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Electromagnetic limit of the optical gain for long-wave IR diffractive optics

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Focal plane collection optics consisting of arrays of microlenses may be used to reduce the physical size of an infrared detector without reduction of its apparent optical size and quantum efficiency. This results in reduction of the thermal generation rate in the detector, allowing improvement of performance of infrared devices operating at elevated temperatures. In this paper, we report on calculations of the optical gain achievable with the use of diffractive microlenses for long-wave (8 – 12 μm) infrared (LWIR) detectors. Fast diffractive lenses are required for this application, and binary optics is the most popular technology of their fabrication. A large fraction of the binary lens surface relief is built of staircase annular structures whose width is of wavelength-scale. Therefore, the electromagnetic theory of gratings has been applied in this paper to calculate the diffraction efficiency of the Fresnel zones for the multi-phase-level lenses. It is shown that electromagnetic effects limit the speed of the LWIR diffractive lenses.

1. Introduction

Focal plane collection optics consisting of arrays of microlenses has been used enabling size reduction of the infrared detector, without reduction of its apparent optical size and quantum efficiency (Fig. 1). This results in reduction of the noisy

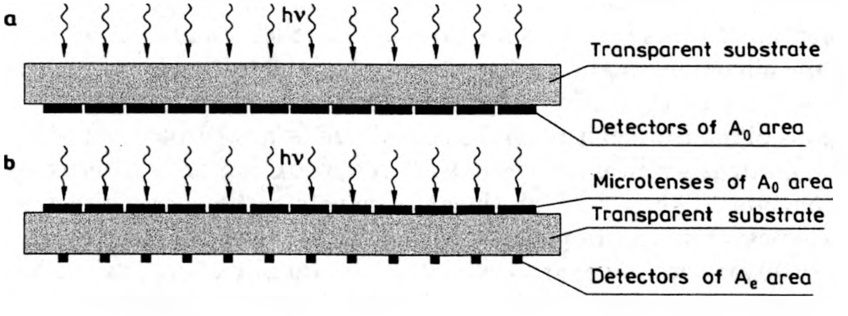

Fig. 1. Schematic cross-section of conventional 2D detector array (a) and array with diffractive microlenses (b).

thermal generation rate in detector operating at elevated temperatures, allowing improvement of its signal-to-noise-performance. The array of refractive or diffractive microlenses can be fabricated on the back side of a detector array structure.

The optical gain G is a factor describing the improvement of the detection system SNR by replacing the detector of the area A_0 in the conventional array by a lens of the same area A_0 and detector with the reduced area A_e. When the noise power is proportional to the detector size, G is equal to the product of the lens effectiveness η and the detector linear size compression

$$G = \eta\sqrt{A_0/A_e} \tag{1}$$

where η is the fraction of optical flux incident on the microlens that reaches the detector. It should be noted that in some cases, the noise is not dependent on the detector size, with the gain in SNR being larger, and directly proportional to the A_0/A_e ratio

$$G = \eta A_0/A_e. \tag{1a}$$

In practice, the optical gain G depends on diffraction efficiency, speed and aberrations of microlenses, as well as on properties of the main optics used in an infrared system.

In this paper, we discuss limitations on the optical gain achievable with the use of diffractive microlenses for LWIR detectors. Section 2 contains the discussion of the validity of the local linear grating approximation and scalar diffraction efficiency formula for the LWIR diffractive microlenses. The electromagnetic calculations of the diffraction efficiency and achievable speed of the LWIR diffractive microlenses (2π- and 4π-phase) are presented in Sec. 3 that is concluded with an estimation of the optical gain in a sample IR system. Section 4 gives the main results.

2. Local linear grating approximation

Binary optics [1], [2] technology is the principal technique for manufacturing multilevel surface-relief diffractive optical elements. Binary optics exploits the ability of VLSI technology to precisely, cost-effectively and reproducibly fabricate masks and transfer the patterns into the substrate surface relief. Figure 1 shows the phase profiles of the binary lenses analyzed in this paper and intended for use in the 250 μm pitch 2D array of $Hg_{1-x}Cd_xTe$ photodiodes.

