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An Analysis of the Transversal Secondary Spectrum in Some Superachromatic Systems

The aim of the paper is to examine the secondary spectrum for some types of the superachromatic systems published earlier. The relation between the secondary spectrum of the paraxial principal rays and the chromatic difference of the image magnitude for two wave lengths λ_F and λ_C has been established. The obtained formula, has been verified by applying it to the previously calculated superachromatic system. The graphs of the chromatic difference of the image magnitude as a function of the field angle for four wave lengths are presented.

The optical systems with corrected longitudinal chromatic aberration, within the wave length range $0.365 \leqslant \lambda \leqslant 1.014\,\mu$m are called superachromatic. The problem of the superachromatic correction in the paraxial region was analysed in the papers [1, 2 ... 6]. There, it has been shown that the superachromatic correction within the paraxial region may be achieved in the systems consisting of as few elements as two or three cemented lenses. These systems appear, however, to have other large aberrations, say, a spherochromatic error; the fact resulting in their limited applicability. In practice, the systems must be more complex and consist of two simplets, at least. As a starting point for this development two-lens superachromats may be recommended for the initial system configuration rather than three-lens simplets. Such a developed system being composed of two quasisymmetric simplets has been designed in the paper [7]. There, each simplet being a superachromat was build up of two cemented lenses.

Superachromatic correction may also be obtained for systems, in which the simplets are not independently superachromats. A system of that kind was calculated in the paper [8]. It was composed of a three-element simplet and a single lens, the latter being located at some distance behind the simplet. This system was designed starting with two two-element simplets: the first one was composed of two connected lenses while the other consisted of two lenses with an air space between them. Each of the orifinal simplets was a superachromat. Next, the first lens of the second simplet was joined to the first cemented simplet.

The goal of the present paper is to examine the secondary spectrum of the principal rays, called further transversal secondary spectrum for the sake of simplicity. The consideration will be restricted to the said types of superachromats.

The secondary spectrum of a system, consisting of two lenses with a normalised focal distance, may be expressed as follows

$$\delta s'_{F\lambda} = -s'^2_F \frac{P_{\lambda 2} - P_{\lambda 1}}{\nu_2 - \nu_1}, \tag{1}$$

(see [9]) where

$\delta s'_F$ — the distance between two axial image points obtained for the wave lenghts λ_F and λ, respectively,

s'_F — the distance of the axial image point for λ_F from the last surface (image distance for λ_F),

$$\nu_C = \frac{n_F - 1}{n_F - n_C} \text{ — relative dispersion,}$$

$$P_\lambda = \frac{n_F - n_\lambda}{n_F - n_C} \text{ — partial dispersion.}$$

As a basic colour for superachromatic correction that corresponding to the wave length $\lambda_F = 0.4861\,\mu$ is usually assumed. A relation analogous to Eq. (1) for the transversal secondary spectrum has been derived in the paper

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[10]. This has been obtained by considering the difference of the paraxial image heights for λ_C and λ_F respectively, resulting in the formula

$$\delta l'_{FC} = \frac{l'_F}{J} \sum y\bar{C}, \qquad (2)$$

$$\bar{C} = \frac{\Delta a}{\Delta\left(\frac{1}{n}\right)} \Delta\left(\frac{\delta n}{n}\right),$$

(see 9) where

l'_F — the distance of the intersection point of the principal ray with the image surface from the optical axis,

$J = nla$ — the Helmholtz-Lagrange invariant,

a — the aperture angle,

y — height of incidence of the principal ray on the first surface of the system,

$\Delta a = a' - a$ — the change of the aperture angle at the surface,

$\Delta\left(\frac{1}{n}\right) = \frac{1}{n'} - \frac{1}{n}$ — change of the reciprocal of the refractive index at the surface,

$\delta n = n_F - n_C$ — the difference of the refractive indices corresponding to λ_F and λ_C respectively,

$\Delta\left(\frac{\delta n}{n}\right)$ — change of the ratio $\frac{\delta n}{n}$ at the surface.

