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Design of Multiplate Mirrors

In Q-switched laser systems mirrors in form of stacks composed of adequately selected plane parallel dielectric plates are applicable on a large scale. Such a stack comprising three identical plates is presented schematically in Fig. 1. This is a multiplate mirror most often applied in laser technique.

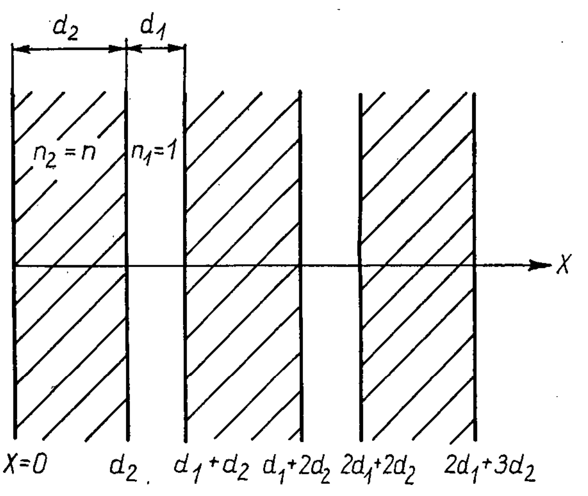

Fig. 1. Circuit of a three-plate stack

Reflection coefficient of an arbitrary system can be calculated if its input impedance Z is known.

Input impedance of a stack consisting of three plates of thickness d_2 and distance d_1 can be obtained by successive transformation of the input impedance of the last plate**) to the point $x = 0$.

Introducing designations:

$$\Delta = \frac{2\pi n d_2}{\lambda}; \quad S = \frac{d_1}{n d_2}$$

reduced input impedance of a stack consiting of three lossless dielectric plates (Fig. 1) is obtained in the form:

$$Z(x=0) = \frac{n^2 A(nA-B\mathrm{tg}\Delta)-nB(nC+D\mathrm{tg}\Delta)+}{nD(nA-B\mathrm{tg}\Delta)-n^2 C(B+nA\mathrm{tg}\Delta)+}$$

$$\frac{+j[D(nC+D\mathrm{tg}\Delta)+nC(nA-B\mathrm{tg}\Delta)]}{+j[n^3 A(B+nA\mathrm{tg}\Delta)+n^2 B(nA-B\mathrm{tg}\Delta)]} \quad (1)$$

where:

$$A = 1-n\mathrm{tg}\Delta\cdot\mathrm{tg}(S\Delta),$$

$$B = n\mathrm{tg}\Delta+\mathrm{tg}(S\Delta),$$

$$C = \mathrm{tg}\Delta+n\mathrm{tg}(S\Delta),$$

$$D = n-\mathrm{tg}\Delta\mathrm{tg}(S\Delta).$$

The parameter characterizing a mirror is its power coefficient of reflection expressed by relation:

$$R = |\varrho^2(x=0)| = \left|\frac{Z(x=0)-1}{Z(x=0)+1}\right|^2. \quad (2)$$

Expressions (1) and (2) for fixed optical thicknesses of plates and distances allow to determine the power coefficient of reflection versus phase shift Δ and thereby versus wavelegth.

Considering the fact that laser systems will be excited in frequency bands where reflection coefficients of a mirror are maximum it is possible to simplify considerably the above expressions and the results thus obtained will be comprised within the admissible error limits.

Reflection coefficient of a multiplate stack is of periodic structure; the structure period and the reflection coefficient value in successive maxima are depending on the parameter S.

By a suitable choice of the assembly it is possible to obtain a significant difference between the maximum reflection coefficient and the reflection coefficient in the neighbouring successive maxima. In this way it is possible to ensure excitement of a laser in these bands where $R = R_{\mathrm{maxmax}}$. Evidently, this will occur for

$$\Delta = (2p+1)\frac{\pi}{2} \quad \text{and} \quad S\Delta = (2k+1)\frac{\pi}{2} \quad (3a)$$

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**) Input impedance of a single dielectric plate is given in [1] and its selective properties in [2].

where p, k — natural numbers,
i.e. when both the plate and the distance can be treated as odd multiples of quarter-wave plates

$$nd_2 = \frac{(2p+1)\lambda}{4}; \; d_1 = \frac{(2k+1)\lambda}{4}. \tag{3b}$$

Such conditions will happen for successive wavelengths for which one could match such natural numbers $p_1, p_2 \ldots, k_1, k_2, \ldots$ that expressions (3) would be satisfied. In these cases resistance of the stack has the real character and equals:

$$Z(0) = n^{-6}$$

and reflection coefficient

$$R_{\text{maxmax}} = \left(\frac{n^6-1}{n^6+1}\right)^2. \tag{4}$$

The parametr S describing the ratio of optical paths across the distance and the plate is by occurrence of conditions (3) a ratio of odd numbers*)

$$S = \frac{d_1}{nd_2} = \frac{2k+1}{2p+1}. \tag{5}$$

Assuming that for determined p and k conditions (3) are satisfied for λ_1, whereas for $p+\Delta p$ and $k+\Delta k$ for the nearest $\lambda_2 > \lambda_1$, then on the ground of relations

$$d_1 = (2k+1)\frac{\lambda_1}{4} = [2(k+\Delta k)+1]\frac{\lambda_2}{4}$$

$$nd_2 = (2p+1)\frac{\lambda_1}{4} = [2(p+\Delta p)+1]\frac{\lambda_2}{4}$$

formula (5) can be represented in the form

$$S = \frac{\Delta k}{\Delta p}. \tag{5a}$$

It is easy to notice that Δk and Δp are also odd numbers.

