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Analysis of an Electron Mirror by Using the Matrix Notation

A method for analysing an electron mirror of given electro-optical parameters is discussed. The mirror is divided into the electric lens region and proper mirror. Electron paths in the matrix notation are presented. Discussion on the usefulness of the presented formulae discribing the mirror surface is based on examples.

1. Introduction

In many devices of electronic optics, e.g. in installations for micro-machining with the use of electron beams, one of the most important problems concerns the reduction of the electro-optical aberration, especially the spherical aberration. Adequate correction of this aberration can not be achieved by using electron lenses only, because of the constancy of the sign of the spherical aberration coefficient. The correction is, hower, possible after including an electron mirror to the system. However, it is difficult to analyse the properties of an electron mirror. Some authors propose to divide the mirror into an area at the entrance, witch is of electron lens character, and the proper mirror, that is the zeroth equipotential surface the cross-section of which is described approximately by a polynominal of the fourth order. An analysis of the electron beam path in the area of the lens and the proper mirror has been carried out by using a matrix notation. Relations determining the shape of the mirror surface which insures correction of given spherical aberration are presented. The usefulness of the presented method of analysis has been discussed on examples.

2. Electro-optical scheme of an electron mirror

The electro-optical mirror is an electro-optical element which by reversing the direction of an electron beam offers the possibility of obtaining an image. In the formalism of light optics it does not correspond to direct reflection but to bending of a light beam in a non-uniform transparent medium, linked with a total internal reflection. Making use of this analogy one should distinguish — in the real *m* of the mirror — a region functioning as a lens in which the bending of electron beams occurs and

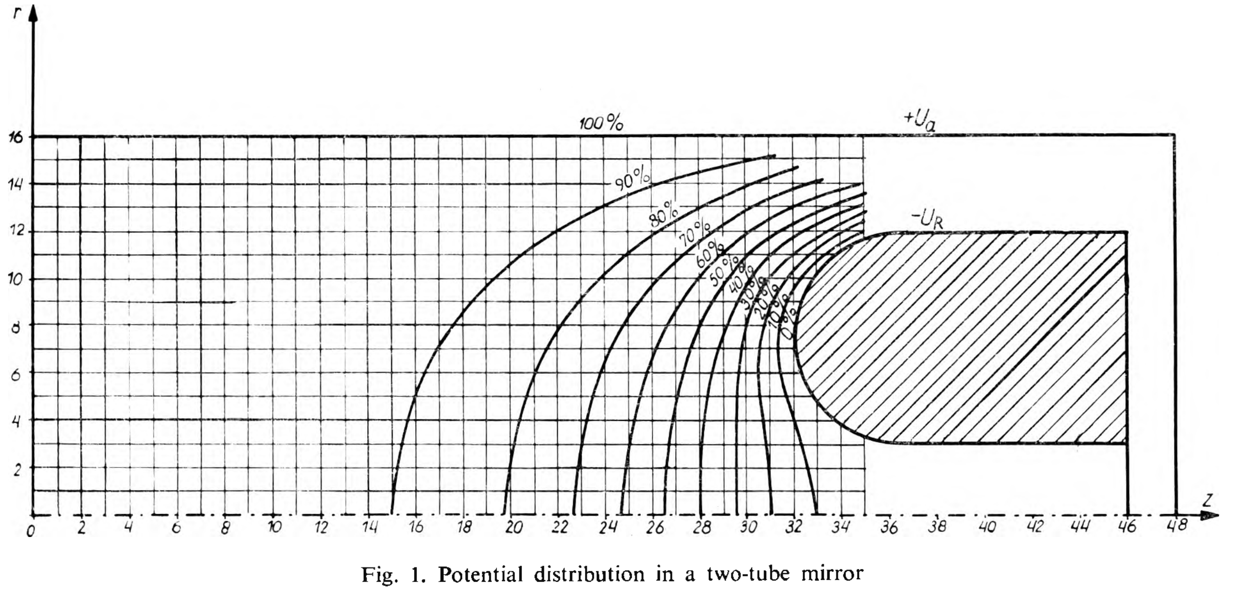

Fig. 1. Potential distribution in a two-tube mirror

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a surface at which the change of the direction of electron movement (electron beam) occurs, i.e. the mirror surface. This scheme can be illustrated by an example of a two-tuble electron mirror [1] presented in figs 1 and 2.

