} n_h!} (\bar{s})^{-3/4}. \tag{20}$$

In the interband optical absorption spectrum of SN, the radius of which fulfils condition (7), each line corresponding to given values of the radial $n_e$ and orbital $l_e$ quantum numbers turns into a series of close lying equidistant levels, corresponding to various values of the main hole quantum number $t_h$. This conclusion follows from (15), (16) and (19), and is a direct consequence of the Coulomb and polarization interactions of an electron and a hole in SN.

We can estimate the value of the overlap integral square $(K(\bar{s},\omega)/A)$ using (19), (20) and the experimental data taken from [3]. For the hole transitions from the equidistant quantum levels: $(n_h = 0; l_h = m_h = 0)$, $(n_h = 1; l_h = m_h = 0)$, $(n_h = 2; l_h = m_h = 0)$, $(n_h = 3; l_h = m_h = 0)$, to the lowest electron size-quantized level $(n_e = 1; l_e = m_e = 0)$, we have

$$\frac{K(\bar{s},\omega)}{A} = \sum_{n_h=0}^{3} L_{n_h}(\bar{s}) \tag{21}$$

where: $L_0 = 7.66\bar{s}^{-3/4}$, $L_1 = 0.5L_0$, $L_2 = 9.4\cdot10^{-2}L_0$, $L_3 = 10^{-2}L_0$. From the above expression, it follows that the main contribution in the light absorption coefficient in a cadmium sulphide SN with the radius $s$ (Eq. (6)) comes from the hole spectral lines corresponding to quantum numbers $(n_h = 0; l_h = m_h = 0)$ and $(n_h = 1; l_h = m_h = 0)$, the transition oscillator strengths of which are dominant. The contribution of higher excited hole lines $(n_h \geqslant 2; l_h = m_h = 0)$ is negligible.

This way, in the framework of the model of the quasi-zero-dimesional system considered it has been shown that the absorption and emission edge of a cadmium sulphide SN is formed by two transitons of comparable intensities.

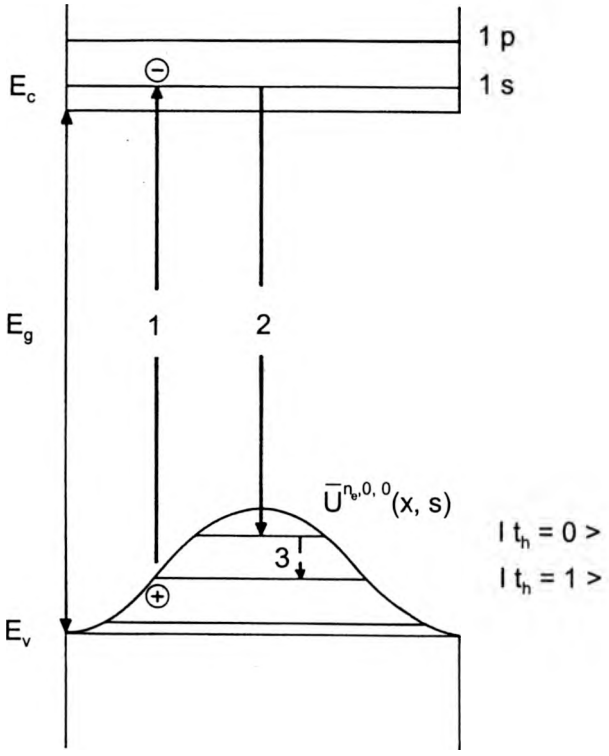
## 6. Nanolaser on heavy hole transition in SNs

We assume that CdS microcrystals have a direct band structure, like bulk CdS. The extrema of the conduction and valence bands are placed in the center of the Brillouin zone. Moreover, CdS is a wide-gap semiconductor ($E_g = 2.58$ eV) in which nonparabolicity effects are weak. At the same time, the dispersion laws near the bottom of the conduction band and the top of the valence band can be considered as parabolic [21].

