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Some Simple Experiments Demonstrating the Significance of the Transfer Function for Incoherent Illumination

A few experiments with Foucault's test that accentuate the significance of the filtration of harmonics are described.

The transfer function for incoherent illumination is the autocorrelation function of the pupil function of the optical system. If we restrict ourselves to the systems free of aberration, then the determination of the transfer function reduces to the simple geometrical calculations.

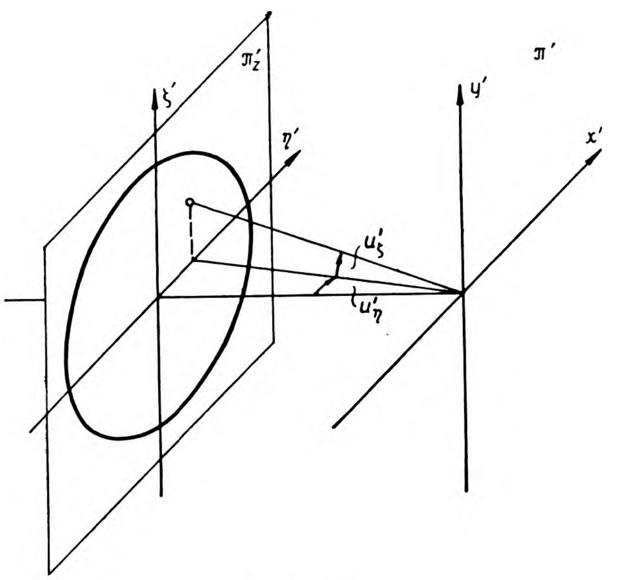

Fig. 1.

Let π' and π_z' (Fig. 1) be the image and pupil planes respectively. In the second plane the shape of the pupil is drawn and it is repeated in Fig. 2. with angular coordinates. If we consider only the line objects with intensity changing for example in x direction and the systems without apodisation, then displacing the shape of the pupil with respect to itself by the distance $\lambda\tilde{x}$ (Fig. 2), the transfer function for spatial frequency $\tilde{x}$ is equal to the ratio of the common area of the two shapes to the total area of the pupil, λ being the wavelength.

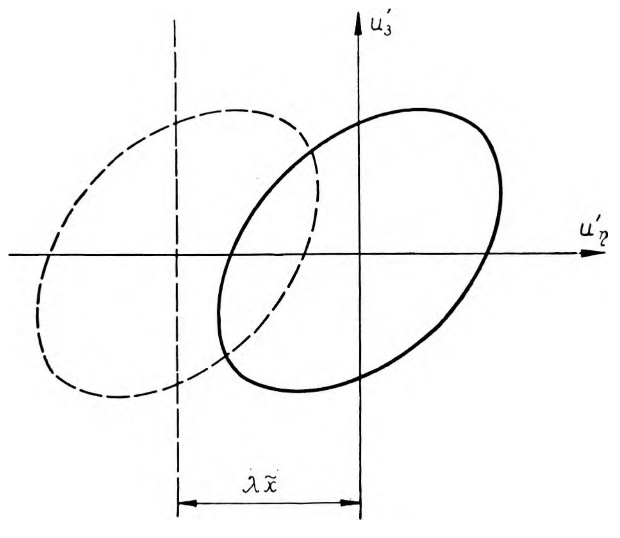

Fig. 2.

The well known transfer function $d_n(\tilde{x})$ for circular and rectangular aperture is plotted in linear coordinates in Fig. 3. $\tilde{x}_g = 2u_0'/\lambda$ the limiting spatial frequency, where u_0' the greatest aperture angle in η' direction. For $\tilde{x} > \tilde{x}_g$ we have $d_n(\tilde{x}) = 0$.

The fact that the transfer function vanishes for the high spatial frequencies is well understood by students, for it is a natural consequence of the resolving power of the optical system. But it is not so evident what will be the influence of decrease of the transfer function on the distribution of light in the image plane.

To clear up se doubts it is convenient to complicate the shape of the pupil to obtain a more complex function.

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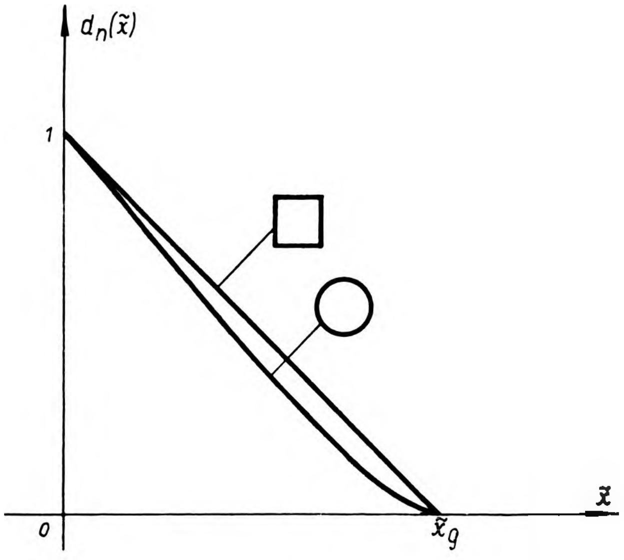

Fig. 3.

