A detailed technical description of the MOBAR 70 meter has been published in the Institute of Electrical Engineering Reports [4].

Deviations of the indicator values for the fluorescent tube radiation were equal to about 10 per cent for the prototype, but it is expected that, when applying the new correction methods (see [5] [6]) the accuracy may be improved.

References

[1] Commision Internationale de l'Eclairage — Publication CIE No. 13 E—1.3.2 1965. Second edition 1971.

[2] Nimeroff I., J. O. S. A. **54**, 1365 (1964).

[3] Somkuti A., Colorimeter. Patent 1,063,023. Great Britain.

[4] Kosek S., *Opracowanie urządzeń pomiarowych do oceny oświetlenia elektrycznego pod względem oddawania barw na stanowiskach pracy w zakładach przemysłowych,* Etap III 1970.

[5] Kosek S., *Przybliżone rozwiązanie problemu korekcji odbiorników fotoelektrycznych.* Przegląd Elektromechaniczny, **47,** No. 11 (1971).

[6] Kosek S., *Dopasowanie zespołu receptorów kolorymetrycznych.* Biul. Informacyjny Instytutu Elektroniki (in press).

*Henryk Wojewoda**

Refraction of Light at a Moving Boundary

The refraction of light at a moving boundary between two homogeneous optically different media is investigated in this paper. In accordance with the Einstein's principle of relativity a relativistic generalization of the Snellius-Descartes formula has been obtained. In particular formulae which determine the optical properties of slowly moving refractive boundary have been derived.

Introduction

The Fermat principle is the fundamental postulate of geometrical optics. The practically applicable geometrical properties of light (rectilinear propagation in optically homogeneous medium, reversibility of light ray, the Snellius-Descartes law) result directly from the Fermat variation principle. This principle is thus of obvious theoretical importance and investigations in this field are much justified.

In particular the problem of relativistic invariance of the Fermat principle deserves attention. The use of the Minkowski space is natural and the most effective in this case. The Fermat formulation can be properly generalized [1]. Its relativistic invariance is then apparent.

For the instrumental optics the Snellius-Descartes laws are of fundamental importance. The purpose of this work is to obtain a relativistic generalization of the refraction law. The boundary of two homogeneous optically different media is moving. We are concerned with the dependence of the refraction angle (reflection angle) on the incidence angle as seen by an observer moving in relation to the boundary.

The relativistic corrections may prove to be important in certain conditions; using the general formulae will then become necessary. The light aberration may be regarded as an experimental confirmation of the obtained results. The movement of reference frame plays an important role here.

The Variational Principle

In the four-dimensional formulation the Fermat principle is expressed by a propagation four-vector [1]. In our opinion this quantity is worthy of special

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notice. Let us recall here its definition and its basic properties.

Let a light wave vector components of the electromagnetic field be determined by the formula:

$$f(\boldsymbol{r}, t) = f_0 e^{-i(\omega t - \boldsymbol{k}\ \boldsymbol{r} + \varphi)} = f_0 e^{i(k_\mu x_\mu + \varphi)} \quad (1)$$

where ω — angular frequency of oscillations, $r = (x, y, z)$ — position vector.
According to the definition of wave vector

$$\boldsymbol{k} = \frac{\omega}{c}\boldsymbol{s} \quad (2)$$

where c — velocity of light, $\boldsymbol{s}$ — unit vector normal to the wave surface.

The wave four-vector k_μ has the form:

$$k_{1,2,3} = k_{x,y,z},\ k_4 = i\frac{\omega}{c}. \quad (3)$$

The world vector x_μ is defined in the following way:

$$x_1 = x, x_2 = y, x_3 = z, x_4 = ic_0 t \quad (4)$$

where c_0 — velocity of light in vacuum.

Thus the light refraction coefficient n satisfies the condition:

$$n = \frac{c_0}{c} \quad (5)$$

The Greek indices $\mu, \nu, \ldots$ assume the values 1, 2, 3, 4; the Latin indices $i, k, l, \ldots = 1, 2, 3$. Repetition index in the product of the same terms indicates summation (Einstein convention).

The wave four-vector components are not independent. It can be [1], that

$$k_\mu k_\mu = 0. \quad (6)$$

In the four-dimensional formulation the variation principle, which determines the propagation of light, assumes the following form:

$$\delta \int k_\mu dx_\mu = 0 \quad (7)$$

(the integration along the world line).
By separating the space and time parts in the integrand (7) we obtain

$$\delta \int (\boldsymbol{k}d\boldsymbol{r} + k_4 dx_4) = 0$$

and finally, taking into account (4) and (5), we have

$$\frac{1}{c_0}\delta \int ndl - n\delta \int dt = 0. \quad (8)$$

Thus expression (7) of the variation principle encompasses two different formulations of the Fermat principle: the principle of extremal time and the principle of extremal optical path [2], [3], [4]. The optical path of a light ray is extremal then and only then when it is the extremal time path.