When developing the theory, the electron and hole bands were assumed to be parabolic. The nonparabolicity parameter $j$ of such bands for EHP energy $E^{t_h}_{1,0}(a)$ (Eq. (17)) in the ground state, which is obtained from the experiments of [3], [19] in SN of size $a \sim a_{ex}$ takes a small value

$$j(a) = \frac{E^{t_h}_{1,1} - E_g}{E_g} \leqslant 8\%.$$

This relation gives a basis for the assumption that the valence and conduction bands are, to a high degree of accuracy, parabolic.




Scheme of the optical transitions of an electron and a hole in a CdS nanocrystal. The electron energy spectrum $E_{n_e,l_e}(s)$ (18) and the hole potential energy $\bar{U}^{n_e,0,0}(x,s)$ (14) are shown; $E_g$ — bandgap width. The electron states in the conduction band are labelled as 1*s* and 1*p*. The hole states in the valence band are denoted by $|t_h = 0\rangle$ and $|t_h = 1\rangle$. Transitions are labelled as follows: 1 — pumping, 2 — creating inverse population, 3 — lasing.

In the Figure, the energy diagram of the optical transitions in CdS SN of radius $a$ fulfilling condition (6) is shown. The electron energy spectrum $E_{n_e,l_e}(a)$ (Eq. (18)) in the conduction band of a SN of the radius a comparable with the linear dimension of quasiparticles in semiconductors is determined only by the size quantization effect.

The energy spectrum $\lambda^{t_h}_{n_e,0,0}$ of heavy hole in the valence band is equivalent to the spectrum of hole carrying out oscillator vibrations with frequency $\bar{\omega}(\bar{s}, n_e)$ (Eq. (16)) in the adiabatic electron potential.

Based on the results presented we can formulate an action scheme for a laser designed on CdS quantum dots grown in boric-silicate glass matrices [3]–[5], [18]–[22]:

i) Pumping: the hole transition from energy level $\lambda^{t_h}_{n_e,0,0}(a)$ to electron state $E_{n_e=1,l_e=0}(a)$ (1*s* state) in conduction band with energy (transition 1 in the Figure)

$$h\nu_1(\bar{s}) = E_g + E_{1,0}(\bar{s}) + \lambda^1_{1,0,0}(\bar{s}). \tag{22}$$

ii) Creation of inverse population on the hole level $|t_h = 0\rangle$ by means of electron transition from the energy level $E_{1,0}(\bar{s})$, with spin flip [23], to the hole energy level $\lambda^0_{1,0,0}(\bar{s})$ (transition 2 in the Figure). The energy of this transition

$$h\nu_2(\bar{s}) = E_g + E_{1,0}(\bar{s}) + \lambda^0_{1,0,0}(\bar{s}). \tag{23}$$

iii) Lasing: transition of the hole in the valence band between equidistant levels $|t_h = 0\rangle \rightarrow |t_h = 1\rangle$ (transition 3 in the Figure) separated by the energy

$$\Delta(\bar{s}) = \hbar\bar{\omega}(n_e = 1, \bar{s}). \tag{24}$$

Under the experimental conditions from [3]–[5] and [18], [19], [22] at liquid nitrogen temperatures ($T = 77$ K), the distance $\Delta E_e$ between electron energy levels ($n_e = 1$, $l_e = 1$) (1*p* state) and ($n_e = 1$, $l_e = 0$) (1*s* state), according to Eq. (18)

$$\Delta E_e = \frac{(u_{1,1}+\pi)(u_{1,1}+\pi)(m_h/m_e)R_h}{s^2} \simeq 480 \text{ meV}$$

where $u_{1,1} = 4.493$ [24]. The distance $\Delta(\bar{s})$ between the hole energy levels (23) is

$$\Delta(\bar{s}) = \hbar\bar{\omega}(n_e = 1, \bar{s}) \simeq 53 \text{ meV}.$$

For SN of the radius $a = 2$ nm, $\Delta(\bar{s})$ is several times greater than the thermal energy $k_B T$ at $T = 77$ K.

In a nanolaser on CdS nanocrystals, the pumping energy $h\nu_1 \simeq 3.11$ eV (22) would correspond to visible light, and lasing at energy $\Delta(\bar{s}) \simeq 53$ meV would occur in the infrared range.

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