As follows from the electron path shape, the lens region of the mirror constitutes an immersion diverging lens, whereas the mirror surface is approximately in line with the equipotential surface 0V. Basic properties of an electron mirror can be determined in accordance with principles of geometrical optics by specifying the positions of the foci and principal planes. Basic optical parametres of two-tube electron mirror are shown in fig. 3.

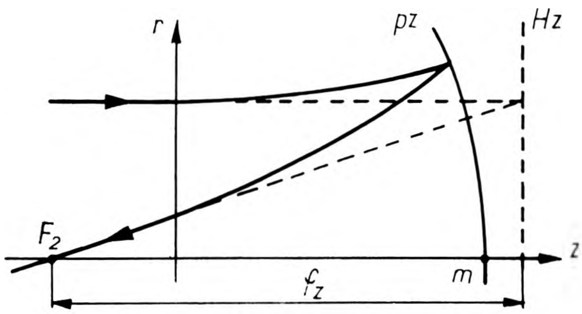

Fig. 2. Electron path in a two-tube mirror

3. Matrix notation for the lens region

Describing the basic optical properties of the lens region by indicating the positions of foci and principal planes facilitates the geometrical construction of the image by using principal rays but is inconvenient for analytical calculations. As a more convenient form of describing these properties the matrix notation may be proposed.

If, for example, an area of field is given, its length being m, (fig. 3) and it acts on passing electrons according to the gaussian optics (i.e. aberrations omitted), then it suffices to know entrance and exit parameters (the radius and path inclination) of two linearly independent paths in order to determine the elements of matrix $[A]$ which describes the properties of this area, i.e. allows the transition from the entrance to exit parameters. In such a case, denoting the entrance radii by r_{p1} and r_{p2} and the entrance inclination by $r_p' = (dr/dz)_p$ for this two paths, and similarly the exit radii and inclinations $r_{k1}, r_{k2}, r_{k1}', r_{k2}'$, one may obtain a set of equations:

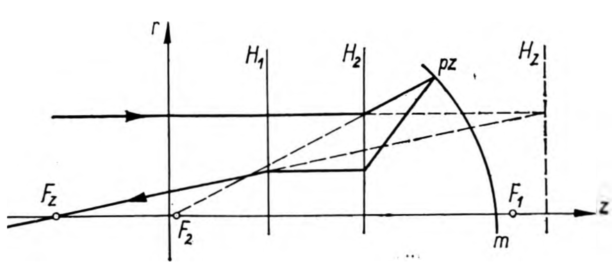

Fig. 3. Arrangement of principal planes and foci in a two-tube mirror

$$\begin{bmatrix} r_{k_1} \\ r_{k_1}' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} r_{p_1} \\ r_{p_1}' \end{bmatrix}$$
$$\begin{bmatrix} r_{k_2} \\ r_{k_2}' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} r_{p_2} \\ r_{p_2}' \end{bmatrix}. \tag{1}$$

Solution of the above set leads to expressions for the matrix elements:

$$a = \frac{r_{k_1} r_{p_2}' - r_{k_2} r_{p_2}'}{r_{p_1} r_{p_2}' - r_{p_2} r_{p_1}'},$$
$$b = \frac{r_{p_2} r_{k_1} - r_{k_2} r_{p_1}}{r_{p_1} r_{p_2}' - r_{p_2} r_{p_1}'},$$
$$c = \frac{r_{k_1}' r_{p_2}' - r_{k_2}' r_{p_1}'}{r_{p_1} r_{p_2}' - r_{p_2} r_{p_1}'}, \tag{2}$$
$$d = \frac{r_{p_2} r_{k_1}' - r_{k_2}' r_{p_1}}{r_{p_1} r_{p_2}' - r_{p_2} r_{p_1}'}.$$

The matrix of the lens region thus determined enables to calculate — in accordance with eqs. (1) — the coordinates of the path at the exit for any entrance coordinates.

The description of the lens region in matrix form or indication of the foci and principal planes differ in form only but contain the same information. It is possible, therefore, to determine the mutual relationships between the two forms of description.