The tensor notation of the variation principle insures its relativistic invariance. Formula (7) has the Lorentz invariance property. Therefore in another inertial frame of reference (primed indices) we have:

$$\delta \int k'_\mu dx'_\mu = 0. \quad (9)$$

Light Aberration

We change the inertial frame of reference. Then the wave four-vector k_μ undergoes the Lorentz transformation. According to the introduced notation we have:

$$k'_1 = \frac{k_1 + i\beta k_4}{\sqrt{1-\beta^2}},\ k'_2 = k_2, k'_3 = k_3,\ k'_4 = \frac{k_4 - i\beta k_1}{\sqrt{1-\beta^2}} \quad (10)$$

where $\beta = \frac{V}{c_0}$, V — velocity of the reference frame K' relative to the frame K.

In the frame K we have

$$k_1 = \frac{\omega}{c}\cos\alpha,\ k_4 = i\frac{\omega}{c}. \quad (11)$$

In the frame K'

$$k'_1 = \frac{\omega'}{c}\cos\alpha',\ k'_4 = i\frac{\omega'}{c}.$$

Expressions (10) on substituting into (11) and (12) give

$$\omega' \cos\alpha' = \frac{\omega(\cos\alpha - \beta)}{\sqrt{1-\beta^2}},\ \omega' = \frac{\omega(1-\beta\cos\alpha)}{\sqrt{1-\beta^2}} \quad (13)$$

and jointly

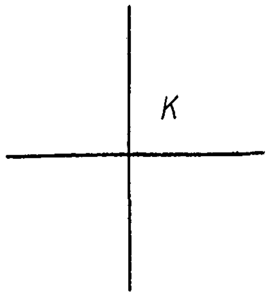
$$\cos\alpha' = \frac{\cos\alpha - \beta}{1 - \beta\cos\alpha}. \quad (14)$$

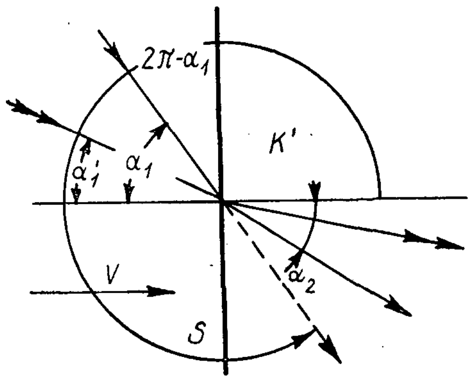
Formula (14) determines the light aberration: relativistic change in the direction of light propagation caused by a change of the inertial frame of reference.

The Law of Light Refraction

Let the reference frame K' be rigidly fixed to a refractive boundary (Fig.). According to our assumption, in frame K' the refraction law in the following form is satisfied:

$$n'_1 \sin\alpha'_1 = n'_2 \sin\alpha'_2 \quad (15)$$





where n_1', n_2' — refractive indices of the adjacent media, measured in the frame K'.

In the reference frame K, because of light aberration, the dependence of refraction angle α_2 on incidence angle α_1 is different. For the incidence (refractive) angle on the basis of (14) we have

$$\cos\alpha' = \frac{\cos\alpha - \beta}{1-\beta\cos\alpha}. \tag{16}$$

The refraction invariant $n'\sin\alpha'$ has in consequence the following expression:

$$n'\sin\alpha' = n'\sqrt{1-\beta^2}\,\frac{\sin\alpha}{1-\beta\cos\alpha}. \tag{17}$$

On introducing the notation

$$n' = \frac{n}{\sqrt{1-\beta^2}} \tag{18}$$

we obtain finally

$$n'\sin\alpha' = \frac{n\sin\alpha}{1+\beta\cos\alpha}. \tag{19}$$

Thus the law of refraction in the frame K is expressed as follows:

$$\frac{n_1\sin\alpha_1}{1-\beta\cos\alpha_1} = \frac{n_2\sin\alpha_2}{1-\beta\cos\alpha_2} = I. \tag{20}$$

Relation (20) is the relativistic form of the Snellius-Descartes formula.

