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Influence of Thermal Stresses Existing in Glass Disks on the Strehl Definition

In the paper the influence of the residual thermal stress in glass disks on the Strehl definition has been examined. The results obtained may be useful for determining the admissible birefringence in optical systems.

1. Introduction

Residual thermal stresses in the optical g'ass cause its birefringence. The first attempt to answer the question of birefringence magnitude-admissible for the optical glass used in production of various optical instruments was undertaken by KOMISSARUK [1]. The author has assumed, however, that the birefringence in the given region of the disk is proportional to the square distance from the disk centre. Such a distribution, for instance, may be observed when the temperature within the sample changes proportionally to the square disk radius. Our earlier measurements have shown that the distribution of birefringence in the disk with internal thermal stresses may be approximated by the function ϱ^3(ϱ — normalized radius in the polar coordinate system associated with the disk). Hence, the subject of this paper was to estimate the influence of the birefringence on the imaging quality in real disks suffering from residual stresses. The analysis has been restricted to the calculation of the Strehl definition on the optical axis, when the glass disk with internal thermal stress is inserted in a parallel light beam.

2. Deformation of the Wave Surface

The main stresses in the glass disks may be decomposed into radial and tangential components. This decomposition results, in turn, in splitting the incident plane wave $\sum_0$ into two waves of deformed surfaces $\sum_r$ and $\sum_\varphi$, respectively (Fig. 1).

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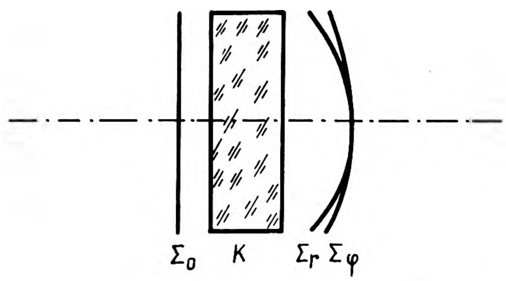

Fig. 1. Splitting of the incident plane wave $\sum_0$ into two deformed surfaces $\sum_r$ and $\sum_\varphi$ by a glass disk with internal thermal stresses

On both surfaces the vibrations of the light vector $\boldsymbol{E}$ are parallel and perpendicular to the disk radius. The deformations of the surfaces $\sum_r$ and $\sum_\varphi$ with respect the reference surface $\sum_0$ denoted by V_r and V_φ, respectively, may by found from the formulas

$$V_r(\varrho) = (n_r - n)d = \Delta n_r(\varrho)d,$$
$$V_\varphi(\varrho) = (n_\varphi - n)d = \Delta n_\varphi(\varrho)d, \quad (1)$$

where

d — thickness of the disk,
ϱ — normalized radius in the polar coordinate system associated with the disk ($\varrho_{\max} = 1$),
n — refractive index at the disk center,
n_φ, n_r — refractive indices for the rays polarized radially and tangentially,
λ — wavelength of the light.

In the paper [2] the curve $w(\varrho)$, describing the birefringence distribution in the disks, has been approximated as follows:

$$w(\varrho) = w(1)\varrho^3,$$

where

$w(\varrho) = n_r(\varrho) - n_\varphi(\varrho)$ — birefringence at a given place of the disk,

$w(1)$ — birefringence at the rim of the disk.

We have shown that the changes in the refractive indices are proportional to the birefringence,

$$\Delta n_r(\varrho) = m_r w(\varrho),$$
$$\Delta n_\varphi(\varrho) = m_\varphi w(\varrho), \tag{2}$$

where m_r and m_φ are material constants (the values of these constants for the selected glass sorts are given in [2]).