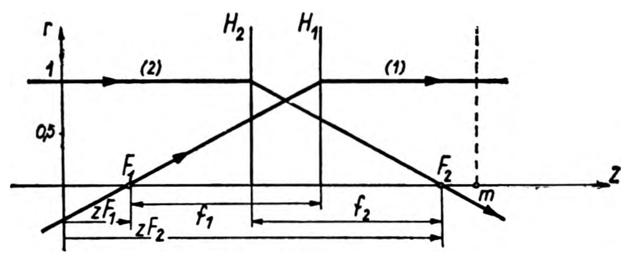

Fig. 4. Arrangement of principal points and paths of principal rays in a thick lens

In fig. 4 the lens region of a length m is shown. Its properties are characterized by marked positions of the principal planes H_1 and H_2 and foci F_1 and F_2. The paths of the principal rays (1) and (2) are also shown in the figure. If one accepts the path coordinates at the entrance and exit of the region according to fig. 4, the set of eqs. (1) assumes the form:

$$\begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} = \begin{bmatrix} a & b \\ \\ c & d \end{bmatrix} \begin{bmatrix} \dfrac{-z_{F_1}}{f_1} \\ \dfrac{1}{f_1} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{m-z_{F_2}}{f_2} \\ \dfrac{1}{-f_2} \end{bmatrix} = \begin{bmatrix} a & b \\ \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix}. \tag{3}$$

The solution of these equations enables us to determine the relations between the matrix elements and the position of the lens principal points as follows:

$$a = \frac{z_{F_2}-m}{f_2}, \qquad b = f_1 + \frac{z_{F_1}(z_{F2}-m)}{f_2},$$
$$c = \frac{-1}{f_2}, \qquad d = \frac{-z_{F_1}}{f_2}. \tag{4}$$

Or in the reversed form:

$$f_1 = b - \frac{ad}{c}, \qquad f_2 = -\frac{1}{c},$$
$$z_{F_1} = \frac{d}{c}, \qquad z_{F_2} = m - \frac{a}{c}. \tag{5}$$

The matrix form of the notation may be illustrated by an example of a two-tube mirror from fig. 1. For a relative voltage of the retarding electrode $U_r/U_a = -0.17$ the entrance and exit coordinates of two paths, which were calculated numerically [2, 3] in the mirror lens region of length $m = 30$ meshes of the calculation network, enable us to compose the following equations:

$$\begin{bmatrix} 0.1365 \\ 0.009463 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1.467 \\ 0.1533 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}. \tag{6}$$

The solution of these equations is the matrix of the lens region

$$A = \begin{bmatrix} 1.365 & 26.61 \\ 0.09463 & 2.876 \end{bmatrix}. \tag{7}$$

Making use of expressions (5) one may obtain the positions of the principal points.

$$f_1 = -14.89, \qquad f_2 = -10.7,$$
$$z_{F_1} = 30.4, \qquad z_{F_2} = 15.6. \tag{8}$$

The resulting focal length of the mirror was in this case equal to $f_z = 136.8$.

Analogous calculations carried out for the lens region of the same mirror with $U_r/U_a = -0.25$ same the following results:

$$[A] = \begin{bmatrix} 1.438 & 28.10 \\ 0.09835 & 2.836 \end{bmatrix} \quad \begin{matrix} f_1 = -13.4, & f_2 = -10.18 \\ Z_{F_1} = 29.2, & z_{F_2} = 15.8 \end{matrix}$$

whereas the resultant focal length of the mirror $f_z = -25$.

The fact should be noted here that with relatively great changes in the voltage between mirror electrodes, the properties of the lens region remained almost unchanged; whereas the resulting parameters of the mirror changed radically. One may, therefore, conclude that the shape of the mirror surface has a decisive influence on the parameters of the mirror, and that the properties of the lens region and the mirror surface can, to a certain extent, be treated separately.

4. Determination of the shape of the mirror surface

In an electron mirror the electron beam — after crossing the lens region — encounters the mirror surface at which it is reflected changing its direction and inclination. In order to find the relation between the shape of the mirror surface and the slope of the reflected ray, one may use the diagram shown in fig. 5.