If the object is at the infinity, then $J = -1$ for the normalized focal length. For such a system consisting of two infinitly thin lenses the transversal secondary spectrum may be expressed by

$$\delta l'_{F\lambda} = l'_F y \frac{P_{\lambda 2} - P_{\lambda 1}}{\nu_{C2} - \nu_{C1}}, \qquad (3)$$

where

$\delta l'_{F\lambda}$ — the distance between points of intersection of the image plane by the principal rays corresponding to λ_F and λ.

From the formula (3) it may be easily seen that if the entrance pupil is identical with the first surface of a system, consisting of two cemented infinitely thin lenses, then the lateral chromatic aberration for an arbitrary wave length is equal to zero. In systems with compensated longitudinal chromatic aberration the lateral chromatic aberration does not depend on the location of the entrance pupil [9]. Thus, without losing the generality of consideration it may be assumed that the position of the entrance pupil is identical with that of the first surface. Incidently, this condition was really fullfilled in the superachromatic systems considered in this paper. If a superchromatic system is a single simplet build up of three lenses or if it consists of several simplets, each of which being a superachromat, then the transverse chromatic aberration should be equal to zero. In Fig. 1 the chromatic

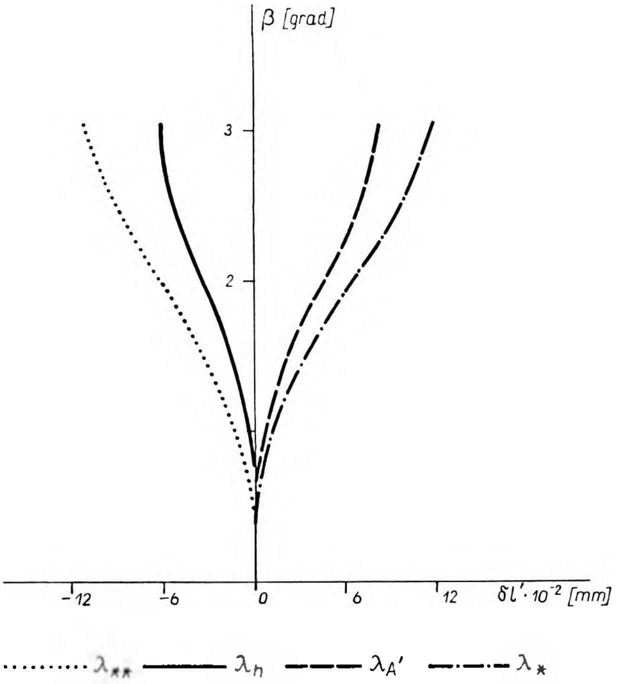

Fig. 1. Chromatic difference of the image magnitude for the superachromatic system consisting of two-lens simplets

difference of the image magnitude is shown as a function of field angle for four wave lengths. This dependance has been calculated for a system consisting of three lenses and designed in the paper [6]. In Fig. 2 a similar relation for the system considered in the paper [7] is represented. As it is easily seen the lateral chromatic aberration is really negligibly small.

Now, we shall consider the lateral chromatic aberration in the system calculated in paper [8]. To start with, we shall analyse a superachromatic system built up of two thin lenses, being located at some distance from each other (Fig. 3). The longitudinal chromatic aberration will be compensated when the following condition is satisfied

