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A NOTE ON THE ROOTS OF THE THEORIES OF PERSONAL INCOME DISTRIBUTION IN THE HISTORY OF ECONOMIC THOUGHT

From the beginning of the nineties, the inequality of income distribution has gained weight in the main topic of discussion, both from a theoretical and applied point of view. The aim of this article is an analysis of the distributive inequality in personal income distribution. The study is carried out trying to underline the contribution to the theory of economic, sociological and anthropological influences. The theoretical evolutive course that has been chosen is the same that has characterized Pareto's interests and that, therefore, starting from the roots, shifting from the functional income distribution to the personal one, brings him to the wording of income distribution law.

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JEL classification: B1

INTRODUCTION

The subject of personal income distribution appears in a period in which economists have began to have reliable statistical data at their disposal; straight after an issue, both formal and substantial, acquires relevance: if the real profile of the function is, or is not, referable to the historical concept of "social pyramid" traced by de Vauban, Say and Leroy Beaulieu.

The Ammon model, connecting the capacity to achieve an income to the distribution of personal skills and so to the accidental errors curve, claims the nonlinearity of the function, showing the way forward to Pareto's theory and to the following contributions of those statisticians, mathematicians and econometricians that have contributed to the development of an elaborate and sophisticated tool for the analysis of reality.

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1. ON THE ROOTS OF PARETO'S INCOME DISTRIBUTION LAW

Differently from the thought of classical economists, in Paretian approach, a manifest and growing interest in personal distribution of incomes is observed at the expense of functional interpretation.

In the first reference to distributive issues, expressed during a conference in April 1886 (Pareto 1886a), which is still inherent in functional dimension, Pareto claims that "*the distribution of national income must assure both to labour and capital the minimum shares for their reproduction: if such conditions will not exist, the available amount of labour and capital would automatically decrease in order to respect the above mentioned constriction (Pareto 1886b). So, only the part of national income that exceeds the sum of these minimum shares can be reallocated, in any case between labour and capital (Pareto 1886b)*".

The subject is revisited in an article of 1891, *Socialismo e Libertà* (Pareto 1891), devoted to a study of *vincolismo* ("…conceived by Pareto as the most frequent form of government, consisting in subtracting, by State intervention, wealth's distribution to the imperium of free competition, with the intent to favour particular social classes…" our translation from Marchionatti, Mornati (2007), p. 5.) ; in this article Pareto identifies a double possible solution "in order to improve the lot of poor people": the first is amenable to a higher increase of national wealth with respect to population one, the second is referable to a distribution, in favour of the less well-off, of consumed income by the well-off excess with reference to the average consumption of the poor. Moreover, he clarifies that unspent income by the rich is not available for reallocation, intended as replenishment of national capital, remembering the Paretian view of 1886.

Pareto, searching for a solution, seems to come out of the classical scheme of functional distribution once and for all to move toward income's personal distribution, although in this first formulation the reference is only towards the two extreme social classes of income earners (Process that four years later, in January 1895, will take to the publication of his first study on the famous incomes law, our translation from Pareto (1895)).

In the late 1880s and early 1890s, Pareto's involvement is geared to estimate the average value of commodities' utility to proceed from empirical laws of application. The aim is to obtain an easy wording of the applied laws, recurring to the empirical law of revenues distribution: $y = H/x^h$ (Pareto 1895a). Therefore, personal income distribution enters in the empirical wording of utility theory, although Pareto will throw aside this kind of study.

From the wording of the initial thoughts on personal distribution to the statement of his incomes law, Pareto takes his cue not only from his previous studies, but also from other authors' thoughts and foreign ideological attitudes; indeed, in Pareto's incomes law we can find referable links and traces to different "sources". Pareto's argumentation develops around two keynotes: social heterogeneity and social selection. He resumes these concepts, through a refined and politically critic processing of an influential anthropologist of the twentieth century, a leading light of the social Darwinism layering: Otto Ammon ("…and here I must use the conditional since Pareto makes no mention of that in the wording of his law" our translation from Marchionatti, Mornati (2007) p. 7.)

