teristic distinctive feature of multiplate mirrors in comparison with multilayer dusted mirrors is their narrow band and periodic structure.

Fig. 4. Spectrum of a neodymium laser with broadband output mirror

Examples of spectra generated by a neodymium laser with multiplate mirrors are given in Figs. 2 and 3. In Fig. 4 radiation spectrum of a laser with broadband mirrors is shown as comparison.

Multiplate mirrors applied to laser systems worked satisfactorily by power densities of the order of 200· MW/cm². These mirrors are applicable also in systems with self-synchronization of modes.

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Adam Dubik, Lech Borowicz, Jerzy Butowtt *

Differentiation of the Object Functions by Means of the Holographic Filters

1. Summary

A method of performing the derivatives of arbitrary orders for the observed objects in the optical frequency range has been presented.

2. Introduction

The purpose of the work is an attempt to solve the problem of the optical information coding in such a way that the delivery of the complete information about the object by means of a minimal number of signals would be possible. Theoretical considerations have been illustrated by coding the object information with the help of points and straight line segments. This notation may be helpful for optical object recognition problems by use of computers or decoding systems.

3. Realization of the Object Function Differentiation. Design of Filters [1]

The following formulas are well-known

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

$$\iint\limits_{-\infty}^{\infty} f(\xi,\eta)\delta[\xi-(x+a),\eta-(x+b)]\,d\xi d\eta$$

$$= f(x+a, y+b).$$

The increments of variables x and y will be denoted hereafter by a symbol h because these are magnitudes of the same order.

The last expression represents the filtering properties of the Dirac function δ [2,3]. $f(x,y)$ is an object function and it may be, for instance, an amplitude distribution in a laser beam, which was transmitted through a photogram of the interesting object.

It is easy to notice that

$$\frac{f(x+h,y)-f(x,y)}{h} = \frac{1}{h}\iint\limits_{-\infty}^{\infty} f(\xi,\eta)\{\delta[\xi-(x+$$

$$+h),\eta-y]-\delta[\xi-x,\eta-y]\}\,d\xi d\eta$$

$$= \frac{1}{h} f(x,y) * K_x(x,y),$$

*) Wojskowa Akademia Techniczna, Warszawa — 49, Poland.

where $K_x(x, y) = \delta(x+h, y)-\delta(x, y)$ while the operation denoted by an asterisk is a correlation.
Finally

$$\frac{\partial f(x, y)}{\partial x} = \lim_{h\to 0}\left[\frac{1}{h} f(x, y) * K_x(x, y)\right].$$

By an analogy to the previous considerations we obtain

$$\frac{\partial f(x, y)}{\partial x}+\frac{\partial f(x, y)}{\partial y} = \lim_{h\to 0}\left[\frac{1}{h} f(x, y) * K_{x+y}(x, y)\right],$$

