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# Non-linear Electro- and Magnetooptic Effects in Light Modulation**

Some non-linear electrooptic and magnetooptic effects have been analyzed and their applicability to the light modulation shown.

## 1. Introduction

Optical properties of crystals immersed in external electric, magnetic and acoustic fields are of much importance from the technological and scientifical points of view. This problem has been the subject of a number of monographs [1–6]. The application of external fields leads to new and interesting optical phenomena, like linear electro-optic (Pockels) and linear magnetooptic (Faraday) effects, as well as to some quadratic, electrooptic (Kerr) and magnetooptic (Cotton–Mouten) effects. The linear electrooptic effects turned out to be the most applicable. They enable to design narrowband interference-polarizing light filters, optical light shutters, measuring devices for long distances and high voltages, as well as devices for film sound recording [7]. The recent development of light modulation techniques is due to the application of linear electrooptic effect [8–10]. It is obvious that the mastery of the light modulation techniques within the whole wavelength range is of a fundamental importance for telecommunication.

The purpose of the present paper is to give a phenomenological approach to light polarization and to discuss the possibility of its modulation, based on the simple electro- and magnetooptic quadratic effects [11].

## 2. Phenomenological theory

When a nonmagnetic material is subjected to a weak monochromatic electromagnetic radiation of electric field strength

$$\boldsymbol{E}(t) = \boldsymbol{E}(\omega, k)\exp[i(\boldsymbol{k}y - \omega t)] + zs, \quad (1)$$

the propagation of light in such a medium is completely described by the tensor of electric permittivity $\varepsilon_{ij}(\omega, \boldsymbol{k})$, which in the most general cases depends on the frequency $\omega$ and wave vector $\boldsymbol{k}$ of the incident radiation. If the medium is subjected to a static uniform electric fields of strength $\boldsymbol{E}^0$ as well as to a static magnetic field of strength $\boldsymbol{H}^0$, the electric permittivity tensor becomes a function of these fields.

If the spatial dispersion of the crystal and the fields $\boldsymbol{E}^0$ and $\boldsymbol{H}^0$ are small enough to be treated as slight perturbations, the electric permittivity tensor may be expanded in a series with respect to these perturbations. We thus obtain:

$$\begin{aligned}\varepsilon_{ij}(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0) &= \varepsilon_{ij}(\omega) + \Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_L + \\ &+ \Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_Q + \ldots, \quad (2)\end{aligned}$$

where

$$\Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_L = 4\pi\{\chi_{ijl}^{\omega}E_l^0 + i[\chi^{\omega\nabla}k_l + \eta_{ijl}^{\omega}H_l^0]\}, \quad (2a)$$

$$\begin{aligned}\Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_Q &= 2\pi\{\chi_{ijlm}^{\omega}E_l^0E_m^0 + \underline{\chi}_{ijlm}^{\omega}H_l^0H_m^0 + \varkappa_{ijlm}^{\omega}k_lk_m + \\ &+ i[2\eta_{ijlm}^{\omega}H_l^0E_m^0 + \underline{\underline{\chi}}_{ijlm}^{\omega}k_lE_m^0] + \ldots\}, \quad (2b)\end{aligned}$$

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** Work sponsored by the Physics Institute of the Polish Academy of Sciences, Warsaw.

denote the linear and quadratic changes in the electric permittivity tensor with respect to the external $\boldsymbol{E}^0$ and $\boldsymbol{H}^0$ fields, respectively. In a nonmagnetic medium and transparent with respect to the frequency $\omega$ the electric parmittivity tensor has to be Hermitian

$$\varepsilon_{ij}(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0) = \varepsilon_{ij}^*(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0), \quad (3a)$$

and invariant with respect to time inversion

$$\varepsilon_{ij}(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)^{t\to -t} = \varepsilon_{ij}(\omega, -\boldsymbol{k}, \boldsymbol{E}^0, -\boldsymbol{H}^0). \quad (3b)$$

Applying the above conditions to (2) we find

$$\begin{aligned}\Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_L &= 4\pi\{\chi_{(ij)l}^{\omega}E_l^0 + i[\chi_{[ij]l}^{\omega\nabla}k_l + \eta_{[ij]l}^{\omega}H_l^0]\},\\ \Delta\varepsilon_{ij}^{\omega}(\boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)_Q &\quad (4a)\\ = 2\pi\{\chi_{(ij)(lm)}^{\omega}E_l^0E_m^0 &+ \underline{\chi}_{(ij)(lm)}^{\omega}H_l^0H_m^0 + \\ &+ \varkappa_{(ij)(lm)}^{\omega}k_lk_m + \\ + i[2\eta_{[ij]lm}^{\omega}H_l^0E_m^0 &+ \underline{\chi}_{[ij]lm}^{\omega}k_lE_m^0] + \ldots\}, \quad (4b)\end{aligned}$$

where $\chi$ and $\eta$ denote polar (electrooptic suspectibility) and axial (magnetooptic suspectibility) tensors of third and fourth ranks; the indices, in which the tensor are symmetric and antisymmetric, are enclosed in the round and square brackets, respectively.