2. THE PERSONAL INCOMES DISTRIBUTION OF AMMON

To understand the breeding ground from which Ammon's theories took shape, it is necessary to consider the basic ideas relating to human nature which started in the nineteenth century. The radical change which occurs in anthropological literature is one of the strongest conditioning factors; especially towards the middle of the 19th century we witness an opposing trend regarding an indubitable principle according to Burke, Rousseau and Smith: the natural equality of human being.

As Marvin Harris noticed, "Towards the middle of the 19th century there is no truth more evident than the fact that Man was created unequal. Nothing brought a more dangerous influence on social science's development. The racial *determinism* was the form which the gathering wave of the science and culture assumed as was breaking on the industrial capitalism' shore" (Harris 1971).

On economic and social side, Ammon's research results in a reproduction, in social hierarchy of primal differences of human being with typical features transmittable through inheritance.

The writing by Ammon on income distribution is *L'ordre social et ses bases naturelles. Esquisse d'une anthropologie* of 1900; here the anthropologist focuses on statistics of Saxony seeing them as the most complete and interesting, being the most developed German region from an industrial point of view (Ammon 1900).

The core table of his work is the following, and it concerns all Saxon income earners including both natural persons and legal entities in 1890:

Table 1

Saxon income earners (1890)

Incomes	Income Earners (IE)	% IE	standardized % of IE
<500 mark	546138 people	38.9	39.9/5 = 7.8
500-800 mark	401439 people	28.6	28.6/3 = 9.5
800-1600 mark	318125 people	22.7	22.7/8 = 2.8
1600-3300 mark	91124 people	6.5	6.5/17 = 0.4
3300-9600 mark	36841 people	2.6	2.64/63 = 0.0
>9600 mark	19402 people	0.7	…..

Source: Ammon op. cit., p. 177

Ammon claims that data has to be represented by a curve (Ammon 1900) where income intervals are reproduced on the X axis in proportion to their size (greater intervals for a large number of taxpayers in a class) and the percentage of income earners is described on the Y axis; the percentage then has to be normalized comparing it to the overall amount of mark in each interval (Calzoni 1961).

Rounding off the histogram's angles, Ammon draws an interpolate curve justified by the fact that income varies following continuous and not discontinuous intervals (Ammon 1900); this curve is known as Social Pyramid (Ammon underlines that to deepen the study of income distribution "il faut….être en état d'apprécier les variations d'une fonction mathématique qui se produisent en conséquence de la variation des grandeurs particulières (Calcul différentiel)", Ammon 1900, pp. 178, 300.)

Ammon's aim was to oust from collective imagination the idea of a pyramid and to replace it with another shape similar to an onion (in Figure 1 the true form of the so-called Social Pyramid). In this way he wanted to emphasize the symmetric structure of society, characterized by a small number of geniuses and inepts at both ends and by a concentration of mediocrity in the middle.

In Ammon's point of view, the different genetic allocation of intelligence and skilfulness is reflected in social stratification as derived by the analysis of income statistics; he suggests one of the first attempts to show the coincidence between the "curve of intelligence" and the "curve of wealth". From the visual comparison of the trends of income and Galton curves (this supposition was already taken into consideration by Galton without any kind of support by income statistics; Galton 1869) , it turns out that upper parts are superimposed, being both specifications of the normal curve. Then asymmetry of the upper part with respect to the lower has been explained by the interruption occurring when the income level is zero (Ammon 1899, p.183, argues the imbalance, affirming that the lower part could have a symmetric trend respectively to the upper one if the curve didn't break in a level of income equal to zero and instead could continue by stretching a curve that depicts also negative incomes, defined as incomes that allow the poorest persons that live on the fringes of society to survive).