The edge regions of the Fresnel lens can be considered as a superposition of the local linear blazed gratings with varying periods. This approximation is required to apply the electromagnetic theory of gratings to calculate the lens diffraction efficiency, and assumes: a) sawtooth phase profile, b) linear shape of grooves, and c) local periodicity (invariance of the groove width). The validity of approximation a) can be inferred from Fig. 2. Approximations b) and c) are illustrated in Fig. 3.

Let us discuss the local periodicity approximation. The radius of the Fresnel zone boundaries is

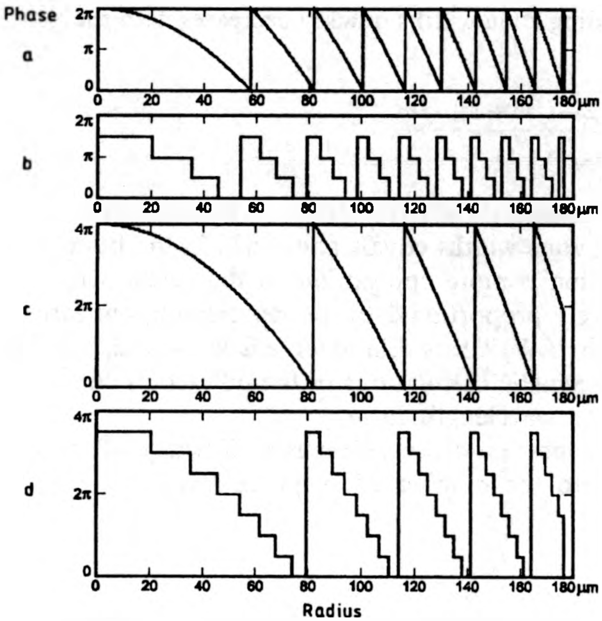

Fig. 2. Phase of diffractive lenses: **a** — Fresnel lens, **b** — four-phase-level approximation, **c** — 4π diffractive lens, **d** — eight-phase-level second-order diffractive lens.

$$r_m = \sqrt{2m\lambda f'/n_{\text{III}} + (m\lambda/n_{\text{III}})^2} \approx \sqrt{2m\lambda f'/n_{\text{III}}} \tag{2}$$

where m is a zone number, f' — the focal length in the lens substrate of refractive index n_{III}, and λ is the wavelength in air. Zone width d_m can be expressed as

$$d_m \approx \frac{r_m}{2m} \propto \frac{1}{\sqrt{m}} \propto \frac{1}{r_m}. \tag{3}$$

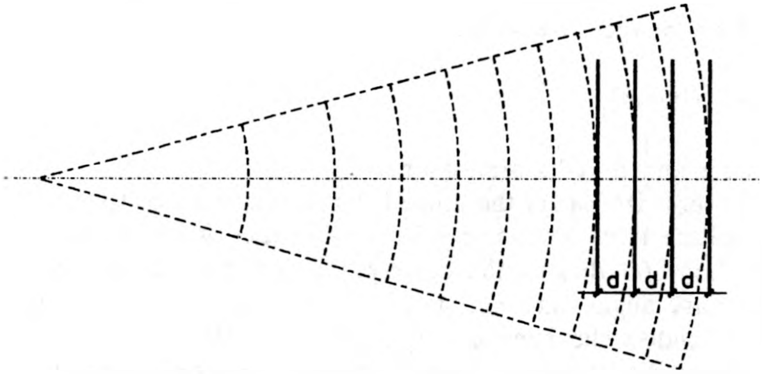

Fig. 3. Local linear grating approximation (solid line) and the boundaries of the Fresnel zones in a section of LWIR lens (dashed-arc line).

The relative difference of the succeeding zone widths quickly decreases with the zone number

$$\left|\frac{d_{m+1}-d_{m-1}}{2d_m}\right| \approx \frac{\sqrt{m-1}-\sqrt{m-2}-\sqrt{m+1}+\sqrt{m}}{2(\sqrt{m}-\sqrt{m-1})}. \tag{4}$$

The LWIR microlenses under consideration have about 10 Fresnel zones. The relative difference of the succeeding zone widths equals about 5% in the boundary region of the lens. This approximation is more appropriate in the visible window, where lens Fresnel numbers, inversely proportional to the wavelength, are larger. However, it allows us to estimate the LWIR lens diffraction efficiency and, what is more important, it enables electromagnetic calculations of the diffraction efficiency dependence on the groove width to wavelength ratio.