The Pockels effect, described in (4a) by $\chi_{(ij)l}^{\omega}E_l^0$ appears only in the 21 crystallographic classes without centre of symmetry; the Faraday effect defined in (4a) by $\eta_{[ij]l}^{\omega}H_l^0$ occurs in all crystals, while natural optical activity, described in (4a) by $\chi_{[ij]l}^{\omega\nabla}k_l$, is observed neither in classes with the centre of symmetry nor in the following classes without centre of symmetry: 4 mm ($C_{4v}$), 3 m($C_{3v}$), 6 mm ($C_{6v}$), $\bar{6}$ ($C_{3h}$), $\bar{6}$ m2 ($D_{3h}$) and $\bar{4}3$ m ($T_d$).

The Kerr effect and electrostriction, the Cotton–Mouton effect and magnetostriction, the change in electric susceptibility (proportional to the square of spatial heterogeneities of the light vector $\boldsymbol{E}(t)$, and the change in electric susceptibility induced by static electric and magnetic fields in the presence of spatial dispersion, are described analogically by the consecutive terms of equation (4b).

By applying the methods of group theory [12–14], we derived relations between the non-zero components of the tensors, occuring in Eqs (4a) and (4b). This allows to express the tensor of electric permittivity by means of appropriate elements of the susceptibility tensor for various configurations of the fields $\boldsymbol{E}^0$ and $\boldsymbol{H}^0$, with respect to the direction of light propagation. Let us assume that the analyzing light propagates along the $z$ direction, parallel to the $z$-axis of the crystal symmetry of the highest multiplicity. Then the difference between the diagonal components $\varepsilon_{xx}(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0) - \varepsilon_{yy} \times (\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)$ defines the optical birefringence, whereas the difference between the nondiagonal components $\varepsilon_{xy}(\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0) - \varepsilon_{xy} \times (\omega, \boldsymbol{k}, \boldsymbol{E}^0, \boldsymbol{H}^0)$ describes the optical rotation of the analyzing light.

The rotation angle of the polarization plane is equal to

$$\theta = \frac{4\pi^2}{n}\{\chi_{xyz}^{\omega\nabla}k_z + \eta_{xyl}^{\omega}H_l^0 + \eta_{xylm}^{\omega} \times H_l^0E_m^0\underline{\chi}_{xyzm}^{\omega}k_zE_m^0\}, \quad (5)$$

where $n = 1/2(n_+ + n_-)$ is the mean refractive index while $n_+$ and $n_-$ are the indices for right and left rotation, respectively.

## 3. Modulation of light polarization

Dielectric crystals exhibiting nonlinear electrooptic phenomana can be used to obtain internal as well as external phase frequency, amplitude [8–10] and polarization modulation. Let us consider the case of external light polarization modulation.

In a crystal subject to a static uniform magnetic field $\boldsymbol{H}^0$ the rotation of the polarization plane according to [5] will be equal to

$$\theta^0 = \frac{4\pi^2}{n}\{\chi_{xyz}^{\omega\nabla}k_z + \eta_{xyl}^{\omega}H_l^0\} \quad (6)$$

and constant in time. If, moreover, an electric signal of amplitude $S^{\omega_s}$ oscillating slowly in time with the circular frequency $\omega_s \ll \omega$ is applied, an additional time-variable rotation takes place equal to

$$\theta^t = \frac{4\pi^2}{n}\{\eta_{xylm}^{\omega}H_l^0 + \underline{\chi}_{xyzm}^{\omega}k_z\} \times S_m^{\omega_s}\sin\omega_s t. \quad (7)$$

Thus, the resultant rotation will be:

$$\theta = \frac{4\pi^2}{n}\{\chi^{\omega\nabla}_{xyz} k_z + \eta^{\omega}_{xyl} H^0_l + [\eta^{\omega}_{xylm} H^0_l + \\ + \chi^{\omega}_{xyzm} k_z] S^{\omega_s}_m \sin\omega_s t\}. \quad (8)$$

During a half-period of the electric signal the rotation changes from $\theta^0 + \theta^t$ to $\theta^0 - \theta^t$.

Assuming various configurations of the fields $H^0_l$ and $S^{\omega_s}_m$ with respect to the light propagation direction, it has been found in which crystals the light polarization modulation can occur. The results obtained are given in Tables 1 and 2.