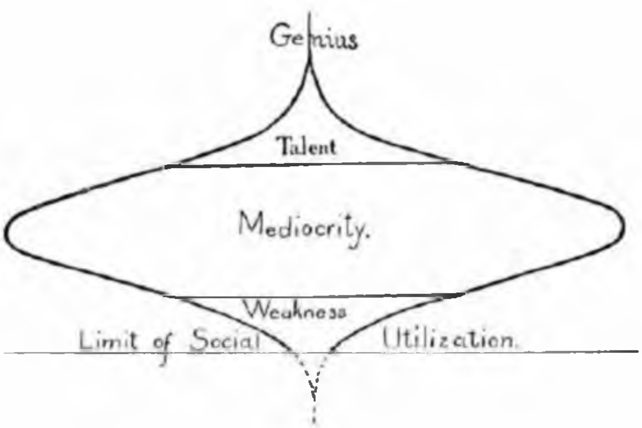

Figure 1. The real form of the so-called Social Pyramid

Source: Ammon op. cit., p. 178

The German anthropologist affirms that by comparing the curves two truths arise: the form of the curve of incomes (except at the base) very nearly coincides with Galton's curve of the distribution of abilities; and the income curve is not symmetrical above and below but corresponds more to what we have called the "true form of the social pyramid" (see Figure 1), which, like the income curve, stands upon a horizontal base line (Ammon 1899). So Ammon concludes, strengthening what was previously said, that a similarity between upper, medium and lower attitude and the equivalent level of income seems to exist (Ammon 1899).

The historical trend of income distribution is materialized in the following table, in which the real structure *ad quem* of income earning classes is compared with the structure that it would have if the growth of the rich population (earners of income), in this case equal to 29.05%, could be uniformly distributed in every class, preserving the unchanged shape of the social pyramid (Ammon 1899).

Table 2

Historical trend of income distribution

Incomes in mark	**IE in 1879**	**IE in 1890 = IE in 1879 * 1,2905**	**IE in 1890**	**Δ between the preovious two columns**	**%Δ compared to 1890**
< 500	560210	722952	546138	-176814	-24.5%
500-800	270246	348752	401439	52687	15.1%
800-1600	165699	213835	318125	104290	48.7%
1600-3300	62140	80192	91124	10932	13.6%
3300-9600	24414	31507	36841	5334	16.9%
> 9600	5293	6831	10402	3571	52.3%
	1088002	1404069	1404069	0	

Source: Marchionatti, Mornati, op. cit. p.21.

Ammon denies the socialist thesis of a greater concentration of big and small income earners at the expense of middle ones; in the observed period he notices a decrease in the numbers in the lowest class. The greatest increase in numbers concerns the class in interval 800-1600 that is obviously out of the well-off. Such observations lead Ammon to vigorously affirm that the income position of the social classes has never improved in such a rapid way: however, these claims are pronounced without underlining the spatial-temporal exiguity on which their validity is based (Marchionatti, Mornati 2007).

Ammon's formulation could be considered the linchpin which Pareto used and improved for his income law.

3. FROM AMMON'S TO PARETO'S APPROACH

Among the two authors' mutual interests there is the choice of the sample used: both Pareto and Ammon examine Saxony for its richness in statistical data on population, however, the first author doesn't confine his analysis to a temporal comparison of inequality coefficients among incomes. Pareto initially uses, as does Ammon, data in closed intervals, afterwards, he throws aside his old principles, innovating with respect to Ammon, utilizing intervals of income closed downwards and opened upwards. Pareto revises the Benini table (1894), by adding the third and fourth columns and ignoring low incomes (Busino 1965).

An example of Paretian revision (Pareto 1896-1897 par. 958) is shown in Table 3.

Table 3

Income distribution (Pareto)

Income in lire	Income earners	X francs	N
<1000	161960		
1000-2000	32518	1000	59486
2000-4000	17202	2000	26968
4000-7000	5502	4000	9766
7000-10000	1867	7000	4264
10000-15000	1087	10000	2397
15000-25000	665	15000	1310
>25000	645	>25000	645

Source: Pareto 1896-1897 par. 958

From now on, the data representation will not be by histograms but by points with coordinates expressed by natural logarithms and observable by interpolation: $\log N = \log A - \alpha \log x$; the parameters are estimated by the Cauchy interpolation method; in the following part there is an example (referred to England 1843) of this computation (Pareto 1896 a, reproduced in Pareto 1982.)