According to the scalar theory, the first-order diffraction efficiency η_1^M of the binary blazed gratings increases with the number M of phase levels

$$\eta_1^M = \left(\frac{\sin\pi/M)}{\pi/M}\right)^2. \tag{5}$$

The diffraction efficiency η_1^2 of two-level blazed gratings is equal to 40.5%, and increases to 81% and 95% for $M = 4$ and 8, respectively. A higher diffraction efficiency, achievable with the larger M, needs a smaller minimal feature width.

The lens f-number, f/D, where f is the effective focal length and D is the lens diameter, depends on the ratio of the edge zone width $d_{\min}$ to the wavelength in air λ

$$\frac{f}{D} = \frac{1}{2}\sqrt{\left(\frac{d_{\min}}{\lambda}\right)^2 - \frac{1}{n_{\mathrm{III}}^2}} \approx \frac{d_{\min}}{2\lambda}. \tag{6}$$

This formula can be obtained from the relation

$$\left.\frac{\partial\Phi(r,f')}{\partial r}\right|_{r=b} d_{\min} = 2\pi \tag{7}$$

where $\Phi(r,f')$ is the phase of the Fresnel lens

$$\Phi(r,f') = \frac{2\pi}{\lambda} n_{\mathrm{III}}(f' - \sqrt{f'^2 + r^2}), \tag{8}$$

r is the radial coordinate and b is its semi-diameter.

The smallest relief feature size, called the critical dimension, is equal to $d_{\min}/M$. The speed of an eight-phase level diffractive microlens in the visible wavelength window is limited to about $f/3$ due to the corresponding critical dimension of 0.5 μm and achievable mask alignment tolerances [1], [2]. Over ten times longer wavelength in the LWIR window alleviates making faster lenses. However, the speed of the LWIR microlenses is limited by the electromagnetic diffraction effects.

Only the central zone of the fast LWIR lens has the width much larger than the wavelength. Diffraction on wavelength-scale structures in the remaining zones depends on polarization and the vector nature of fields since d_{min}/λ is close to 1. Waveguiding and interference phenomena inside the structure may lead to rapid changes of the transmitted and reflected fields with the incidence angle and wavelength of the illumination. Diffraction efficiency of the edge zones of a fast Fresnel lens cannot be correctly predicted using the scalar diffraction theory [3], which does not take these effects into account. According to scalar theory the diffraction efficiency (Eq. (5)) does not depend on the d_{min}/λ ratio. Therefore, rigorous methods of electromagnetic integral [4]–[6] or differential [7], [8] equations should be applied to predict the diffraction efficiency of the fast LWIR lenses.

3. Diffractive LWIR lenses – electromagnetic estimation of the minimum zone width

Electromagnetic methods of integral equations can be used to evaluate non-periodic diffractive structures. However, only the calculations of 1D (cylindrical) lenses have been reported to date [4], [5], due to their extremely large numerical cost. Much faster electromagnetic methods of differential equations are sufficient to calculate diffraction efficiency of the diffractive lens edge zones using the local linear grating approximation. Periodicity of the diffractive structure, being a basic assumption of differential equations methods, is locally fulfilled in this case, as has been shown in Sec. 2.

The eigenvalue method [1], [7] and the scattering-matrix algorithm [9] have been applied in this paper to estimate the minimum zone width of the LWIR four-phase-level Fresnel lens of acceptable diffraction efficiency. The minimum zone width determines the highest numerical aperture of an effective Fresnel lens. The local linear grating approximation has been applied, *i.e.*, a sector of the zone has been considered to be a period of the one-dimensional four-level blazed diffraction grating.

The geometry of the diffraction problem under consideration (the phase profile is simplified) is shown in Fig. 4. Plane wave of incidence angle θ is coming from region I of the refractive index n_{I}. Corrugated region II has the depth h, and substrate region III has the refractive index n_{III}.