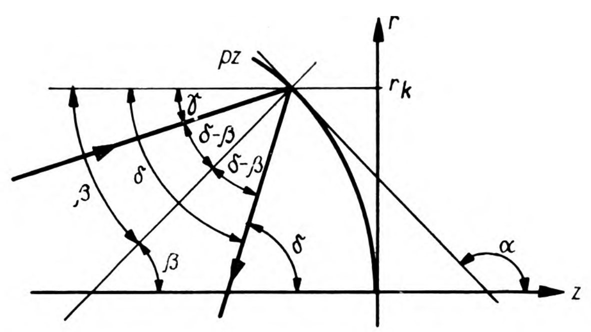

Fig. 5. Diagram of an electron reflection at the mirror surface

Denoting the radius and the path slope of incident electron at the reflection point by r_k, r_k', and the reflected electron by r_z, r_z', one obtains:

$$r_k = r_z,$$
$$r_k' = \tan\gamma, \qquad r_z' = \tan\delta. \tag{10}$$

The slope of tangent to the mirror surface at this point may be denoted as:

$$s = \tan\alpha, \tag{11}$$

and the normal slope:

$$n = -\frac{1}{s} = \tan\beta. \tag{12}$$

As follows from fig. 5, the angles marked there comply with the equations:

$$\delta = 2\beta - \gamma. \tag{13}$$

Hence

$$\tan\delta = \tan(2\beta-\gamma) = \frac{\tan 2\beta - \tan\gamma}{1+\tan 2\beta \tan\gamma}. \tag{14}$$

In turn

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}. \tag{15}$$

By inserting relations (10), (12), and (15) into formula (14) and rearranging them, one can obtain:

$$\tan\delta = r_z' = \frac{-2s - r_k' s^2 + r_k'}{s^2 - 2sr_k' - 1}. \tag{16}$$

Now we may consider the problem of determining the mirror surface shape that would allow to obtain specified values of the electron path parameters r_w, r_w' at the exit of the mirror with assumed properties of the lens region written in the form of matrix [A]. The electron mirror represented by the lens region of length m described by matrix [A] and mirror surface pz is shown in fig. 6.

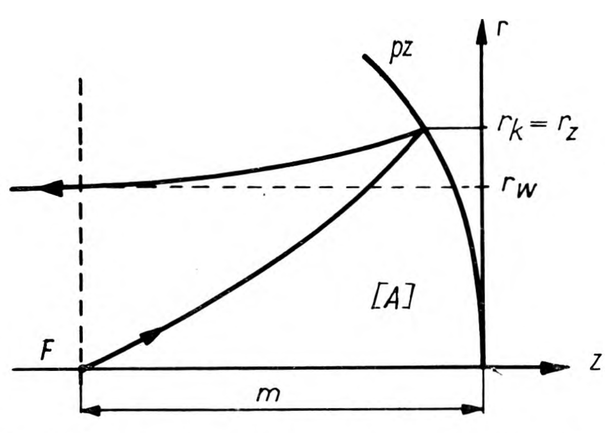

Fig. 6. Electron mirror with focus on the lens region boundary

The paths of electrons entering the mirror region intersect each other at point F which lies at the border of the lens region, the paths of outgoing electrons being parallel to the axis. The parameters of electron ray incident upon the mirror surface are determined by the relation:

$$\begin{bmatrix} r_k \\ r_k' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} r_p \\ r_p' \end{bmatrix}. \tag{17}$$

Since $r_p = 0$, we obtain

$$\begin{bmatrix} r_k \\ r_k' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ r_p' \end{bmatrix} = \begin{bmatrix} br_p' \\ dr_p' \end{bmatrix}. \tag{18}$$

The process of electron reflection at a mirror surface may be interpreted as the action of an "operator" Z_s which conserves the radius $r_z = r_k$ at the reflection but changes the inclination r_k' for r_z' in accordance with relation (16).

The action of the "operator" Z_s may, therefore, be written down in the following form:

$$Z_s \begin{bmatrix} r_k \\ r_k' \end{bmatrix} = Z_s \begin{bmatrix} br_p' \\ dr_p' \end{bmatrix} = \begin{bmatrix} br_p' \\ \dfrac{s^2 dr_p' + 2s - dr_p'}{1+2sdr_p' - s^2} \end{bmatrix}. \tag{19}$$

Since the entrance electron path — following its reflection — constitutes the beginning of the exit path of parameters r_w, $r_w' = 0$, one may also write:

$$\begin{bmatrix} br_p' \\ \dfrac{s^2 dr_p' + 2s - dr_p'}{1+2sdr_p' - s^2} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} r_w \\ 0 \end{bmatrix} = \begin{bmatrix} ar_w \\ cr_w \end{bmatrix}. \tag{20}$$

It follows from the above equation that:

$$\frac{s^2 dr_p' + 2s - dr_p'}{1+2sdr_p' - s^2} = cr_w = \frac{bc}{a} r_p'. \tag{21}$$