Table 4

Interpolation method: England 1843

x (in £)	N	$\log_{10}$x	$\nabla \log_{10}$x	$\log_{10}$N	$\nabla \log_{10}$N
150	106637	2.17609	0.91853*	5.027908	−1.35574*
200	67271	2.30103	0.79359*	4.827828	−1.15566*
300	38901	2.47712	0.61750*	4.589961	−0.91779*
400	25472	2.60206	0.49256*	4.406063	−0.73389*
500	18691	2.69897	0.39565*	4.271633	−0.59946*
600	13911	2.77815	0.31647*	4.143358	−0.47119*
700	11239	2.84510	0.24952*	4.050728	−0.37856*
800	9365	2.90309	0.19153*	3.971508	−0.29934*
900	7923	2.95424	0.14038*	3.89889	−0.22672*
1000	7029	3.00000	0.09461*	3.846894	−0.17472*
2000	2801	3.30103	0.20641	3.447313	−0.22486
3000	1566	3.47712	0.38250	3.194792	−0.47738
4000	1040	3.60206	0.50744	3.017033	−0.65514
5000	701	3.69897	0.60435	2.845718	−0.82645
10000	208	4.00000	0.90538	2.318063	−1.36011
50000	8	4.69897	1.60435	0.90309	−2.76908
		Σ(log10x)/16 = 3.09425	$\Sigma\nabla \log_{10}$x =8.42080	Σ($\log_{10}$N)/16 =3.67217	$\Sigma\nabla \log_{10}$N =−12.62608

Source: Pareto (1982), pp. 340, 341

Where:
$\nabla \log_{10}x = (\log_{10}x-3.09425)^1$;
$\nabla \log_{10}N = \log_{10}N - 3.67217^2)$;
$\alpha = (\Sigma\nabla \log_{10}N) / (\Sigma\nabla \log_{10}x)$ in this case: $|\alpha| \approx 1.50$.

The trend of income distribution is similar to a convex curve towards the origin (Busino 1965): from this point onwards Pareto will contest to partisans of the Social Pyramid concept the representative forcefulness of its shape for income distribution and he will replace it with another figure "which has the shape of an arrowhead".

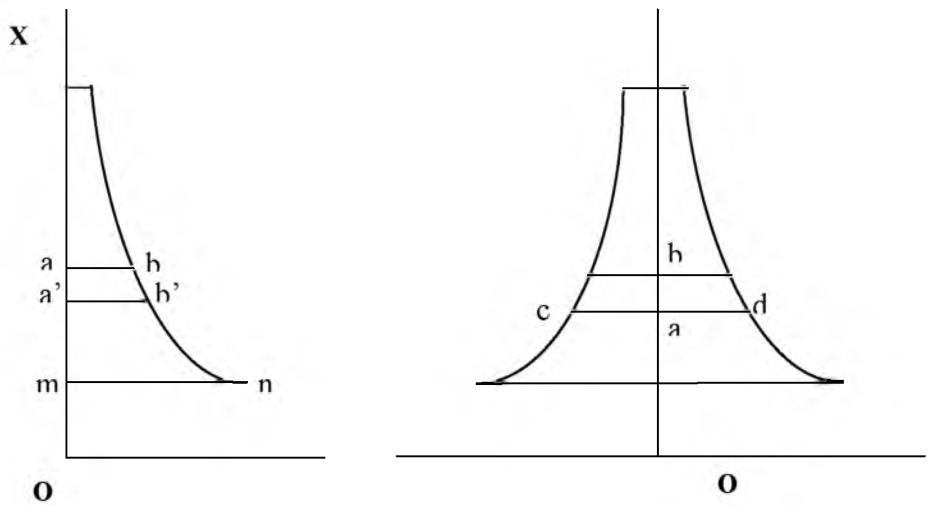

Figure 2. Arrow's diagram

Source: Pareto 1906, p. 271

The y-axis is for incomes while the x-axis is for individuals, therefore the poorest classes are placed on the bottom of Pareto's Diagram and the richest on the top, close to the vertex.