$$\frac{\varphi_1}{\nu_{C1}} + h^2 \frac{\varphi_2}{\nu_{C2}} = 0, \qquad (4)$$

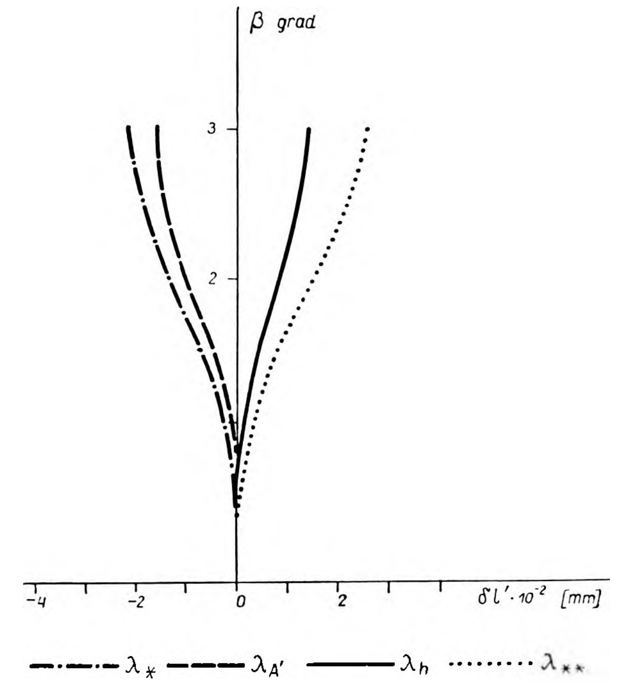

Fig. 2. Chromatic difference in the image magnitude for the three-lens simplets

where

φ — focussing power of the lens,

h — height of incidence of the axial ray on the second lens, assuming that the height of the incidence on the first surface $h_1 = 1$.

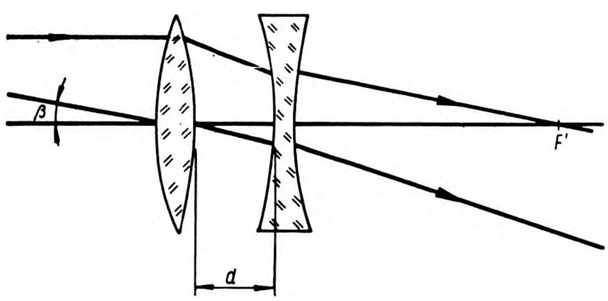

Fig. 3. Ray trace in the system composed of two thin lenses

We shall assume that: the aperture angle $a_1 = 0$, the height of incidence of the principal ray $y_1 = 0$ and the field angle $\beta_1 = 1$. For a system with normalized focal lenght $a'_p = 1$ after the last surface of the system. Chromatic difference of the image magnitude is expressed by the formula (2)

$$\delta l'_{FC} = -l'_F \sum y\bar{C}, \tag{5}$$

φ_2 may be estimated from (4)

$$\varphi_2 = -\frac{\nu_{C2}\varphi_1}{\nu_{C1}h^2}. \tag{6}$$

Substituting the appreciated value of φ_2 into (5) and letting

$$y_1 = 0 \qquad y_2 = -d \qquad \bar{C}_2 = \frac{-h\varphi_2}{\nu_{C2}}, \tag{7}$$

we obtain

$$\delta l'_{FC} = l'_F \frac{d\varphi_1}{h\nu_{C1}}. \tag{8}$$

Below we restrict our considerations to the case, when the focussing power of the first lens φ_1 will be of the same value as in the case of a cemented simplet. Then

$$\varphi_1 = \frac{\nu_{C1}}{\nu_{C1} - \nu_{C2}}. \tag{9}$$

Now, on the base of the formula

$$h = 1 - d\varphi_1 \tag{10}$$

and (9), we get by substituting (9) and (10) to (8)

$$\delta l'_{FC} = \frac{l'_F d}{\nu_{C1}(l-d) - \nu_{C2}}. \tag{11}$$

The received formula determines the lateral chromatic difference of the image magnitude for the spectral lines λ_F and λ_C respectively, in the superachromats consisting of two lenses with an air space between them.