Expression (21) can be transformed to a second order equation:

$$s^2 + 2s\frac{a - bcdr_p'^2}{r_p' V} - 1 = 0, \tag{22}$$

where

$$V = ad + bc. \tag{23}$$

Solution of this equation has the form:

$$s_{1,2} = \frac{dr}{dz} = \frac{bcdr_p'^2 - a}{r_p' V} \pm \sqrt{1 + \left(\frac{a - bcdr_p'^2}{r_p' V}\right)^2}. \tag{24}$$

Inserting $r = r_k = br_p'$ and rearranging the solution with a physical sens, results in:

$$\frac{dr}{dz} = \frac{cdr^2 - ab}{rV}\left[1 + \sqrt{\left(\frac{rV}{ab - cdr^2}\right)^2 + 1}\right]. \tag{25}$$

Now, if we assume

$$\left(\frac{rV}{ab - cdr^2}\right)^2 \ll 1 \tag{26}$$

and apply the approximate formula:

$$\sqrt{1+x} \simeq 1 + \frac{x}{2}, \quad |x| \ll 1, \tag{27}$$

then eq. (25) assumes the following form:

$$\frac{dr}{dz} = 2\frac{cdr^2 - ab}{rV} - \frac{1}{2}\frac{rV}{ab - cdr^2}. \tag{28}$$

Relation (28) is a differential equation for a mirror surface. Its solution may the following approximate form:

$$z = Ar^2 + Br^4. \tag{29}$$

Hence

$$\frac{dz}{dr} = 2Ar + 4Br^3. \tag{30}$$

Comparison of (28) and (30) leads to the relation:

$$\frac{1}{2Ar+4Br^3} = 2\frac{cdr^2-ab}{rV} - \frac{1}{2}\frac{rV}{ab-cdr^2}, \tag{31}$$

from which the constants A and B can be found:

$$A = -\frac{1}{4}\frac{V}{ab},\ B = -\frac{1}{8}\frac{Vcd}{a^2b^2} - \frac{1}{32}\frac{V^3}{a^3b^3}. \tag{32}$$

The second term in the expression for B follows from not allowing for the influence of the mirror surface size on the radius r_k (the mirror surface was assumed as almost flat) and should, in the end, be omitted. Thus in the first two expension terms approximation the equation for the mirror surface assumes the form:

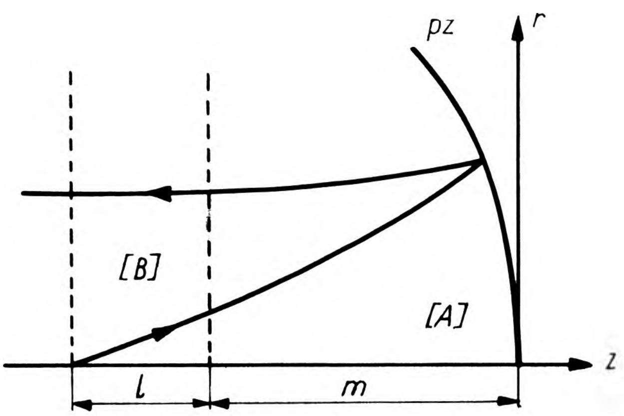

Fig. 7. Electron mirror with the drift space

$$z = -\frac{1}{4}\frac{ad+bc}{ab}r^2 - \frac{1}{8}\frac{(ad+bc)cd}{a^2b^2}r^4. \tag{33}$$

The above given equation of the mirror surface applies only to a special case of mirror — its focus F lies on the boundary of the lens region. The applicability range of this equation may be extended to cases of the mirror focus being at the distance l from the lens region boundary. In accordance with fig. 7 the region of length l is a drift space with properties described by matrix $[B]$:

$$[B] = \begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix} \tag{34}$$

The properties of both regions jointly are described by matrix $[C]$ which is a product of the lens region matrix $[B]$ and that of the drift space

$$[C] = \begin{bmatrix} a^* & b^* \\ c^* & d^* \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & l \\ 0 & 1 \end{bmatrix}. \tag{35}$$

It now suffices to insert the elements of matrix $[C]$ instead of $[A]$ into eq. (33) in order to obtain an equation for the mirror surface for an arbitrary focus location.