Pareto's arrow is a rotation solid, obtained by the complete rotation of the income curve around the y-axis: any single circle section of this arrow has

[1] The * is for a negative result in absolute value
[2] The * there is a change of sign if the result correspond to values initially negative of $\log_{10}x$

no other analytical meaning except that to represent, with its semi-diameter, a frequency of earners of income (Marchionatti, Mornati 2007).

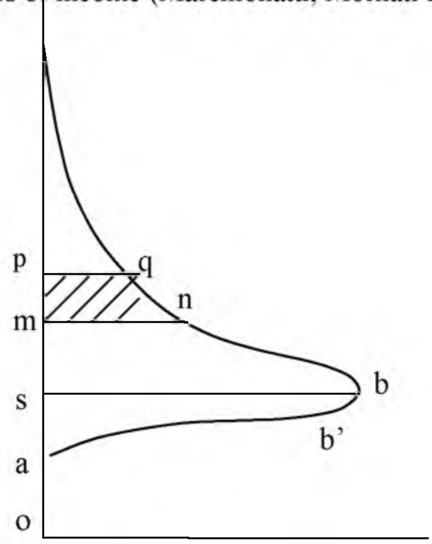

Figure 3. Top's diagram

Source: Pareto 1906, p. 271

Pareto, already in the *Cours*, has given up the arrow's diagram to replace it with the top's shape; he initially considers equivalent the two diagrams but in the following works he permanently replaces the arrow with the top. Pareto observes that wealth distribution doesn't assume the pyramid shape but the form of a solid "which has the shape of an arrowhead or a top if better" Pareto (Cours, vol. II, p. 346). The diagram's shape is not the result of a random process because, being so, it may coincide with the same probability curve (or error curve); instead, the distribution curve is completely different from the previous one and is the result of a universal law.

At the end of the inductive part of his research, Pareto, as does Ammon, dedicates himself to the explanation of the presented law, however, the effort made by Ammon to convert the distribution curve into a normal one is totally absent.

Pareto clearly brings out that the income curve is asymmetric compared to the Gaussian one due to human diversity (Pareto 1896-1897). He clarifies that "the curve is not symmetric around the mean, the top part is very long while the bottom is almost flat, with a strong asymmetrical trend compared to the upper. But from this we can not conclude that there is not symmetry among the individuals' qualities that move away from the mean. Indeed between two men that equally distance themselves from the mean of qualities, the man with exceptional aptitudes towards gaining money can obtain high earnings, while the man with opposite qualities (exceptional negative qualities) cannot fall, without disappearing, below the minimum revenue that is needed to sustain life" (Pareto 1906).

The causes that move income distribution, according to Pareto's Law, are considered to be "Forces of Nature" depending on human nature rather than on the same organized society.

In the part of the Manual of Political Economy on Population (Pareto1906), the author, at the beginning of the chapter deals with social heterogeneity: "*as we noticed before (II,102) society is not homogeneous; and anybody, not wanting to close his eyes, has to grant that men differ from one another physically, morally and intellectually. To proper inequalities of human beings correspond economic and social inequalities which can be noticed among all those people, since ancient times up to present day and anywhere; in such way, that aspect being always present, human society can be considered a community with hierarchy*".

4. ANALYTICAL IMPLICATIONS OF INCOME LAW

The greater development of Paretian analysis compared to Ammon's concerns the implications that we can derive from statistics examination, these implications lead Ammon to the confutation of the socialist argument of income concentration.

Pareto and Tullio Martello's correspondence (Mail to Tullio Martello at 15 December 1895 (Pareto 1973), pp. 273-279) brings out that the author of income law deduces that progressive tax does not necessarily absorb all taxable income and likewise that in a proportional tax system "the well-off contribution to public expenditure is much less than that of the poor, while their level of expenditure is higher." (Pareto 1895a).