We will describe the main steps of the eigenvalue method for the TE polarization case. Inside the structure the electric field E_y, parallel to the grooves, is assumed to be the sum of propagating (γ_n real) or evanescent (γ_n imaginary) modes of amplitudes a_n and b_n, for the forward (positive z-direction) and backward (negative z-direction) waves, respectively

$$E_y(x,z) = \sum_{n=1}^{\infty} \{a_n \exp(i\gamma_z z) + b_n \exp(-i\gamma_n z)\} \sum_{l=-\infty}^{\infty} P_{ln} \exp(i\alpha_l x) \tag{9}$$

where

$$\alpha_l = \frac{2\pi}{\lambda} n_{\mathrm{I}} \sin\theta + \frac{2\pi l}{d}. \tag{10}$$

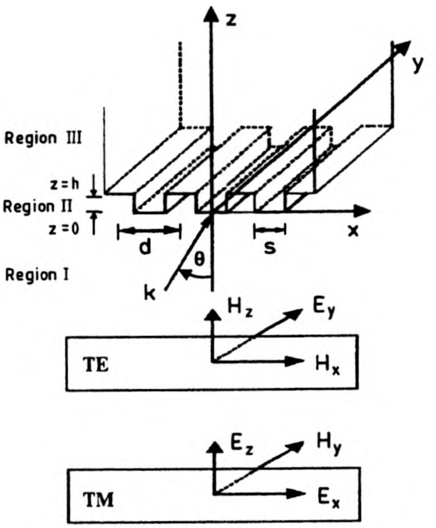

Fig. 4. Geometry of the diffraction problem.

The field distribution (9) is pseudo-periodic in the x-direction. Matrix P_{ln} couples each mode (numbered by n) to field harmonics in the x-direction (numbered by l). Postulated separable solution (9) is substituted into the Helmholtz equation

$$\frac{\partial^2}{\partial x^2}E_y(x,z)+\frac{\partial^2}{\partial z^2}E_y(x,z)+k^2\hat{\varepsilon}_r(x,z)E_y(x,z)=0, \tag{11}$$

that is derived from the Maxwell equations. After Fourier expansion of a relative permittivity $\hat{\varepsilon}_r(x,z)$, an eigenvalue problem is obtained. The eigenvalues γ_n^2 and eigenvectors P_{ln} ($n = 1, \ldots \infty$) are obtained by its numerical solution. Subsequently, amplitudes a_n and b_n are calculated from the boundary conditions at $z = 0$ and $z = h$. Finally, the diffraction efficiencies of transmitted and reflected orders are calculated from the fields at $z = h$ and $z = 0$, respectively.

The first-order diffraction efficiency of the four-phase-level grating, being the local approximation of the edge zone of the four-phase-level diffractive lens, was calculated for TE and TM polarizations versus the ratio of the period d to wavelength λ. The following values of the optical parameters are assumed in calculations: $\lambda = 9$ μm, $n_{\mathrm{I}} = 1$, $n_{\mathrm{III}} = 2.7$, $\theta = 0°$, $-20°$, $20°$. Usually, the detector-microlens array is placed in the vicinity of focal plane of an objective. The value of the oblique incidence angle corresponds to the maximum aperture angle of the objective of numerical aperture equal to 0.34. Figure 5 shows the calculation results averaged over both polarizations.

Resonance domain oscillations of diffraction efficiency in the first order shown in Fig. 5 are less evident than for a visible light diffraction on, *e.g.*, glass grating. This is due to a lower ratio of the depth of grating to the wavelength in air equal to $1/(n_{\mathrm{III}}-n_{\mathrm{I}})$, and lower diffraction angle corresponding to a spatial frequency for the