The electron mirror is not an imaging element in the full sens of the word, as it allows only to obtain images of point objects and this merely for a specified location of the image and object. Any change of the location requires a correction of the shape of the mirror surface in order to avoid image errors.

The applicability range of formula (33) should, for this reason, be extended to the case when the object and the image are at a finite distance from the mirror, as shown in fig. 8. In this case point object P lies on the mirror axis at lens region boundary, whereas the point image 0 at the distance l^* from this region. The parameters of the entrance path at the mirror surface are determined, as before, by eq. (18) and as before, eq. (19) determines the action of the mirror Z_s "operator" on the parameters by reflection. The parameters of the exit paths at the mirror surface and at the mirror exit are linked the equation:

$$\begin{bmatrix} r_z \\ r_z' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} r_w \\ r_w' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} r_w \\ Er_w \end{bmatrix} = \begin{bmatrix} (a+bE)r_w \\ (c+dE)r_w \end{bmatrix} \tag{36}$$

where:

$$E = \frac{r_w'}{r_w} = \frac{1}{l^*}. \tag{37}$$

In accordance with the condition of coincidence of the entrance path after reflection formula (19) should be compared with (36) at the beginning of the exit path. This results in:

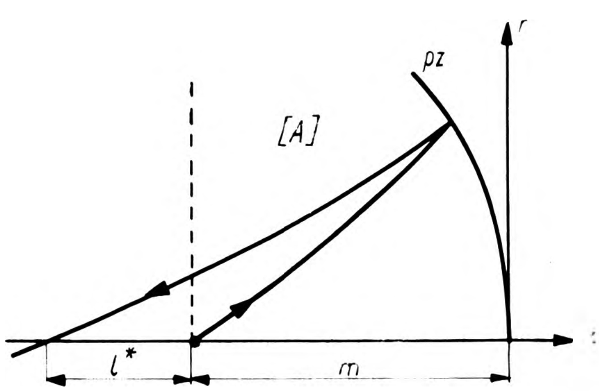

Fig. 8. Ray paths in an electron mirror with a finite image distance

$$\begin{bmatrix} br_p' \\ \dfrac{s^2 dr_p' + 2s - dr_p'}{1+2sdr_p' - s^2} \end{bmatrix} = \begin{bmatrix} (a+bE)r_w \\ (c+dE)r_w \end{bmatrix}. \tag{38}$$

The above equation may be transformed to a second order equation:

$$s^2\left[dr_p' + \frac{b(c+dE)}{a+bE} r_p'\right] + s\left[2 - 2\frac{b(c+dE)d}{a+bE} r_p'^2 + \right.$$
$$+\left[-dr_p' - \frac{b(c+dE)}{a+bE} r_p'\right] = 0 \tag{39}$$

from which, by the some procedure as the previous one, an equation for the mirror surface may be obtained:

$$z = -\frac{1}{4}\frac{(a+bE)d+b(c+dE)}{(a+dE)b} r^2 -$$
$$-\frac{1}{8}\frac{[(a+bE)d+b(c+dE)](c+dE)d}{(a+bE)^2 b^2} r^4. \tag{40}$$

When object P is located not at the lens region boundary but at an arbitraty distance l from the region, then according to (35) the elements of matrix $[C]$ should be substituted for those of matrix $[A]$ into the given equation of the mirror surface.

5. Allowance for spherical aberration of an electron mirror

Electron mirrors may be characterized by a great spherical aberration. As regards the two-tube mirror shown in fig. 1 numerical calculations indicate that sperical aberration of the lens region is not great. Hence it may be infered that the shape of the mirror surface decides the spherical aberration error. Assuming a specified spherical aberration error of the mirror and neglecting the errors of the lens region, the shape of the surface may be calculated as follows:

Referring the spherical aberration error to the object we may, according to fig. 9, assume that it displaces the object from the lens region by a distance Δl dependent on the slope of the slope of the entrance path. Treating the region of length Δl as a drift space described by matrix $[B]$, we may describe that region together with the lens region, according to (35), by matrix $[C]$.

$$[C] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & \Delta l \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & a\Delta l+b \\ c & c\Delta l+d \end{bmatrix}. \tag{41}$$