However, the most treated subject, in this first stage of analytical development, is the "quantitative" resume of the study on redistribution effects.

A sort of such computation was already conducted by Paul Leroy Beaulieu in his most famous essay: *Essai sur la rèpartition des richesses et sur la tendance à une moindre inègalitè des conditions*. In this work, Leroy Beaulieu, in reference to Saxon statistics of 1878, has affirmed that if all incomes greater than 7850 francs (or rather all incomes "moyen, gros et très gros") would be confiscated and if the income would be distributed among those whose belongings were less than 7850 francs, these ones would have seen to increase their own income by only 14-15%, in the hypothesis that such a distribution would not have meant a reduction of national income. According to Leroy Beaulieu, this hypothesis, typical of socialist thought, is considered absurd because such distribution would have discouraged the activity of industrial barons, dealers and engineers of the country (Leroy Beaulieu 1881).

Pareto, without any reference to the French economist, repeats the exercise of the Saxon case in 1886 (Pareto 1895b), proving that "la liquidation sociale" (a share of excess part of the medium income is turned into capital by the well-off and therefore can not be shared by the socialist States) would be very modest and, most of all, lower than the income increase which taxpayers could benefit with the abolition of customs protectionism (in this case Pareto does not distinguish between poor and well-off).

The most important deduction of Pareto law is a blend of analytical skilfulness and political involvement that characterized his studies at least until *Cours* publication. He defines the reduction of inequality in income distribution through the decrease of α parameter. Pareto enunciates two different ways based on incomes equality: well-off impoverishment and enhancement of the poor. Leroy Beaulieu' assertion (1881, p. 47), in which incomes of less privileged increase most quickly than the rich incomes, suggests to Pareto the choice of the second definition. So, after this clarification from income law: $N=A/x^{\alpha}$, we can assume the following formula:

$$R/P = \alpha h/(\alpha-1) = z$$

Where:
R = sum of taxable incomes
P = overall number of income earners
h = minimum registered income

From that it is possible to deduce

$$dz=[\ \alpha/(\alpha-1)]dh-[h/(\alpha-1)^2]d\alpha$$

The necessary and sufficient condition to have a reduction in income inequality (α reducing) and/or an increase in minimum income (h increase) consists of a growth of per capita income (Pareto1896-1897).

Pareto denies the socialist assumption of inequality in incomes increase, moving from the combination of α reduction and h gain which is occurring thanks to technological advance.

However, Pareto quickly specified that nothing can guarantee the continuation of such phenomenon as production increase due to technological advance could be more balanced than continuous wealth destructions made by "Socialisme d'état" (Pareto 1896-1897); in that way he contradicts Ammon and most of all Leroy Beaulieu's optimism.

5. SUMMARY AND FINAL REMARKS

The acquisition of an established agreement on the distributive aspect, resulting from statistical data on personal income distribution referred to different times and places, allows to assume that a social model of income distribution exists and that its fundamentals can not be troubled, as de Sismondi claims (1819) "…without causing dangerous tensions".

The subject gains central importance both for social and political implications which have constituted the object of important contributions of analysis. For these reasons, the analysis of the negative impact of the inflationary trend on personal income distribution becomes crucial, as well as the estimate, in terms of effectiveness, of economic policy interventions implemented to balance the effects of this negative impact.

Over and above that, the analysis of personal income distribution leads to the fore the effects, in terms of society welfare, of a different distribution of wealth among the people. This analysis, rejecting the extreme structures of absolute egalitarianism and of maximum inequality, leads to the fore the policy objective to guarantee the lower inequality consistent with the efficiency constriction of the economic system.

From this point of view the taxes (charges) can be seen not only as an instrument to assure adequate resources to the public expenditure, but also to contribute to the reduction of inequality in distribution, maintaining its correct structural aspect.

Concluding, microeconomic analysis also can take advantage of this particular approach: the technological advances affect demand for skilled jobs indeed, due to their nature, bear on personal income distribution. In this way, the various topics of economic science regain their core unity, putting a person at the centre of analysis.

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