The relation $w(\varrho) = w(1)\varrho^3$ is not a unique function which approximates satisfactorily the set of experimental points. Similar property has been also stated in the following polynomial

$$w(\varrho) = w(1)(0.6\varrho^4+0.4\varrho^2). \tag{3}$$

The approximate equally well the set of experimental points within the error of measurement. It should be noted, however that for further calculations the formula (3) is more convenient. If we assume that $w(\varrho)$ is described by (3) then the eq. (2) is not fulfilled rigorously, but the deviation from linearity is negligible. Anyway any deviation has not been observed when determining the coefficients m_r and m_φ experimentally. In view of eqs. (2) and (3) the equations (1), take the form

$$V_r(\varrho) = w(1)\, m_r d(0.6\varrho^4+0.4\varrho^2) = V_r(1)(0.6\varrho^4+0.4\varrho^2),$$
$$V_\varphi(\varrho) = w(1) m_\varphi d(0.6\varrho^4+0.4\varrho^2) = V_\varphi(1)(0.6\varrho^4+0.4\varrho^2). \tag{4}$$

In the sequel the wave surface deformations will be described with the help of $W(\varrho)$ and $R(\varrho)$, defined as follows

$$W(\varrho) = \frac{1}{2}[V_r(\varrho)+V_\varphi(\varrho)],$$
$$R(\varrho) = V_r(\varrho) - V_\varphi(\varrho). \tag{5}$$

From the equations (4) and (5) and the condition $m_r = m_\varphi + 1$ (see[2]) it follows that

$$W(\varrho) = mR(\varrho) \tag{6}$$

and in particular

$$W(1) = mR(1), \tag{6a}$$

where

$$m = 1/2(m_r+m_\varphi).$$

3. Strehl Definition

Let us assume that the glass disk inserted into an optical system is struck by a light beam travelling parallely to the optical axis and polarized in the OY direction. It is worth noting that this case is general one, due to rotational symmetry. If we denote by A the amplitude of the wave incident on the disk, the components of disturbance after passing through the plate (see Fig. 2) are

$$A_r = A\cos\varphi \cdot e^{ikV_r}$$
and
$$A_\varphi = A\sin\varphi \cdot e^{ikV_\varphi} \tag{7}$$

respectively. The disturbance within the whole pupil is coherent (fixed polarization plane) and hence, the resulting vector on the X-axis is equal to

$$S_x = A\int_0^1\int_0^{2\pi} \sin\varphi\cos\varphi\,[e^{ikV_r} - e^{ikV_\varphi}]\varrho d\varrho d\varphi = 0,$$

while on the Y-axis is equal to

$$S_Y = A\int_0^1\int_0^{2\pi} \cos^2\varphi\, e^{ikV_r}\varrho d\varrho d\varphi + A\int_0^1\int_0^{2} \sin^2\varphi\, e^{ikV_\varphi}\varrho d\varrho d\varphi = A\pi\Big[\int_0^1 e^{ikV_r}\varrho d\varrho + \int_0^1 e^{ikV_\varphi}\varrho d\varrho\Big]$$

respectively.

Hence, the resulting intensity J (Strehl definition) is

$$J = |S_X+S_Y|^2 = C\Big(\int_0^1 \varrho e^{ikV_r}d\varrho + \int_0^1 \varrho e^{ikV_\varphi}d\varrho\Big)\times \Big(\int_0^1 \varrho e^{-ikV_r}d\varrho + \int_0^1 \varrho e^{-ikV_\varphi}d\varrho\Big), \tag{8}$$

where C — normalizing constant.

It should be expected that the change in the references sphere will improve the quality of imaging. Therefore, we assume

$$V_r'(\varrho) = V_r(1)(0.6\varrho^4+0.4\varrho^2)+D\varrho^2,$$
$$V_\varphi'(\varrho) = V_\varphi(1)(0.6\varrho^4+0.4\varrho^2)+D\varrho^2, \tag{9}$$

where D — defocusing parameter,

then

$$R'(\varrho) = R(1)(0.6\varrho^4+0.4\varrho^2),$$
$$W'(\varrho) = W(1)(0.6\varrho^4+0.4\varrho^2)+D\varrho^2 = W(1)(0.6\varrho^4+D'\varrho^2), \tag{10}$$

where

$$D' = 0.4+\frac{D}{W(1)}.$$

By transforming (8) and taking account of (6a) and (10) we obtain the following formula for the Strehl definition.