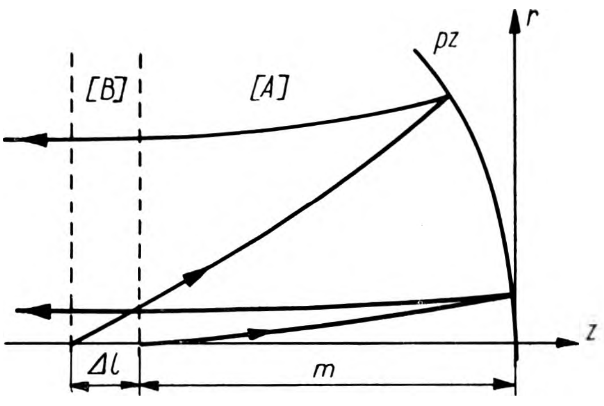

Fig. 9. Ray paths in a electron mirror with a spherical aberration error

The magnitude of the spherical aberration error may be defined by the relation:

$$\Delta l = C_s r_p'^2, \tag{42}$$

where C_s is the coefficient of the spherical aberration. The slope of the entrance path r_p' is connected with the radius $r = r_k$ of the reflection on the mirror surface by the equation:

$$\begin{bmatrix} r_k \\ r_k' \end{bmatrix} = \begin{bmatrix} a & a\Delta l+b \\ c & c\Delta l+d \end{bmatrix}\begin{bmatrix} 0 \\ r_p' \end{bmatrix}. \tag{43}$$

Hence:

$$r = (a\Delta l+b)r_p', \tag{44}$$

whereas relation (42) assumes the form of a third order equation:

$$\Delta l = C_s \frac{r^2}{(a\Delta l+b)^2}. \tag{45}$$

Utilizing (16) and (41) together with an equation analogous to (20) for entrance paths and proceeding as before we obtain the following equation for the mirror surface:

$$z = -\frac{1}{4}\frac{(ad+bc)+2ac\Delta l}{ab+a^2\Delta l} r^2 -$$
$$-\frac{1}{8}\frac{[(ad+bc)+2ac\Delta l](cd+c^2\Delta l)}{(ab+a^2\Delta l)^2} r^4. \tag{46}$$

This equation may be solved numerically by inserting relation (45) into it.

Within the range where the spherical aberration is small enough to fulfil the inequality

$$|a\Delta l| \ll |b| \tag{47}$$

the relation (45) may be written as:

$$\Delta l = C_s\left(\frac{r}{b}\right)^2. \tag{48}$$

Then, by making some simplifications, the mirror surface equation can be rearranged to:

$$z = -\frac{1}{4}\frac{ad+bc}{ab}r^2 - \frac{1}{8}\frac{(ad+bc)cd}{a^2b^2}r^4 + \\ + \frac{C_s(ad-bc)}{4b^2}r^4. \tag{49}$$

It should be that the first two terms of the equation are identical with equation (33) and describe a mirror surface free of aberration. The third term is a correction term that increases or decreases the mirror surface curvature depending on the sing of the coefficient of spherical aberration.

6. Conclusions and examples of the matrix analysis of an electron mirror

The discussion presented here may be useful for determining the shape of mirror electrodes in order to achieve specified electrooptical parameters. For this purpose, the field intensity distribution should be determined in its region, next the parameters of the lens region should be established by using numerical methods. These parameters written down in a matrix form, and assumed to be constant for small field changes, enable us to determine the shape of the mirror surface for arbitrary exit parameters of the mirror.

For example, the lens region matrix of the two-tube mirror mentioned in section 2 has, for the relative voltage of the retarding electrode $U_r/U_a = -0.17$, and in accordance with earlier performed calculations, the form:

$$[A] = \begin{bmatrix} 1.365 & 26.61 \\ 0.09463 & 2.876 \end{bmatrix}. \tag{50}$$

When denoting by l^* the distance between the image and the lens region, and also by l the distance between the region and the object, the case of converging a divergent beam send forth by a point on the lens region boundary into a parallel beam shown in fig. 6, corresponds to distances $l^* = \infty$ and $l = 0$. The equation of the mirror surface obtained by inserting the elements of matrix $[A]$ into eq. (33) has the form:

$$z = -4.54\cdot 10^{-2}r^2 - 1.74\cdot 10^{-4}r^4. \tag{51}$$

The shape of the mirror surface described by the equation corresponds to curve a which is shown in fig. 10 in linear-logarithmic coordinates. If we assume as shown in fig. 7, the focusing of the parallel beam at a certain distance from the lens region, e.g. $l = 100$, the matrix $[C]$ which determines the properties of the lens and the drift space will, according to (35), be equal to:

$$[C] = \begin{bmatrix} 1.365 & 163.1 \\ 0.09463 & 12.339 \end{bmatrix}. \tag{52}$$

On inserting the elements of matrix $[C]$ into eq. (33) we obtain in this case the following equation for the mirror surface:

$$z = -3.6\cdot 10^{-2}r^2 - 9.45\cdot 10^{-5}r^4, \tag{53}$$

which corresponds to curve b in fig. 10.