For the spectral line λ the formula (11) will be written as follows

$$\delta l'_{F\lambda} = \frac{l'_F d}{\nu_{\lambda 1}(1-d) - \nu_{\lambda 2}}. \tag{12}$$

From the well-known relation

$$\nu_\lambda = \frac{\nu_C}{P_\lambda} \tag{13}$$

the formula (12) may be rewritten as follows

$$\delta l'_{F\lambda} = \frac{l'_F d}{\frac{\nu_{C1}}{P_{\lambda 1}}(1-d) - \frac{\nu_{C2}}{P_{\lambda 2}}}. \tag{14}$$

The glasses used for the superachromat satisfy the relation

$$P_{\lambda 1} = P_{\lambda 2} = P_\lambda. \tag{15}$$

Thus, the formula (14) may be written in the form

$$\delta l'_{F\lambda} = \frac{l'_F d P_\lambda}{\nu_{C1}(1-d) - \nu_{C2}}. \tag{16}$$

Chromatic difference of the image magnitude in the two-lens superachromat with an air space calculated exactly and estimated from paraxial formulas

β [grad]	$\delta l'_{F\lambda}$ [mm]							
	*		A'		h		**	
	1	2	1	2	1	2	1	2
3	0.01052	0.01043	0.00771	0.00773	−0.00570	−0.00567	−0.01016	−0.01003
2.6	0.00910	0.00902	0.00667	0.00668	−0.00493	−0.00490	−0.00878	−0.00867
2.1	0.00742	0.00737	0.00544	0.00546	−0.00402	−0.00402	−0.00716	−0.00708

From (16) and (11) we can easily get

$$\delta l'_{F\lambda} = P_\lambda \delta l'_{FC}. \tag{17}$$

The formula (17) determines the relation between the secondary spectrum of the paraxial principal rays and the lateral chromatic difference for the two wave lengths λ_F and λ_C (in the discussed type of the system). This formula will be veryfied by applying it to the previously obtained system consisting of two lenses made of fluorite and LaK 11, respectively. The focal length of the system is equal to 100 mm while the air space is 3.1 mm. In table the values of the lateral chromatic aberration calculated exactly (the first column) and estimated from Eq. (12) (the second column) have been collected. It is easily seen that for the assumed field angle the agreement is very good. Note that the assumed value for the field angle is the maximal admissible for that type of the system.

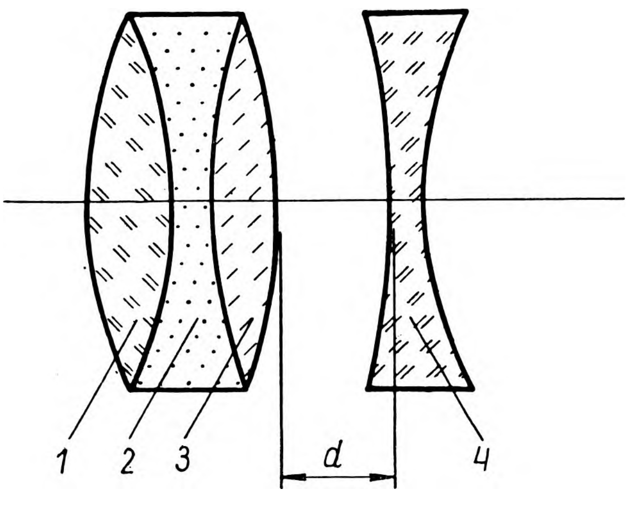

Fig. 4. Layout of the superachromat composed of two nonsuperachromatic simplets

Now, we shall analyse the secondary spectrum for the principal rays in the case of a system slightly generalised with respect to that considered in [8]. The layout of the system is presented in Fig. 4.