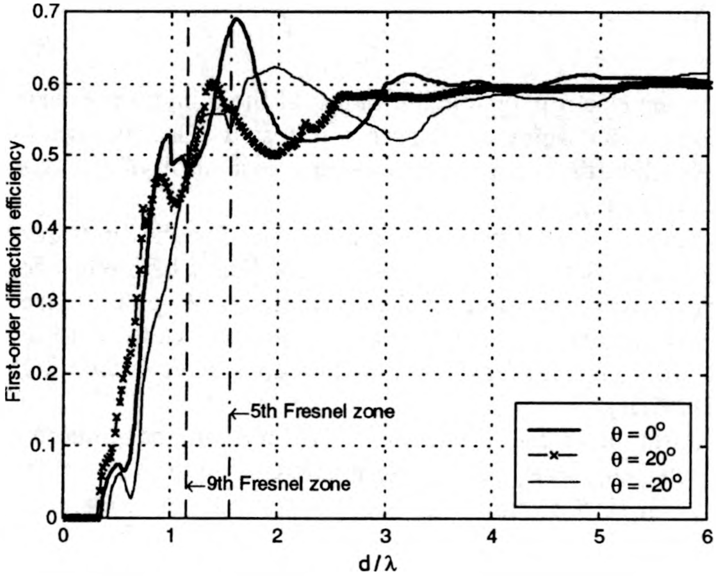

Fig. 5. Zonal efficiencies of the four-level Fresnel lens averaged over both states of polarization.

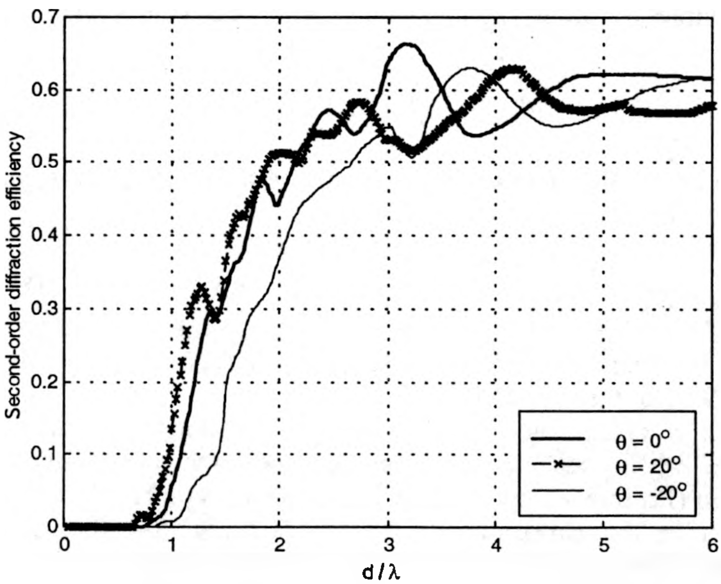

Fig. 6. Zonal efficiencies of the 8-level 4π-phase blazed grating averaged over both states of polarization.

high-index infrared materials. The result is that less energy is reflected at the medium boundaries and the evanescent waves contribute less to an energy exchange between modes.

The main conclusion derived from the results shown in Fig. 5 is that the minimum value of the lens-edge-groove width d should fulfil the following relation to prevent excessive loss of η:

$$d_{\min}/\lambda \approx 1.15. \tag{12}$$

An attempt to overcome the electromagnetic limitation of the minimum Fresnel zone width using the second-order diffractive lenses was tested. Electromagnetic calculations of the 8-phase-level grating being a staircase approximation of 4π-phase blazed grating are presented in Fig. 6.

Although the groove width is now twice as large as that for the first-order grating, no benefits can be found. Both the second-order cut-off and high-efficiency values of d/λ increased almost twice compared to the conventional 2π-phase blazed grating, and cancelled improvements due to the increased groove width. Another disadvantage of the higher-order gratings for the application considered is their high angular and chromatic selectivity.

The electromagnetic limitation of the d/λ ratio sets a limit on the numerical aperture of the diffractive lens $\mathrm{NA}_{\mathrm{out}} = n_{\mathrm{III}}\sin\theta_{\mathrm{out}}$ illuminated through the objective of numerical aperture $\mathrm{NA}_{\mathrm{in}} = n_{\mathrm{I}}\sin\theta$. From the grating equation for the edge of the lens

$$n_{\mathrm{III}}\sin\theta_{\mathrm{out}} = n_{\mathrm{I}}\sin\theta + m\frac{\lambda}{d} \tag{13}$$

where m stands for the diffraction order, the following relation can be obtained for $m = 1$

$$\mathrm{NA}_{\mathrm{out}} = \mathrm{NA}_{\mathrm{in}} + \frac{\lambda}{d_{\min}}. \tag{14}$$

It allows the evaluation of the detector size compression factor $\sqrt{A_0/A_e}$ in formula (1) for the diffraction limited system

$$\sqrt{\frac{A_0}{A_e}} = \frac{\mathrm{NA}_{\mathrm{out}}}{\mathrm{NA}_{\mathrm{in}}} = 1 + \frac{\lambda}{d_{\min}\mathrm{NA}_{\mathrm{in}}}. \tag{15}$$

For the following values of parameters: $\mathrm{NA}_{\mathrm{in}} = 0.34$, $d_{\min}/\lambda \approx 1.15$, and $\eta = 0.6$, the maximum optical gain $G_{\max}$ is equal to 2.1.