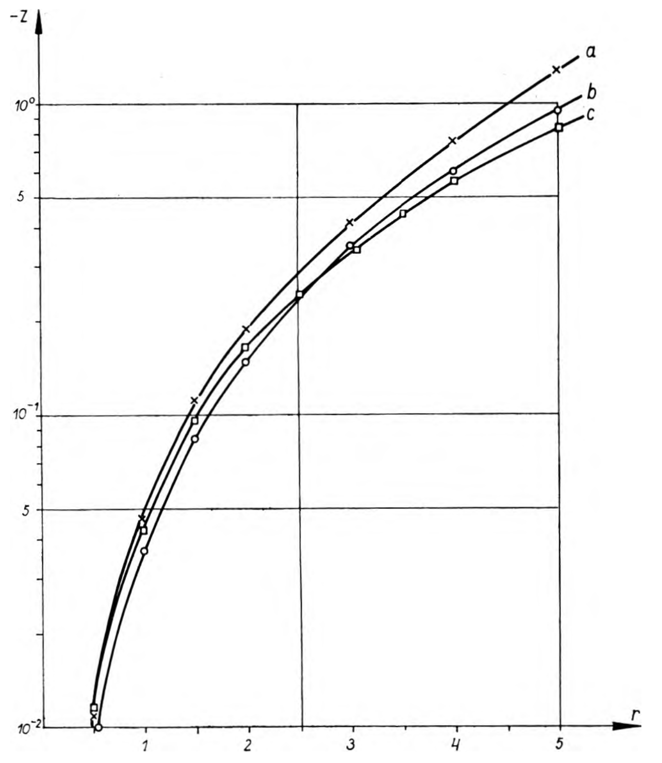

Fig. 10. Shape of the mirror surface,
—curve a — $l = 0$, $l^* = \infty$, $C_s = 0$
—curve b — $l = 100$, $l^* = \infty$, $C_s = 0$
—curve c — $l = 0$, $l^* = \infty$, $C_s = 10^3$

When the image is formed at a finite distance from the mirror fig. 8 i.e. $l = 0$, $l^* = 100$, the mirror surface equation may be obtained from relations (37) and (40) in the form:

$$z = -4.59\cdot 10^{-2}r^2 - 1.87\cdot 10^{-4}r^4. \tag{54}$$

The obtained equation is very much like eq. (51) and the corresponding shape of the mirror surface is similar to that presented by curve a in fig. 10. Curve c shown in the same figure corresponds to a mirror surface calculated from eq. (49) with the assumption of a constant spherical aberration of the mirror $C_s = 10^3$ and constant object and image positions $l = 0$, $l^* = \infty$. This is described by the equation:

$$z = -4.54\cdot 10^{-2}r^2 - 1.74\cdot 10^{-4}r^4 + 7.02 \cdot 10^{-4}r^4. \quad (55)$$

The mirror surface that introduces spherical aberration for small radii r almost coincides with the non-aberration surface.

Since the mirror surface corresponds approximately to the equipotential surface 0V, its shape is determined by the field distribution required to reach specified electro-optical parameters of the mirror. The knowledge of the mirror surface shape enables us to carry out a correction of the electrode shape in preliminary designs of an electron mirror necessary for reaching specified electro-optical parameters (e.g. by modeling on a resistance network). It should however, be noted, that in the considered two-tube mirror, -cf. presented examples even small changes in the mirror surface shape cause a radical change in final parameters. To obtain accurate results the correction procedure must, therefore, be carried out with great precission.

Анализ электронного зеркала при употреблении матричной записи

Обсуждается метод анализа электронного зеркала с заданными электронно-оптическими параметрами. Зеркало разделили на область электрической линзы и основное зеркало, представляя пробег электронов в матричной записи. Пригодность представленных формул, описывающих зеркальную поверхность, обсуждалась на примерах.

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Received, April 22, 1972.