Table 1

Crystallographic classes predicted to exhibit optical rotation proportional to $\boldsymbol{H}^0 \boldsymbol{S}^{\omega_s}$

| Direction $k$ | $S^{\omega_s}$ | $H^0$ | $\eta^{\omega}_{(ij)lm}$ | Crystallographic classes |
|---|---|---|---|---|
| $k_z$ | $S^{\omega_s}_z$ | $H^0_z$ | $\eta^{\omega}_{xyzz}$ | 1, 2, 3, 4, 6, 2 mm, 4 mm, 6 mm, 3 m |
| | $S^{\omega_s}_x$ | $H^0_z$ | $\eta^{\omega}_{xyzx}$ | 1, m |
| | $S^{\omega_s}_x$ | $H^0_y$ | $\eta^{\omega}_{xyyx}$ | 1, 2, 222, 4, $\bar{4}$, 422, $\bar{4}$2 m, 3, 32, 6, 622, 23, 432, $\bar{4}$3 m, $y$, $y_h$ |
| $k_x$ | $S^{\omega_s}_x$ | $H^0_x$ | $\eta^{\omega}_{yzxx}$ | 1, m |
| | $S^{\omega_s}_y$ | $H^0_x$ | $\eta^{\omega}_{yzxy}$ | 1, m |
| | $S^{\omega_s}_z$ | $H^0_y$ | $\eta^{\omega}_{yzyz}$ | 1, 2, 3, 4, 6, 2 mm, 4 mm, 6 mm, 3 m |

Table 2

Crystallographic classes predicted to exhibit optical rotation proportional to $\boldsymbol{k}\boldsymbol{S}^{\omega_s}$

| Direction $S^{\omega_s}$ | $k$ | $\chi^{\omega}_{(ij)lm}$ | Crystallographic classes |
|---|---|---|---|
| $S^{\omega_s}_x$ | $k_z$ | $\chi^{\omega}_{xyzx}$ | 1, 3, 32, $\bar{1}$, $\bar{3}$, 3 m, $\bar{3}$ m |
| | $k_x$ | $\chi^{\omega}_{yzxx}$ | " — " — " |
| $S^{\omega_s}_y$ | $k_z$ | $\chi^{\omega}_{xyzy}$ | 1, 3, $\bar{1}$, $\bar{3}$, |
| | $k_x$ | $\chi^{\omega}_{yzxy}$ | " — " — " |
| $S^{\omega_s}_z$ | $k_z$ | $\chi^{\omega}_{xyzz}$ | 1, 2, 3, 4, 6, $\bar{4}$, $\bar{1}$, m, 2/m, 4/m, $\bar{3}$, $\bar{6}$, 6/m |
| | $k_x$ | $\chi^{\omega}_{yzxz}$ | " — " — " — " |

In the classes (Tables 1 and 2) marked with a continuous line no optical activity occurs, whereas in those denoted with a broken line no magnetooptic Faraday effect may be expected.

From Tables 1 and 2 it follows that for classes 2 mm, 4 mm, 3 mm and 6 mm only a linear magnetooptic effect with the modulating component will occur provided, however, that the light travels along the $z$-axis.

## 4. Conclusion

The above analysis is aimed at drawing attention to some nonlinear electrooptic and magnetooptic effects, which may be proved applicable to the light modulation. In particular, the feasibility of light modulation based on changes in electric susceptibility induced by the simultaneous action of a modulating electric signals $\boldsymbol{S}^{\omega_s}$ and a static and uniform magnetic field $\boldsymbol{H}^0$, is indicated. This phenomenon should be well observable in the ferromagnetic crystals, especially in the adsorption band.

Recently, Kubota has observed a laser beam intensity-induced rotation of the analysing high polarization plane, in presence of a static magnetic field in ZnS and CdS semiconductors [15].

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The author wishes to express his true indebtedness to Professor Stanisław Kielich for suggesting the topics as well as for his highly valuable and helpful interest. The author wishes to thank Professor Jerzy Kołodziejczyk for enabling him to participate in the Second Polish-Czechoslovakian Optical Conference.

## Le rôle des phénomènes électro et magnéto-optiques non linéaires pour la modulation de la lumière

On a analysé certains phénomènes electro-et magnéto-optiques et on a montré comment ils peuvent être utilisés. On a examiné en particulier la possibilité de la modulation de la polarisation de la lumière en partant d'un changement de la susceptibilité électrique provoqué par une influence simultanée du signal électrique modulant $\boldsymbol{S}^{\omega_s}$ et du champ magnétique homogène constant $\boldsymbol{H}_0$.

**Роль нелинейных электро- и магнитооптических явлений в модуляции света**

Проанализированы некоторые электро- и магнитооптические явления; показана возможность их применения, особенно же возможность модуляции поляризации света на основе изменения электрической восприимчивости, вызванного одновременным влиянием электрического сигнала, модулирующего $S^{\omega_s}$ и постоянного однородного магнитного поля $\boldsymbol{H}_0$.

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*Received, March 14, 1974*