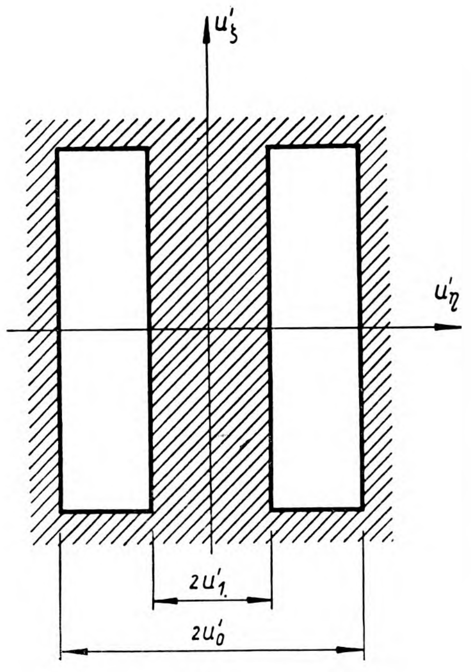

Fig. 4.

Let the pupil form be composed of two rectangles (Fig. 4). If $u'_0 < 3u'_1$, then the transfer function has two frequency bands (Fig. 5). Extreme frequencies of bands according to Fig. 5 satisfy the relations

$$\tilde{x}_1 = \frac{u'_0 - u'_1}{\lambda}, \qquad \tilde{x}_2 = \frac{2u'_1}{\lambda}, \qquad \tilde{x}_g = \frac{2u'_0}{\lambda}.$$

For $u'_0 = 3u'_1$ we have $\tilde{x}_1 = \tilde{x}_2$.

Let us consider now the image of Foucault's tests with different spatial frequencies (Fig. 6). We shall be concerned according to the direction of x' axe with the vertical lines only. The number of lines in one test (10 lines) is sufficiently large for our consideration to accept that

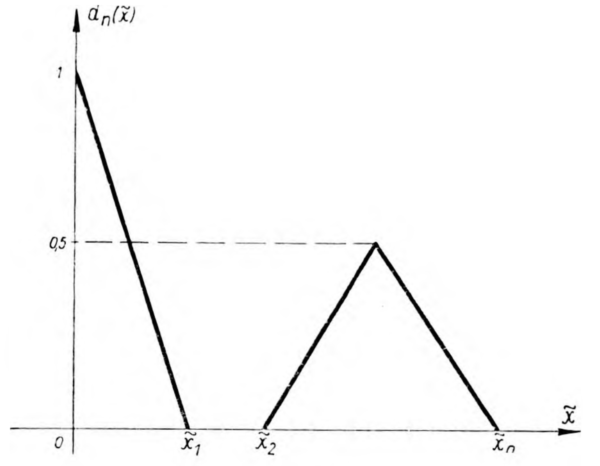

Fig. 5.

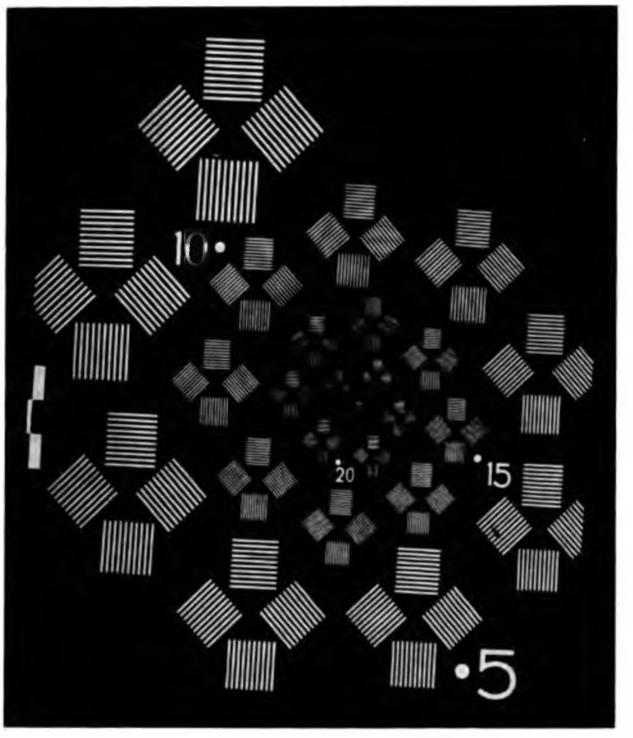

Fig. 6.

the spectrum of objects is discreteness and the frequencies of harmonics are equal to $0, \tilde{x}_{0i}, 3\tilde{x}_{0i}, 5\tilde{x}_{0i}, \ldots$, where $\tilde{x}_{0i} = 1/X_{0i}. X_{0i}$ is the period of test i. The absence of even harmonics results from the equal width of the bright and dark lines. On Fig. 7 the distribution of frequencies according to the test number is plotted. In the lower part of the diagram the transfer function is drawn for two shapes of the pupil. Its dimensions are given in the figure.