The inverse Lorentz transformation is the Lorentz transformation with a parameter $-\beta$. This fact allows us to express in a simple way the angle α by the aberration angle α', namely

$$\cos\alpha = \frac{\cos\alpha' + \beta}{1+\beta\cos\alpha'}. \tag{21}$$

The path of light ray encountering a moving boundary of two optically different media can thus be successively calculated from the following formulae:

$$\cos\alpha_1' = \frac{\cos\alpha_1 - \beta}{1-\beta\cos\alpha_1}, \tag{22}$$

$$\sin\alpha_2' = \frac{n_1'}{n_2'}\sin\alpha_1', \tag{23}$$

$$\cos\alpha_2 = \frac{\cos\alpha_2' + \beta}{1+\beta\cos\alpha_2'}. \tag{24}$$

The refractive angle α_2 can be expressed explicitly by the incidence angle α_1. Making use of the "relativistic invariant of refraction — I" (20) we obtain

$$\cos\alpha_2 = \frac{\sqrt{1-\left(\frac{I}{n_2'}\right)^2}+\beta}{1+\beta\sqrt{1-\left(\frac{I}{n_2'}\right)^2}}. \tag{25}$$

In the nonrelativistic approximation (first order terms in respect to β) the Snellius-Descartes formula in the frame K assumes the form:

$$\frac{n_2\sin\alpha_2}{n_1\sin\alpha_1} = 1+2\beta\sin\frac{\alpha_1+\alpha_2}{2}\sin\frac{\alpha_2-\alpha_1}{2} = 1+\Delta. \tag{26}$$

The "kinetic correction — Δ" vanishes for the trivial case $\alpha_2 = \alpha_1$ (no refraction) and for $\beta = \frac{V}{c} = 0$.

In the same approximation ($\beta^2 \to 0$) we have

$$\cos\alpha_2 = \sqrt{1-\left(\frac{I}{n_2'}\right)^2}+\beta\left(\frac{I}{n_2'}\right)^2. \tag{27}$$

The optical properties of moving mirror can be investigated in a similar way [5], [6]. The reflection angle, according to (14), satisfies the relation:

$$\cos\alpha' = \frac{\cos\alpha+\beta}{1+\beta\cos\alpha}. \tag{28}$$

Thus the following relation between the reflection angle α_2 and the incidence angle α_1 is the relativistic generalization of the law of reflection:

$$\frac{\cos\alpha_1-\beta}{1-\beta\cos\alpha_1}=\frac{\cos\alpha_2+\beta}{1+\beta\cos\alpha_2} \tag{29}$$

or after simple rearrangement

$$\frac{\sin\alpha_1}{\cos\alpha_1-\beta}=\frac{\sin\alpha_2}{\cos\alpha_2+\beta}. \tag{30}$$

Relations (20) and (25) are the solution of the stated problem; they determine the optical properties of a moving refraction boundary.

References

[1] Landau L., Lifchitz E., *Théorie du champ*, Moscou, 1966, Éd. Mir.

[2] Born M., Wolf E., *Principles of optics*, London-New York-Paris-Los Angeles, 1959, Pergamon Press.

[3] Maréchal A., *Traité d'optique instrumentale* (Imagérie géométrique — Aberrations), Tome 1, Paris, 1952.

[4] Tudorowski A. I., *Teoria optičeskich priborov*, Moskva-Leningrad, 1948, Izd. Akad. Nauk ZSRR.

[5] Mc Crea W. H., *Relativity physics*, London 1947, Methuen and Co. LTD.

[6] Tamm I. E., *Osnovy teorii električestva*, Moskva 1947, Gostechizdat.

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New Developments in Optics

The most precise measurements have been offered by optics for a long time. The invention of laser and the growing use of the computer technique have increased the measuring precision by some orders of magnitude. The greatest progress has been noted recently in measurements of time. Now, the times of order of picoseconds are easily measured. With two lasers and a microwave generator, stabilized by a molecular generator, it is possible now to determine frequencies up to $6\cdot10^{13}$ Hz. Carrying out the independent measurements of frequency and of the wave-lenth allows us to determine the velocity of light with a precision unattainable so far.

One of the fundamental sources of information is photography. The simple photogram contains more than 10^4 bits of information. The aerial fotographs of great areas taken by a modern camera contain much more information. Two years ago, in the newspapers published in the USA there appeared a photograph taken from the height of about 13.5 km; on which two golf balls were distinctly seen. It is quite evident that the abundance of information in the photograms of great areas is due to the fact that there are not two identical surfaces that are emitting or reflecting light in the same way. The cameras for long-range photography are usually linked with a programmed computer, memorizing millions of bits. The immense multitude of information contained in such a photogram can be analyzed in a proper way by a computer only. A photograph taken by the long-range camera can contain more information than an encyclopaedia written on 500 pages.

In recent years a new method for optical investigations has been developed in Mahrburg [4, 5, 6]. It is believed that the method will have great influence on further progress in the field of pure as well as applied optics. The above method suggested by Blodgett and Langmuir as early as in 1931 [3], has been applied only recently in optics of monomolecular layers. They are formed from some organic, asymmetric compounds; their shape is commet alike and there is attached a long chain to the ball. The monomolecular layers are usually formed from the following substances: arachide acid, palm acid, the complex compound of europium or some pigments, e.g. cyanin. Especially durable layers are formed from salt of arachide acid CdC_{20}. The monomo-

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