$$J = 4\left\{\left[\int_0^1 \varrho\cos\frac{k}{2}R(1)(0.6\varrho^4+0.4\varrho^2)\times\right.\right.$$

$$\left.\times\cos kmR(1)(0.6\varrho^4+D'\varrho^2)\,d\varrho\right]^2+\left[\int_0^1 \varrho\cos\frac{k}{2}R(1)\times\right.$$

$$\left.\left.\times(0.6\varrho^4+0.4\varrho^2)\sin kmR(1)(0.6\varrho^4+D'\varrho^2)d\varrho\right]^2\right\}. \quad (11)$$

In the case, when the difference in optical paths at the rim of the disk is small ($R(1) \leqslant \lambda/4$), the trigonometric functions in the formula (11) may be replaced by the first terms of the expansion. By neglecting terms of the order higher then two we have

$$J = 1-\frac{3}{50}k^2R^2(1)-2k^2R^2(1)\times$$

$$\times m^2\left(\frac{4}{250}+\frac{D'}{20}+\frac{D'^2}{24}\right). \quad (12)$$

By requiring the derivative of the expression (12) to be equal to zero the condition for the maximum intensity value (at the plane of best imaging) is obtained as

$$D' = -0.6. \quad (13)$$

Hence

$$D = -W(1).$$

In view of (13) the formula (12) has the form

$$J_{max} = 1-k^2R^2(1)\frac{30+m^2}{500}, \quad (14)$$

where J_{max} denotes the Strehl definition in the plane of best focusing.

The relation (14) being an approximate formula is accurate enough for practical applications. E.g. for $m = 6$ and $R(1) = 0.1\lambda$ obtained from (14) is by about 1% smaller than the exact value (calculated from the formula (11), for $R(1) = 0.25\lambda$ the error approaches 4%.

The estimation of admissible birefringence in the optical instruments may be based on the formula (14). According to the Rayleigh cryterion we may admit that the value of J drops to 0.8. Setting $m = 6$, which is a value typical of optical glasses [2], we get the condition $R(1)_{max} = \lambda/5$. In practice, however, the wave aberrations of an optical system are conditioned by many other factors (e.g. by technological, constructional, and mounting errors). Therefore, the admissible value $R(1)$ is considerably lower. For high birefringences (i.e. such, that the difference in the optical path at the disk rim $R(1) > \lambda/4$ the numerical calculations performed according to the formula (11) were carried out with the help of a ODRA 1204 computer. The results obtained for $m = 6$, are presented in the graphs (Figs. 2–5). These results are correct only in case,

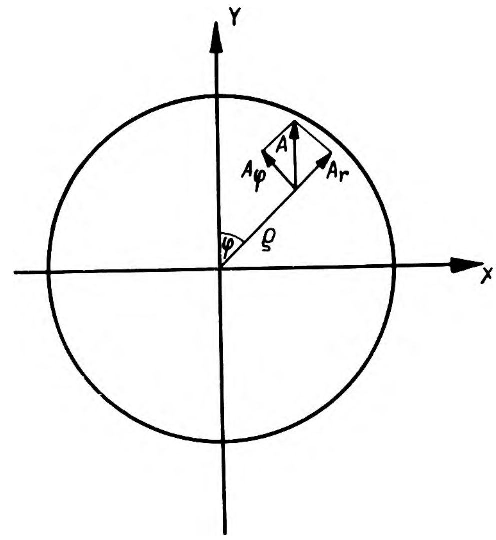

Fig. 2. The change in state of polarization of a plane wave after passing a glass disk thermally stressed.
A – amplitude of the plane wave incident on the glass disk and polarized in the O_x direction, A_r, A_φ – local amplitudes of the wave polarized radially and tangentially, after passing throught the disk

when the glass disk is illuminated by a monochromatic and parallel light beam exclusively. The graph 3 presents the dependence of the Strehl definition J on the defocusing parameter D, whereas the dependence of the Strehl definition upon the difference in optical paths at the disk rim $R(1)$ is shown in graph 4. From the both graphs it follows that the change in focusing of the system improves considerably the imaging quality (it increases the Strehl definition).