The test shown in Fig. 6 is strictly speaking the image of Foucault's test photographed by a rectangular pupil. Its width in η' direction is so taken that $\tilde{x}_g = 2u_0'/\lambda$. Consequently the vertical lines of test 25 are not resolved because the first harmonic of this test coincides with the limit frequency of the optical system. Evidently the distinction of horizontal lines is caused by a greater dimension of the pupil in ξ' direction.

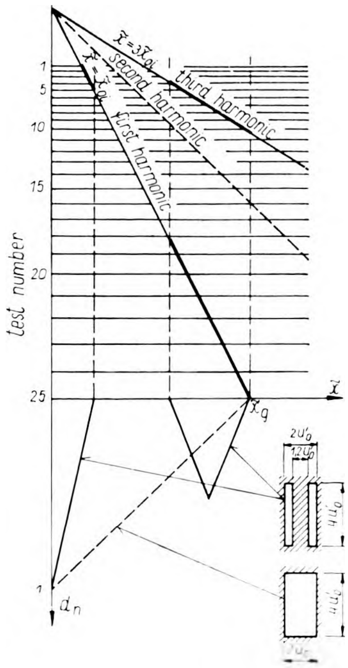

Fig. 7.

The image of the test shown in Fig. 8 is photographed by a pupil composed of two rectangulars. The harmonics transferred to the image plane for different numbers of the test are plotted with a thick line (Fig. 7).

The first harmonic is transferred for tests 1–4 and 18–24, the third harmonic – for tests 4–10. In that way, we have for tests 4–10 a three times greater frequency of vertical lines as compared to the horizontal ones. Tests 11–17 can not be solved. The visible structure of tests 11–14 is caused by the second harmonic of the test. It means that the width of dark and bright lines of the carried out test are not exactly equal.

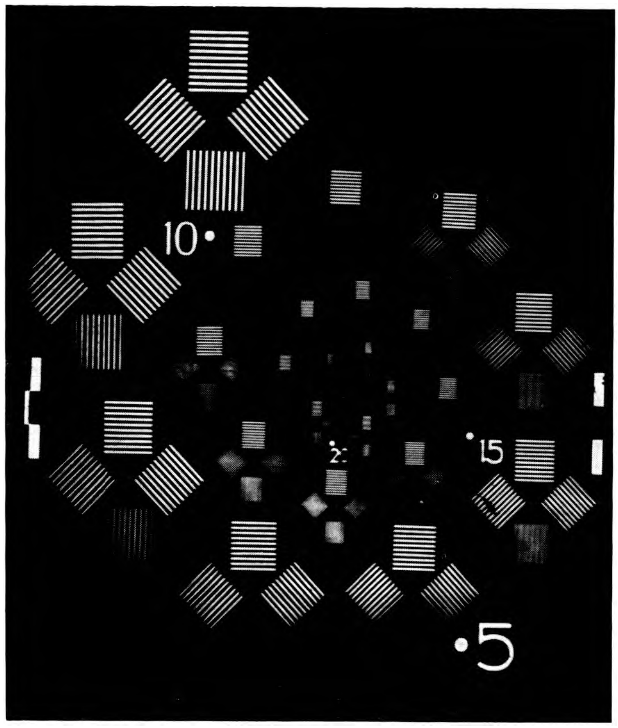

Fig. 8.

In other experiments, to emphasize the second harmonic, one can use a test with a different width of bright and dark lines. It is very useful for students to get acquainted with the analitical examination of the composition of the harmonics for the Foucault's tests of different periods as well as the transfer function shown in Fig. 9.

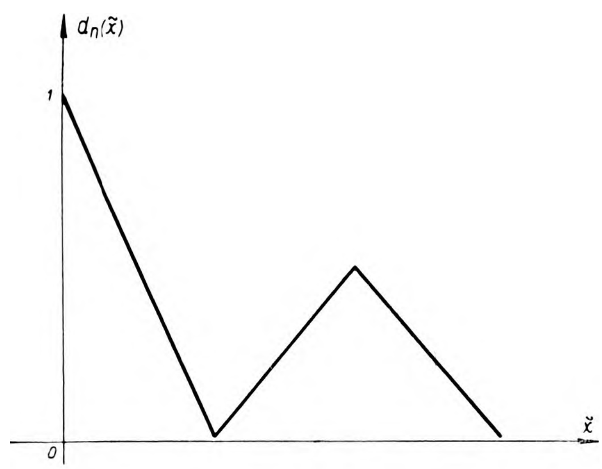

Fig. 9.