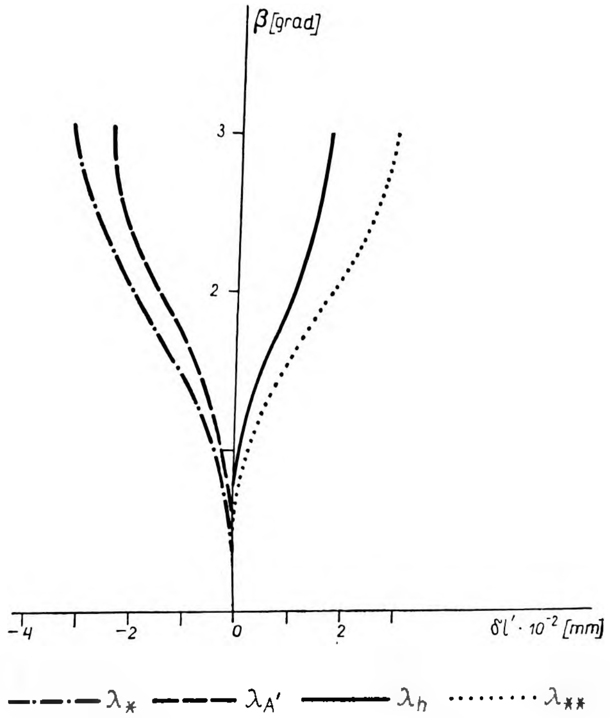

Fig. 5. Chromatic difference of the image magnitude composed of two non-superachromatic elements with an air space

The focussing power of the particular lenses fulfils the relation

$$\frac{\varphi_1}{\nu_{C1}} + \frac{\varphi_2}{\nu_{C2}} = 0, \tag{18}$$

$$\frac{\varphi_3}{\nu_{C3}} + h^2 \frac{\varphi_4}{\nu_{C4}} = 0, \tag{18}$$

$$\varphi_1 + \varphi_2 = \psi.$$

In the system published in paper [8] ψ was assumed to be equal to 0.5.

The focal length of the system is normalized to 1. By the same reasoning as that applied when deriving (12) and (17) we can obtain a formula for $\delta l'_{FC}$ and the transvers secondary spectrum respectively

$$\delta l'_{FC} = \frac{l'_F d(1-\psi)}{\nu_{C3}(1-d) - \nu_{C4}(1-d\psi)} \tag{19}$$

and finally

$$\delta l'_{F\lambda} = P_\lambda \, \delta l'_{FC}. \qquad (20)$$

The chromatic difference of the image magnitude for this system is presented in Fig. 5. The above considerations have shown that it is unnecessary to calculate the lateral chromatic aberration of magnification for different wave lengths, because the knowledge of this quantity for two colours, for instance λ_F and λ_C is sufficient to estimate easily the lateral chromatic aberration for an arbitrary wave length from the formula (20).

Analyse du spectre secondaire latéral quelques systèmes super-achromatiques

Le travail a pour but d'examiner le spectre secondaire des rayons principaux pour quelques types de systèmes super-achromatiques déjà publiés. On a établi la relation pour le spectre secondaire des rayons principaux paraxiaux en fonction de la différence de grandeur de l'image des deux raies λ_F et λ_C. On a examiné la relation obtenue dans le cas d'un système super-achromatique déjà calculé. On a également présenté les graphiques de la différence chromatique de grandeur de l'image en fonction de l'angle du champ pour quatre longueurs d'onde.

Анализ поперечного вторичного спектра некоторых суперахроматических систем

Целью этой работы было исследование вторичного спектра главных лучей некоторых типов суперахроматических систем, опубликованных раньше. Выведена зависимость величины вторичного спектра параксиальных главных лучей от хроматической разности величины изображения для двух линий λ_F и λ_C. Полученная зависимость проверена на вычисленной суперахроматической системе. Приводятся также диаграммы хроматической разности величины изображения в зависимости от угла поля зрения для четырёх длин волн.

References

[1] HERZBERGER M. Mc CLURE N., Applied Optics **2** (1963), 553.

[2] STEPHENS R., JOSA **50** (1960), 1016.

[3] DRUCK H., Optik **23**, (1966), 523.

[4] SCHULTZ H., Optik **25**, (1967), 203.

[5] SCHULTZ H., Optik, **25**, (1967), 208.

[6] GAJ M., NOWAK J., Z. N. P. Wr. **12**, (1967), 11.

[7] GAJ M., NOWAK J., Optik **29**, (1969), 321.

[8] GAJ M., NOWAK J., Optica Applicata, in press.

[9] SLUSAREV G. G., *Metody rasčeta optičeskich system*, Leningrad 1969.

[10] NEFEDOV B., Optiko-mechaničeskaja promyšlennost **3**, 1970, 14.