When $p \gg \Delta p$ and $k \gg \Delta k$, the following relation is valid:

$$\Delta\lambda = \Delta p \cdot \Delta \lambda_p = \Delta k \cdot \Delta\lambda_k \tag{6}$$

where $\Delta\lambda$ is the interval between resonances of the stack, whereas

$$\Delta\lambda_p = \frac{\lambda^2}{2nd_2} \text{ and } \Delta\lambda_k = \frac{\lambda^2}{2d_1}$$

are respective intervals between successive resonances of plates and distances. Therefore, in order to obtain the maximum reflection coefficient (R_{maxmax}) resonances of plates and distances should agree for the same wavelengths. By appropriate choice of Δp and Δk (physically nd_2 and d_1) one can suitably shape generation bands of laser systems.

The essential parameter of a mirror is the width of the maximum reflection band. The entire reflection band can be calculated by trying to find zero loci of $R = R(\Delta)$ function nearly the R_{maxmax} point. Analysis of the numerator of expression (2) reveals that in these points the following condition should be satisfied

$$(1+n)^2\cos\Delta\,(1+S)+(1-n)^2\cos\Delta\,(1-S) = \pm 2n. \tag{7}$$

As the parameter S in practice does not differ substantially from unity, then the second term of condition (7) is a slow-variable function with amplitude smaller at least by one order of magnitude. Expression (7) can be therefore rewritten in the following simple form:

$$\cos\Delta\,(1+S) \simeq \pm\frac{2n}{(n+1)^2}. \tag{7a}$$

Making use of (7a) and assuming that the band in the middle of height is equal to the half of the band at its base one can write that

$$\delta\lambda \simeq \frac{\Delta\lambda_p}{2\pi(1+S)}\left[\arccos\left(-\frac{2n}{(n+1)^2}\right) - \arccos\left(\frac{2n}{(n+1)^2}\right)\right] \tag{8}$$

and e. g. for glass BK—1 ($n = 1.5$) this formula is reduced to

$$\delta\lambda \simeq \frac{1}{6}\cdot\frac{\Delta\lambda_p}{1+S}. \tag{8a}$$

Fig. 2. Spectrum of a neodymium laser with output mirror of glass BK—1; $d_1 = d_2 = 1.32$ mm; $\Delta\lambda = 8.51$ Å

Fig. 3. Spectrum of a neodymium laser with output mirror of glass BK—1; $d_1 = d_2 = 1.0$ mm; $\Delta\lambda = 29.96$ Å

It is easy to notice that the width of reflection band is by about one order of magnitude smaller than interval between resonances of the plate. The charac-

* It has been assumed that optical thicknesses of the plate and of the distance are large as compared with the wavelength.

teristic distinctive feature of multiplate mirrors in comparison with multilayer dusted mirrors is their narrow band and periodic structure.

Fig. 4. Spectrum of a neodymium laser with broadband output mirror

Examples of spectra generated by a neodymium laser with multiplate mirrors are given in Figs. 2 and 3. In Fig. 4 radiation spectrum of a laser with broadband mirrors is shown as comparison.

Multiplate mirrors applied to laser systems worked satisfactorily by power densities of the order of 200· MW/cm^2. These mirrors are applicable also in systems with self-synchronization of modes.

References

[1] von Hippel A. R., Dielektryki i fale, PWN, 1963.
[2] Snitzer E., Appl. Optics, Vol. 5, No. 1, 1966.

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Differentiation of the Object Functions by Means of the Holographic Filters

1. Summary

A method of performing the derivatives of arbitrary orders for the observed objects in the optical frequency range has been presented.

2. Introduction

The purpose of the work is an attempt to solve the problem of the optical information coding in such a way that the delivery of the complete information about the object by means of a minimal number of signals would be possible. Theoretical considerations have been illustrated by coding the object information with the help of points and straight line segments. This notation may be helpful for optical object recognition problems by use of computers or decoding systems.

3. Realization of the Object Function Differentiation. Design of Filters [1]

The following formulas are well-known

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

$$\iint\limits_{-\infty}^{\infty} f(\xi,\eta)\delta[\xi-(x+a),\eta-(x+b)]\,d\xi d\eta$$

$$= f(x+a, y+b).$$

The increments of variables x and y will be denoted hereafter by a symbol h because these are magnitudes of the same order.

The last expression represents the filtering properties of the Dirac function δ [2,3]. $f(x,y)$ is an object function and it may be, for instance, an amplitude distribution in a laser beam, which was transmitted through a photogram of the interesting object.

It is easy to notice that

$$\frac{f(x+h,y)-f(x,y)}{h} = \frac{1}{h}\iint\limits_{-\infty}^{\infty} f(\xi,\eta)\{\delta[\xi-(x+$$

$$+h),\eta-y]-\delta[\xi-x,\eta-y]\}\,d\xi d\eta$$

$$=\frac{1}{h}f(x,y) * K_x(x,y),$$

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