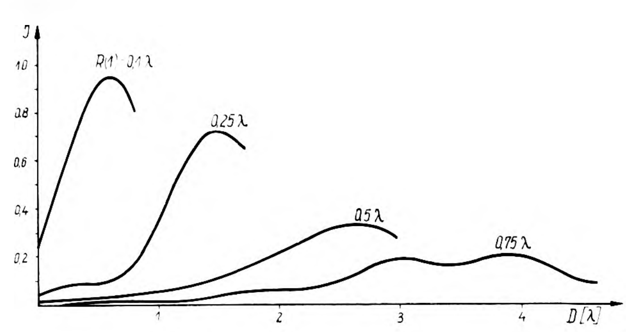

Fig. 3. The dependence of Strehl definition on the refocusing parameter D for the selected values of the optical path differences at the rim of the disk $R(1)$ ($m = 6$)

The values of the defocusing parameter, which give the maximum Strehl definition for the fixed $R(1)$ are presented in graph 5.

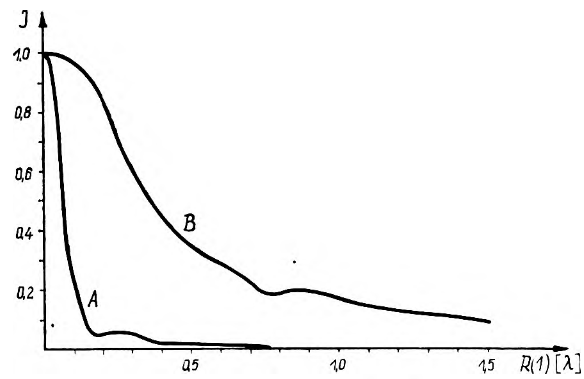

Fig. 4. The dependence of the Strehl definition on the optical path difference at the disk rim $R(1)$ the curve A corresponds to the Gaussian plane ($D = 0$), the curve B corresponds to the best focus plane (calculations made for $m = 6$)

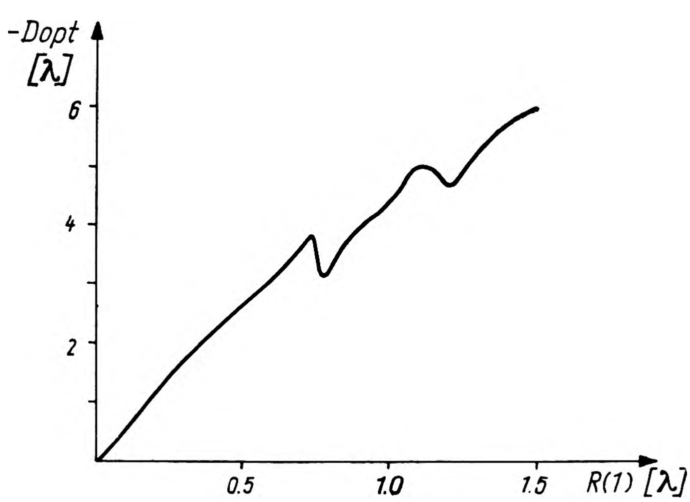

Fig. 5. The relation between D_{opt} and optical path difference at the rim of the disk $R(1)$ (for $m = 6$). D_{opt} — value of the parameter D which gives the maximum Strehl definition for fixed $R(1)$

4. Conclusion

The influence of the residual thermal stresses in glass disks on the Strehl definition has been analysed. For low birefringence value ($R(1) \leqslant \lambda/4$) an analytic formula has been obtained, while the calculations for higher birefringence being made on a computer.

Influence des contraintes thermiques dans les roulettes de verre sur la luminosité Strehl

On a présenté l'influence des contraintes thermiques résiduelles dans les roulettes de verre sur la luminosité Strehl. Les résultats obtenus peuvent être utilisés à la mesure de la tolérance de la biréfringence dans les ensembles optiques.

Влияние температурных напряжений в стеклянных дисках на светлоту Штреля

Представлено влияние остаточных температурных напряжений в стеклянных дисках на светлоту Штреля. Полученные результаты могут быть использованы при допуске двухпреломления в оптических системах.

References

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