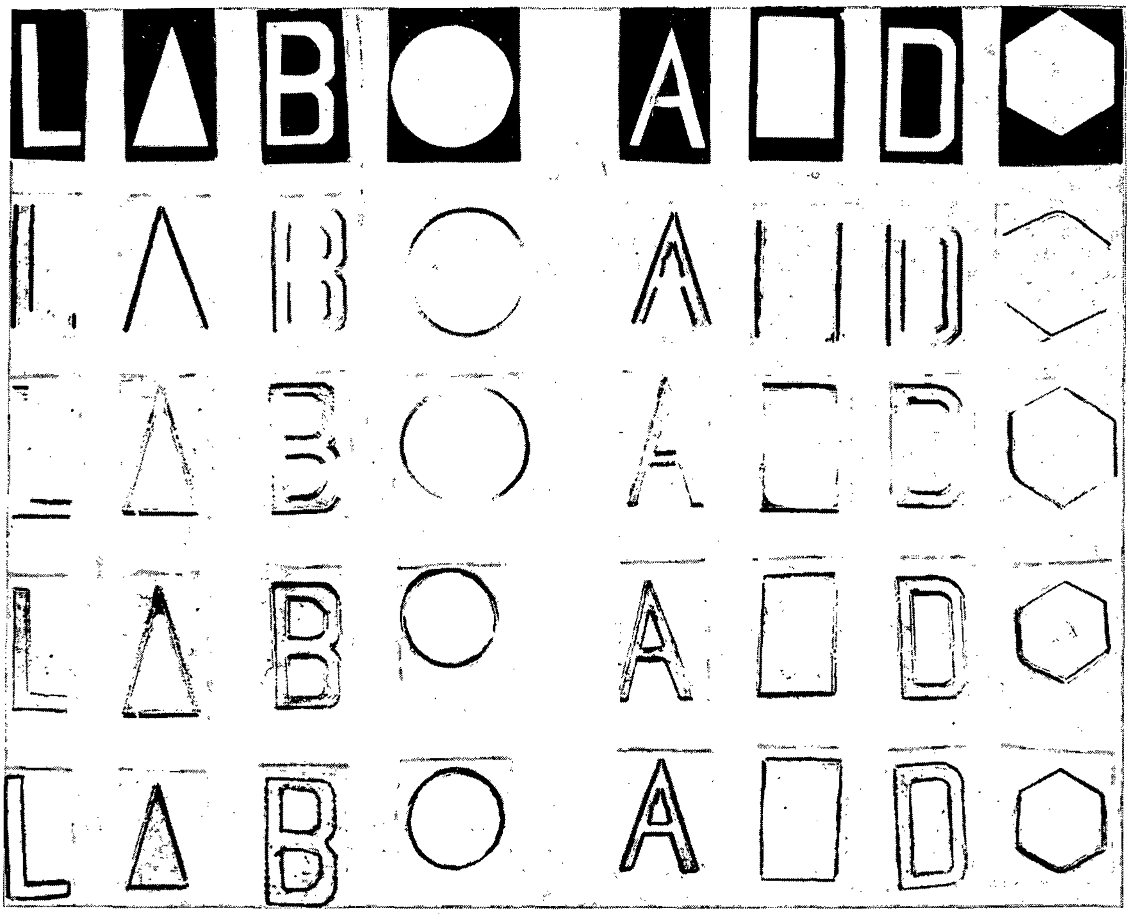

Fig. 1. The object function $f(x,y)$ in the form of letters and geometrical figures as well as the results of the operations $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, $\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial y}$, $\frac{\partial}{\partial x}+\frac{\partial}{\partial y}$ performed on the functions

Analogically

$$\frac{\partial f(x, y)}{\partial y} = \lim_{h\to 0}\left[\frac{1}{h} f(x, y) * K_y(x, y)\right].$$

The transfer functions $G_x(f_x, f_y)$ and $G_y(f_x, f_y)$ of the filters $K_x(x, y)$ and $K_y(x, y)$ can be obtained as follows

$$G_x(f_x, f_y) = F[K_x(x, y)] = F[\delta(x+h, y)-\delta(x, y)] = \exp 2\pi i h f_x - 1$$

$$G_y(f_x, f_y) = F[K_y(x, y)] = F[\delta(x, y+h)-\delta(x, y)] = \exp 2\pi i h f_y - 1$$

where the symbol $F[\ ]$ denotes the Fourier transforming and f_x and f_y are the spatial frequences. Below we will use the notation

$$2\pi f_x = \omega_x,\ 2\pi f_y = \omega_y.$$

where the frequency response K_{x+y} of the filter has the form

$$G_{x+y}(\omega_x \omega_y) = \exp i\omega_x h + \exp i\omega_y h - 2.$$

For producing the higher order derivatives, for instance $\frac{\partial^2 f(x, y)}{\partial x^2}$ and $\frac{\partial^2 f(x, y)}{\partial x \partial y}$ the filters with the following frequency responses are needed

$$G_{xx}(\omega_x, \omega_y) = -\exp i\omega_x h - \exp(-i\omega_x h)+2$$

$$G_{xy}(\omega_x, \omega_y) = \exp i(\omega_x h+\omega_y h) - \exp i\omega_x h - \exp(-i\omega_y h)+1.$$

In the case of the expression

$$\frac{\partial^2 f(x, y)}{\partial x^2}+\frac{\partial f(x, y)}{\partial y}$$

Fig. 2. The object functions *f(x,y)* in the form of some letters and a rectangle and the result of an operation $\frac{\partial}{\partial x \partial y}$ performed on the functions

the corresponding filter has the form

$$G(\omega_x, \omega_y) = -\exp i\omega_x h - \exp(-i\omega_x h) + \exp i\omega_y h + 1.$$

The filters of these types are realized by taking successive holograms of the Fourier images of differently located Dirac functions δ when using a phase shifting plate with the shift equal to π. For instance, to obtain a filter realizing an operation $\frac{\partial}{\partial x}$ the production is determined by the following mathematical relationship

$$|\{F[\delta(x+h, y)] + F[\delta(x, y+a)]\}|^2 + |\{F[\delta(x, y)] + \\ + \exp(i\pi) \times F[\delta(x, y+a)]\}|^2 = 4 + [\exp i\omega_x h - 1] \exp \times \\ \times (-i\omega_y a) + [\exp(-i\omega_x h) - 1] \exp i\omega_y a.$$

The above expressions contain a term $\exp(i\omega_x h) - 1$. Thus a filter realizing the operation $\frac{\partial}{\partial x}$ may be recorded on a light sensitive material. The expression $\exp(-i\omega_y(a$ indicates that a filter of the required form is spatially separated from the luminance distribution represented by the remaining terms.

4. The Experimental Results

To verify the considerations presented some filtration experiments for optional letters and geometrical figures were carried out. The results of some operations chosen from the following

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x \partial y}, \frac{\partial}{\partial x} + \frac{\partial}{\partial y}, \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y}$$

performed on the objects are presented in Figs 1 and 2. It can be seen, that when applying the holographic filters the unwanted information transformed about the object can be eliminated. The information transformed in the described way may be introduced with the help of the photodetectors matrix to a computer.

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