Formula (15) assumes a diffraction limited system. Aberrations will further reduce the optical gain, which can be illustrated by the following example. Electromagnetic limitation (12) implies that for the 250×250 μm square microlens, the shortest focal length in the substrate can be $f' = 500$ μm. Substrate-side numerical aperture for the corner of that lens and normal plane wave illumination is $\mathrm{NA} = n_{\mathrm{III}}\sin(19.5°) = 0.9$. 78% of the lens surface belongs to a circle of 125 μm radius. The d/λ ratio is equal to 1.55 for the boundary of that high-efficiency region containing the first five zones. This lens is shown in Fig. 2**b**.

The optical gain obtainable with this microlens is calculated according to formula (1). The evaluated system consists of the Petzval objective of 0.4λ rms wave aberration, square diffractive lenses and the square detectors placed on the back surface of the lenslet substrate.

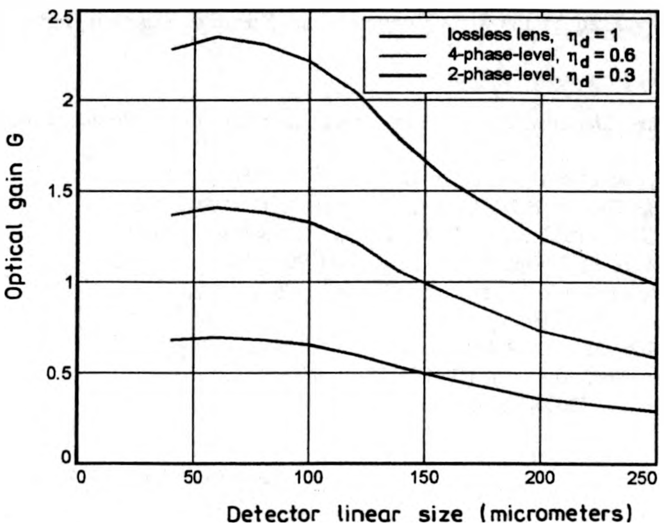

Fig. 7. Optical gain for the square detectors illuminated through the Petzval objective and diffractive microlenses.

Diffractive lens efficiency η consists of two factors: η_d and η_c, where $\eta_d = 0.6$ denotes the fraction of the incident energy transmitted into the first order (this value was taken from the electromagnetic calculations), and η_c is the fraction of the first-order optical flux incident onto the detector (η_c was evaluated using ray-tracing). Antireflection coatings can cover both the microlens array and the detector without microlenses, and thus they do not influence the optical gain. Microlens fabrication errors reduce the optical gain, but they were not taken into account in the calculations.

The achievable optical gain is shown in Fig. 7. Practical benefits from using diffractive lenses can only be obtained for more than 4-phase-level lenses.

4. Conclusions

The speed of infrared lenses for $\lambda \approx 9$ μm is limited by the electromagnetic reduction of diffraction efficiency of lens edge zones despite the long wavelength and high refractive indices. The minimum width of the four-phase-level Fresnel lens edge zone of acceptable diffraction efficiency $\eta_1 = 0.5$ is limited to 1.15λ. This lens is almost twice as slow as a refractive aplanatic lens of the same diameter and material. The optical gain achievable using first-order 4-phase-level diffractive lenses in the practical long-wave IR FPA systems is less than 2. Higher-diffraction-order designs do not allow for faster lenses and suffer from chromatic and angular selectivity.

Further work will be carried out with the matrices of refractive aplanatic microlenses made with the binary optics technology. Preliminary results for arrays of 50 μm pitch are promising [10].

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